Lab 2

In this lab, we will practice with Mathematica the following topics:

- Find the limits, derivatives, and antiderivatives of a function.
- Evaluate exactly or numerically a definite integral.
- Visualize and evaluate the area of the region between two curves.
- Compute and visualize inverse functions.

1 Reminder about getting access

There are two ways to get free access to Mathematica:

A) Install three free components: *Wolfram Engine*, *JupyterLab*, and *WolframLanguageForJupyter*. You can use the unlimited computing power of Mathematica on your own computer, with Jupyter Notebook acting as a user interface. The instruction is here:

https://web.engr.oregonstate.edu/~phamt3/Resource/Wolfram-Mathematica-with-JupyterLab.pdf

B) Use the cloud-based version of Mathematica: https://www.wolframcloud.com
In this option, you are limited to about 8 minutes of computation per month. Files stored on the cloud will be deleted after 60 days.

2 Find limits, derivatives, and antiderivatives

(1) Type `Limit[(Cos[x]-1)/x^2, x->0]`, then `Shift+Enter`.

(2) Type `a = Limit[(1+1/x)^x, x->Infinity]`, then `Shift+Enter`.
Type `N[a,6]`, then `Shift+Enter`.
Type `N[a,10]`, then `Shift+Enter`.
Type `N[a,15]`, then `Shift+Enter`.

(3) Find the limit

$$\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$$

Note that in Mathematica, e^x is typeset as `E^x` or `Exp[x]`.

(4) Type `D[Cos[x], x]`, then `Shift+Enter`.
Type `D[Cos[x], {x,2}]`, then `Shift+Enter`.
Type `D[Cos[x], {x,3}]`, then `Shift+Enter`.

(5) Find the third derivative of the function $f(x) = e^{\cos(x^2)}$.

(6) To find an antiderivative of a function, we use the command **Integrate**. Try

```
Integrate[x^2, x]
f[x_]:=x-1
Integrate[f[x], x]
```

(7) Try

```
f[x_]:=Integrate[x/Sqrt[1+2x],x]
f[x]
D[f[x],x]
```

(8) Find an antiderivative of the function $x^2 \sin x$. Double check by differentiating the result.

3 Evaluate definite integrals

To evaluate the exact value of a definite integral, we use the command **Integrate**. Sometimes, it is impossible to get the exact value (some integrals are really tricky!) In that case, Mathematica may take a long time trying to compute. If you are using the Wolfram Cloud, you should terminate the execution (by pressing the combination Alt + .) if Mathematica takes longer than 10-20 seconds. Otherwise, you might soon run out of the precious 8 minutes quota of the month. If you are using Jupyter Notebook, you don't have to worry about this issue.

If the command **Integrate** is taking too long to return a value, terminate it and try the command **NIntegrate** instead. It will instantly give you an approximate numerical value of the definite integral.

(9) Try `Integrate[1/x,{x,1,4}]`

(10) Try

```
NIntegrate[E^(x^2),{x,0,2}]
NIntegrate[E^(x^2),{x,0,2},WorkingPrecision->8]
```

(11) Find the exact value of $\int_0^{\pi^2} \cos\sqrt{x}dx$.

(12) Approximate the integral $\int_0^{\pi^2} \cos(\sqrt{x} - x)dx$ up to 10 digits after the decimal point.

4 Region between two curves

To highlight the region between two curves, we use the command **Plot** and the option *Filling*.

(13) For example, to highlight the region between the line $y = x$ and the parabola $y = x^2$ when $0 \leq x \leq 1$, try the following:

```
Plot[{x,x^2},{x,0,1},Filling->True]
Plot[{x,x^2},{x,0,1},Filling->True,PlotLegends->Automatic]
```

(14) Plot the curves $y = \cos x$ and $y = \sin x$, where $0 \leq x \leq 3\pi$, and highlight the region between them.

(15) The area between the two curves is $\int_0^{3\pi} |\cos x - \sin x|dx$. Notice the absolute value symbol. In Mathematica, the absolute value of a number b is written as `Abs[b]`. Evaluate the exact area between the two curves.

(16) To find the points of intersection between the curves $y = \cos x$ and $y = \sin x$, we need to solve the equation $\cos x = \sin x$ where $0 \le x \le 3\pi$. Try the following:

```
Solve[Cos[x]==Sin[x] && 0<=x<=3*Pi, x]
NSolve[Cos[x]==Sin[x] && 0<=x<=3*Pi, x]
```

(17) The values you have just found are the x-coordinates of the intersection points. Can you find the y-coordinates of those points?

5 Compute and visualize inverse functions

(18) Let $f(x) = \frac{2x+1}{3x-2}$. To find the inverse of this function, we set $y = \frac{2x+1}{3x-2}$ and solve for x. Try

```
Solve[y==(2x+1)/(3x-1), x]
```

You can notice that there is only one value of x. Thus, the function has an inverse and the inverse function is $f^{-1}(y) = \frac{1+y}{-2+3y}$.

(19) Let $f(x) = \frac{x^2}{x-2}$. Does the function have an inverse on $\mathbb{R}\backslash\{2\}$?

(20) Usually, it is difficult to find an inverse function. For example, the function $f(x) = 2x + \sin x$ is one-to-one (because $f'(x) = 2 + \cos x > 0$), but it is extremely difficult to solve for x from the equation $y = 2x + \sin x$. However, we can still draw the graph of the inverse function by mirroring the the graph of $f(x)$ across the line $y = x$. Try the following:

```
p1 = ParametricPlot[{x, 2x + Sin[x]}, {x, 0, 5}, PlotStyle -> Blue];
p2 = ParametricPlot[{2x + Sin[x], x}, {x, 0, 5}, PlotStyle -> Red];
p3 = ParametricPlot[{x, x}, {x, 0, 7}, PlotStyle -> Dashed];
Show[p1, p2, p3, PlotRange -> Full]
```

6 To turn in

Submit your implementation of Exercises (1) - (20) as a single pdf file.