

**Final Problems 1**

Due Friday Sep 29

1. Show that the following language is regular (which may not involve explicitly constructing a DFA):

$$\{w \in \{a, b\}^* \mid \text{first 3 characters of } w \text{ are equal to the last 3 characters of } w\}$$

2. Give a DFA for the following language:

$$\{w \in \{a, b\}^* \mid \text{the last character of } w \text{ appears at least 2 times in } w\}$$

Explain in plain language the meaning of every state.

3. Consider the alphabet  $\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \dots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ . We can consider strings in this alphabet to consist of a “top, middle, and bottom row.” Let  $A$  be the set of strings whose top, middle, and bottom rows encode binary numbers  $x, y, z$ , respectively, where  $x+y = z$ . For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in A \text{ but } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \notin A$$

Show that  $A$  is regular.