

CS 261 – Data Structures

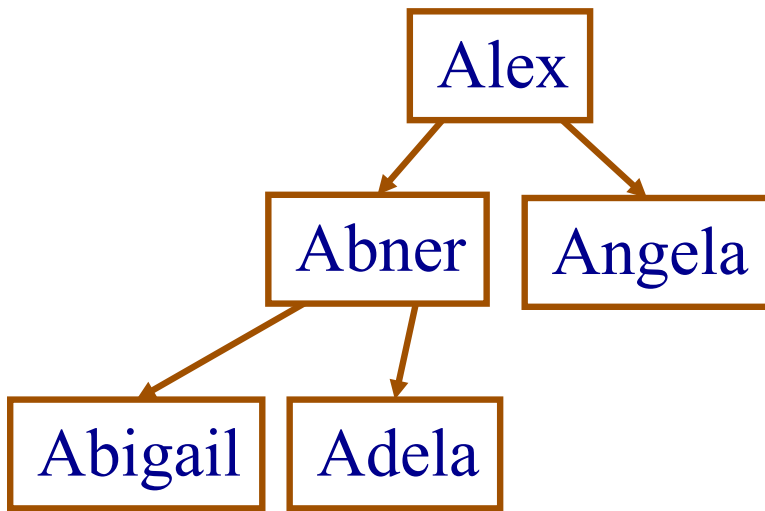
AVL Trees

AVL Implementation

```
struct AVLNode {  
    TYPE          val;  
    struct AVLNode *left;  
    struct AVLNode *right;  
    int           hght; /* Height of node*/  
};
```

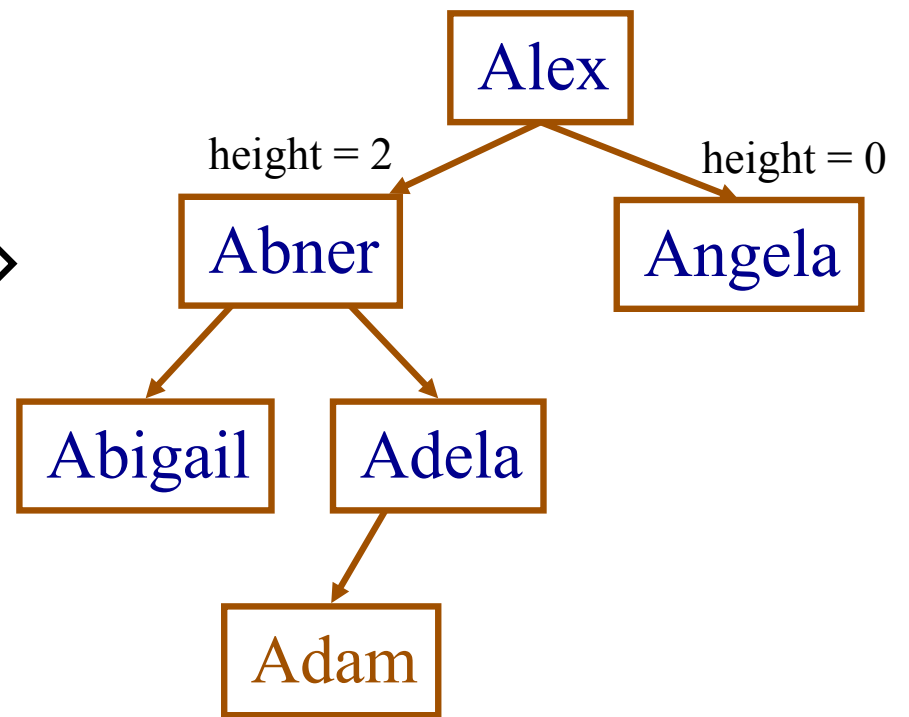
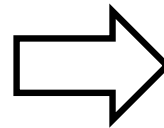
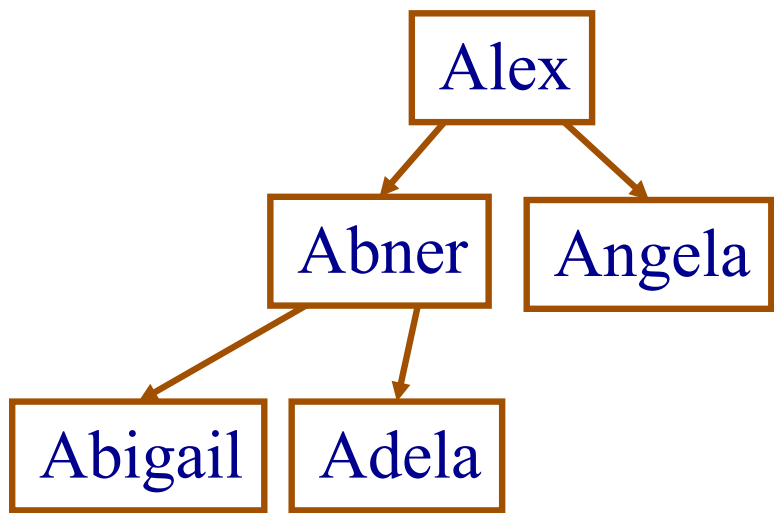
Add

Adam

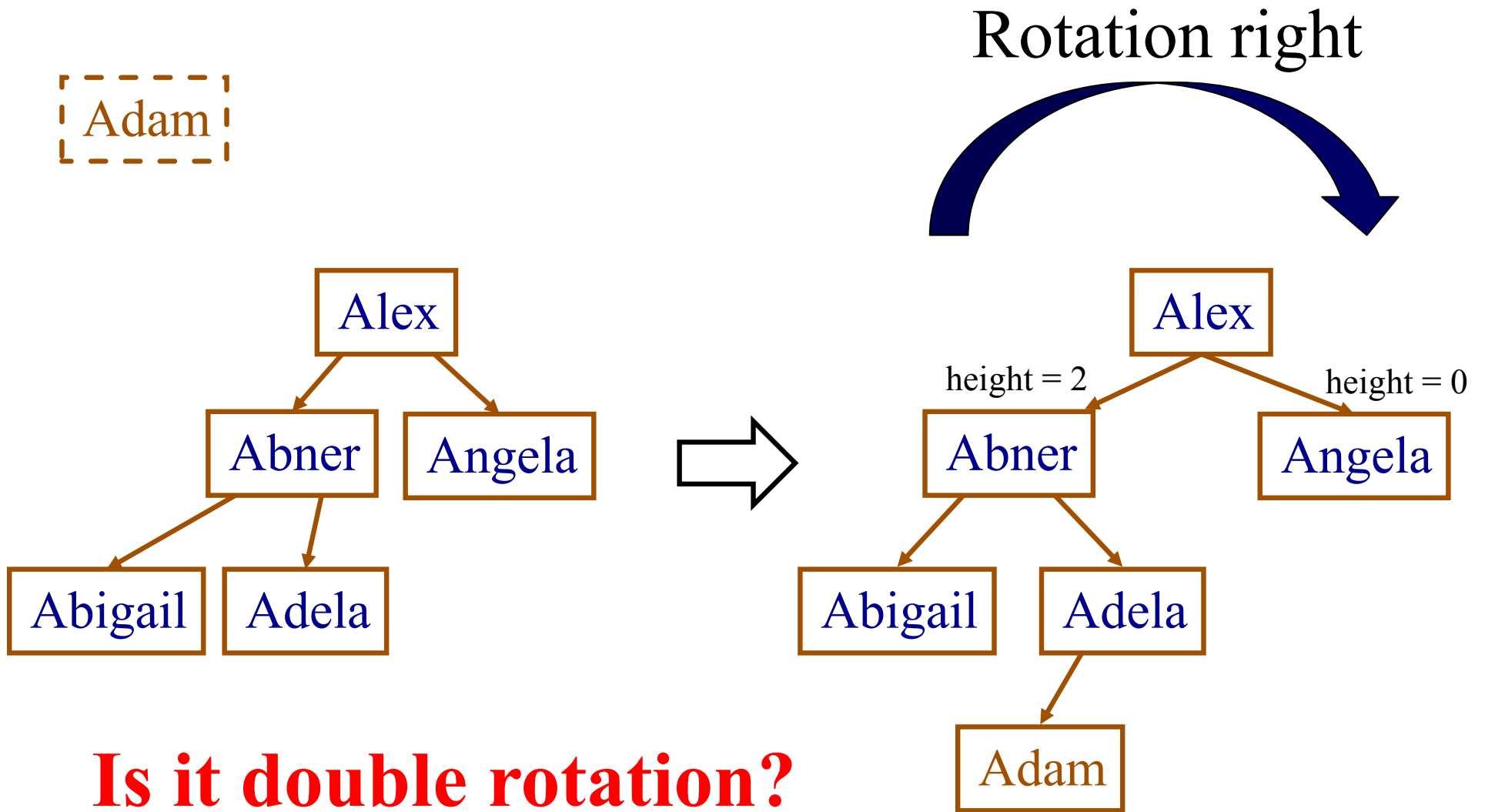


Add – Insert at the Leaf Level

Adam



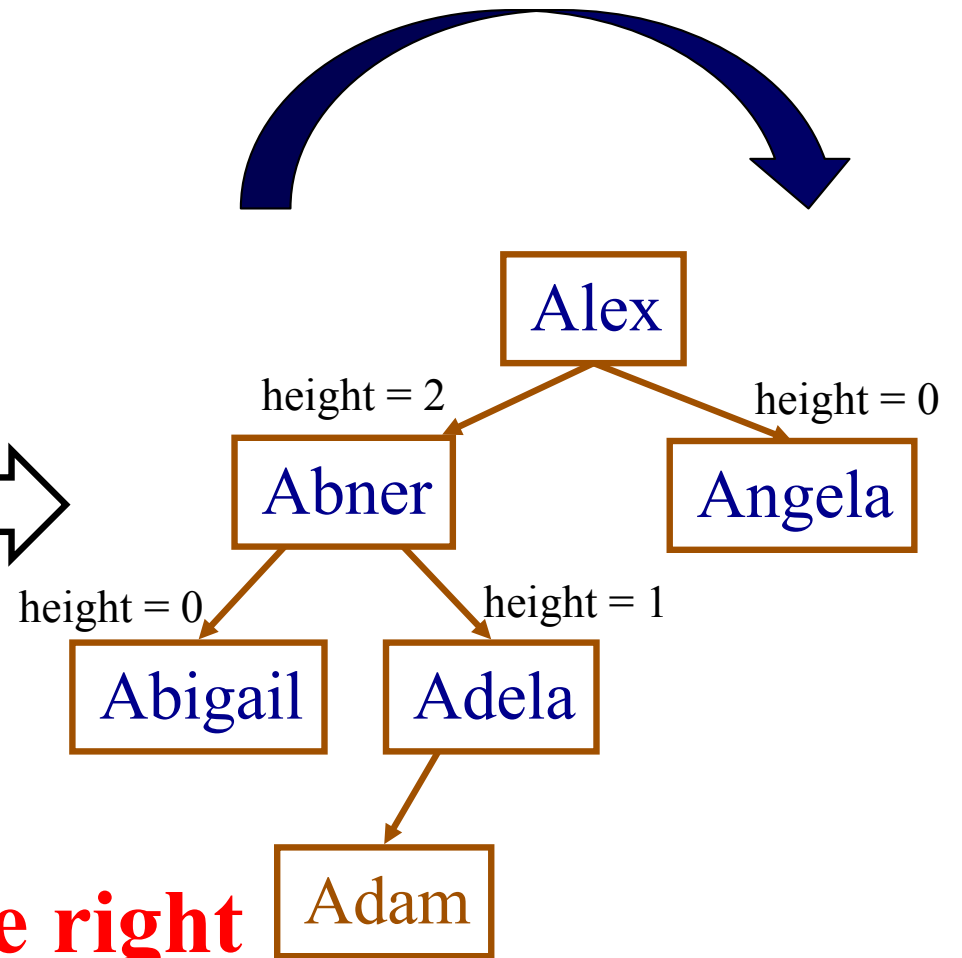
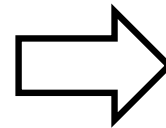
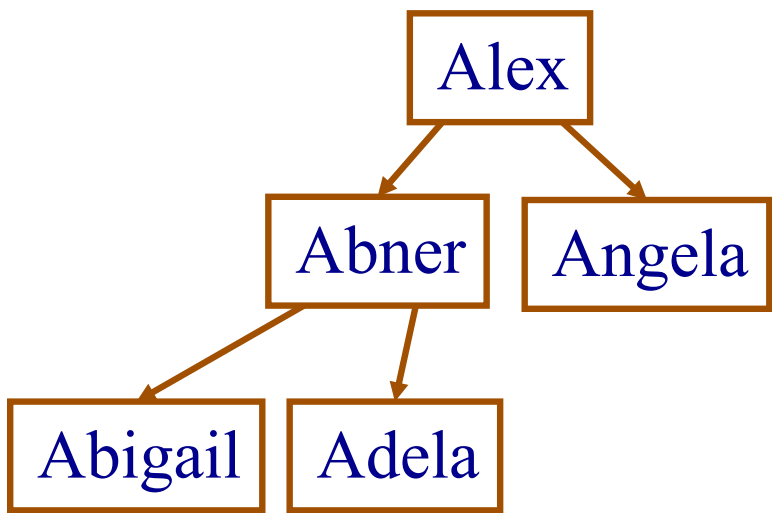
Add – Insert at the Leaf Level



Add – Insert at the Leaf Level

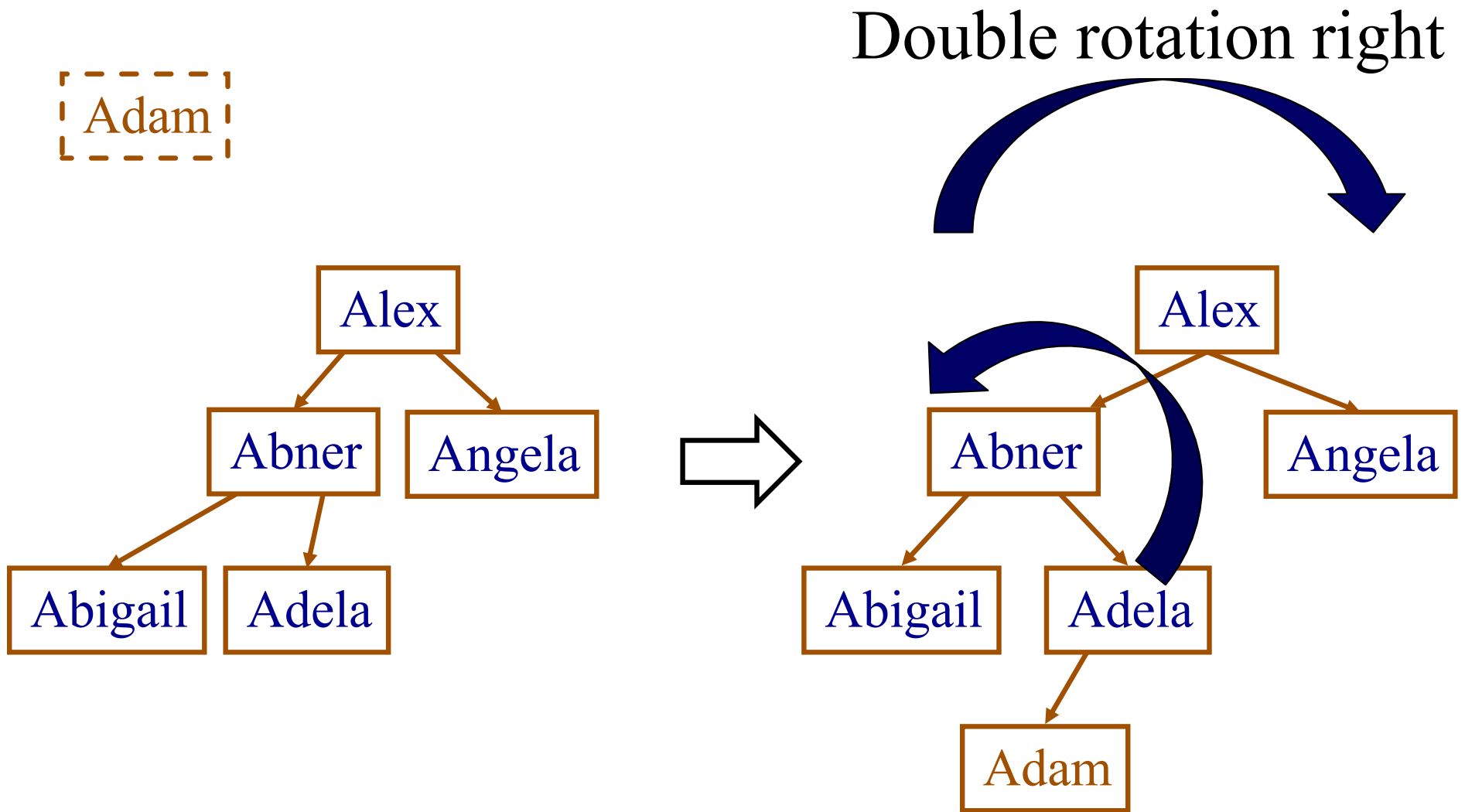
rotation right

Adam



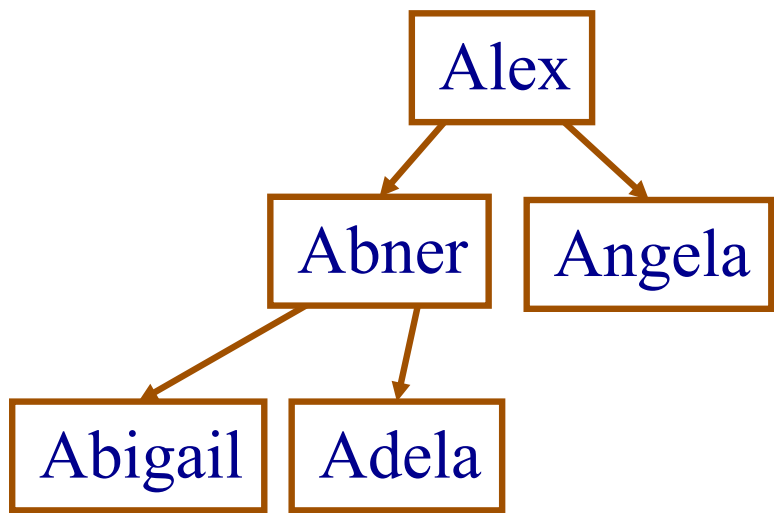
left child is heavy on the right

Add – Insert at the Leaf Level

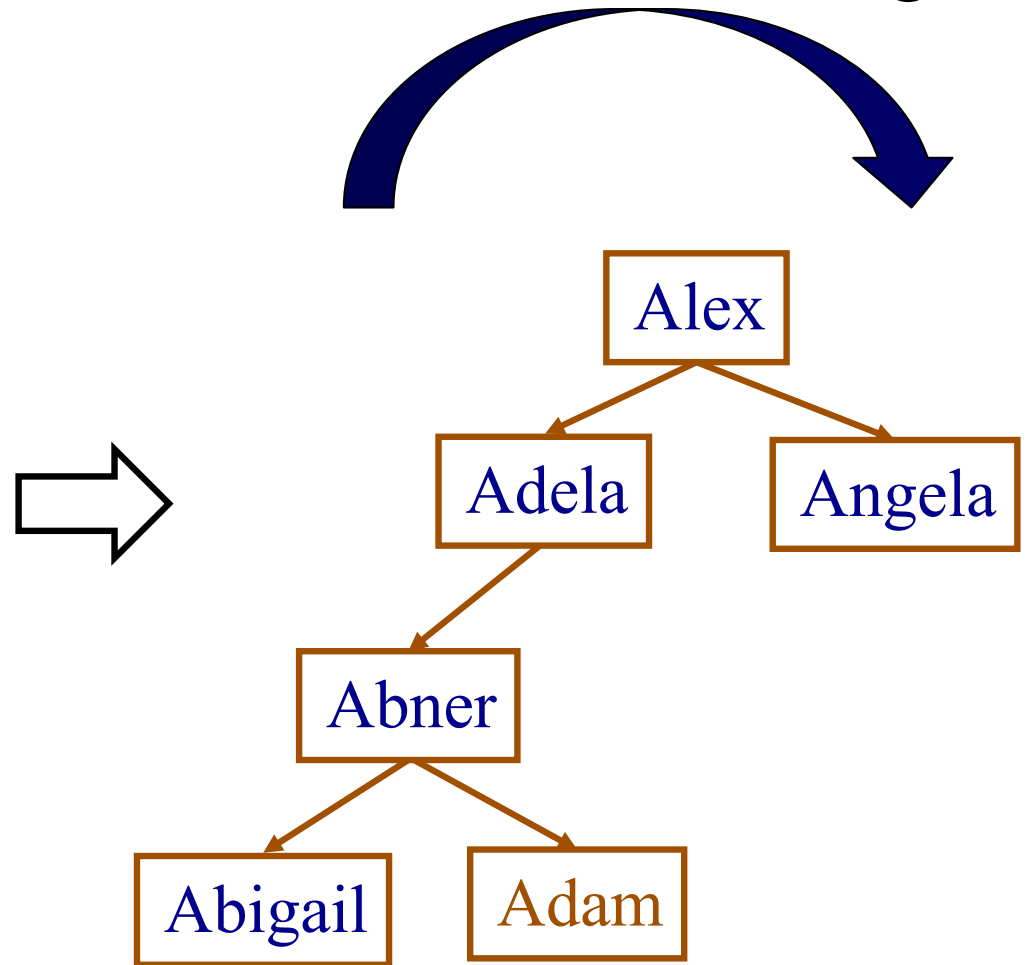


Add – Insert at the Leaf Level

Adam

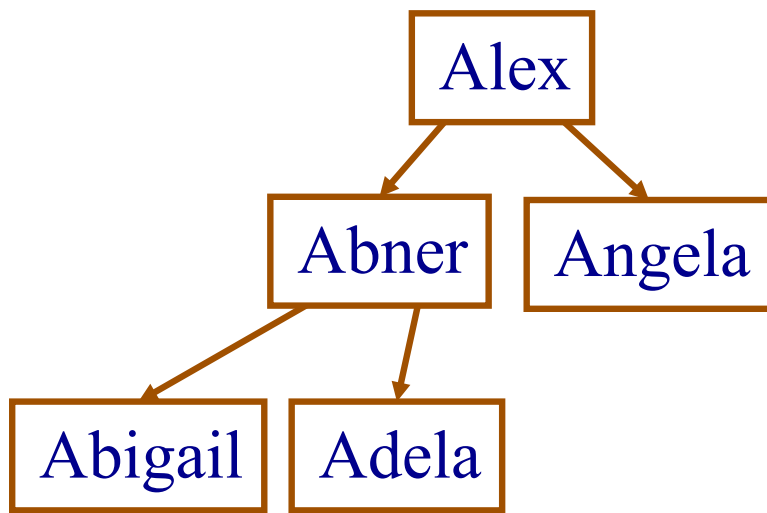


Double rotation right

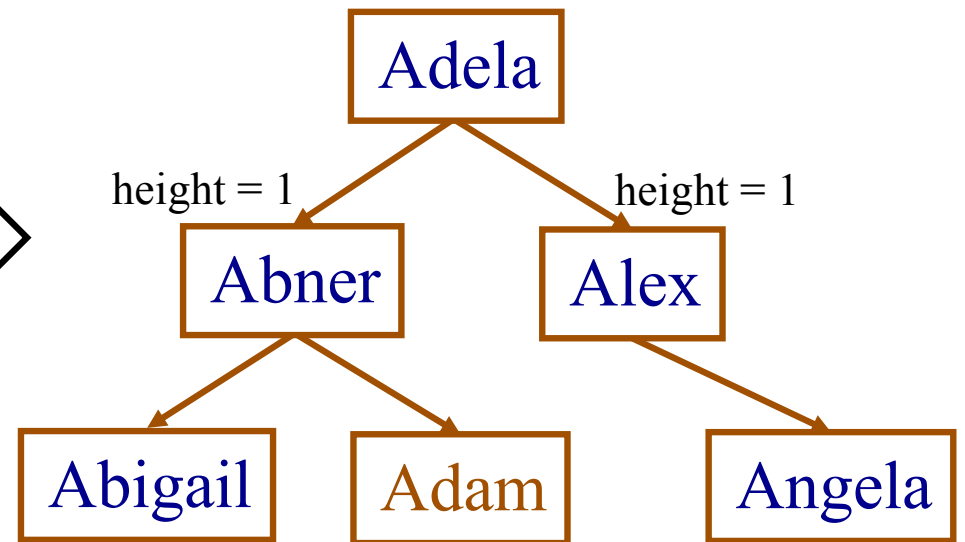
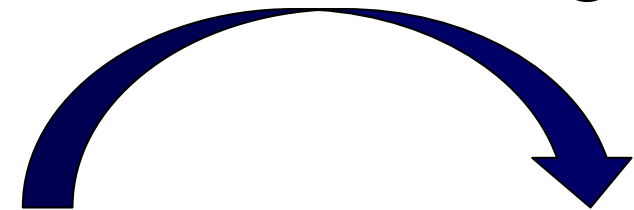
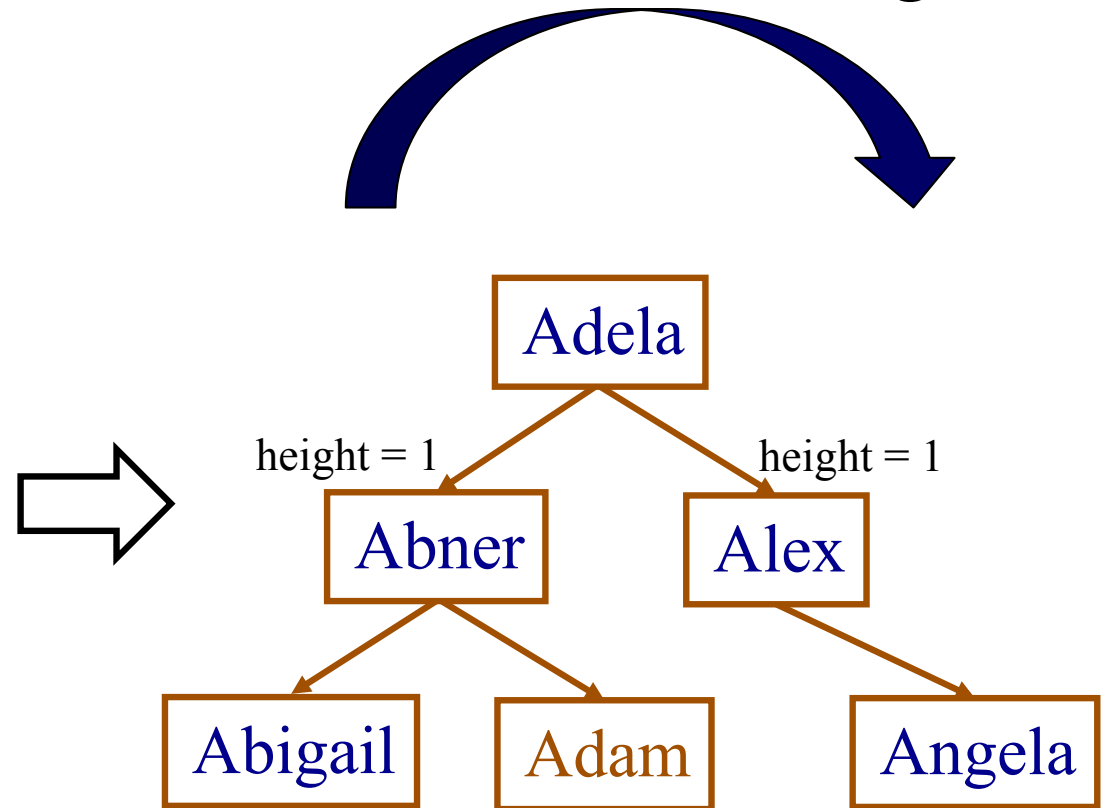


Add – Insert at the Leaf Level

Adam

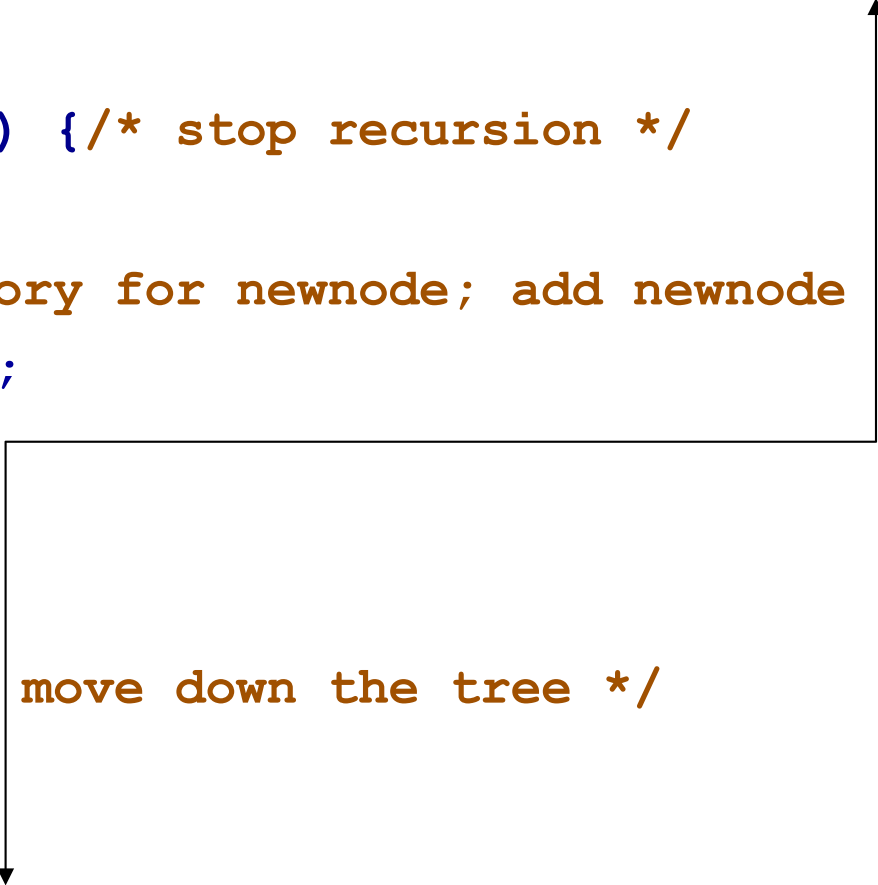


Double rotation right



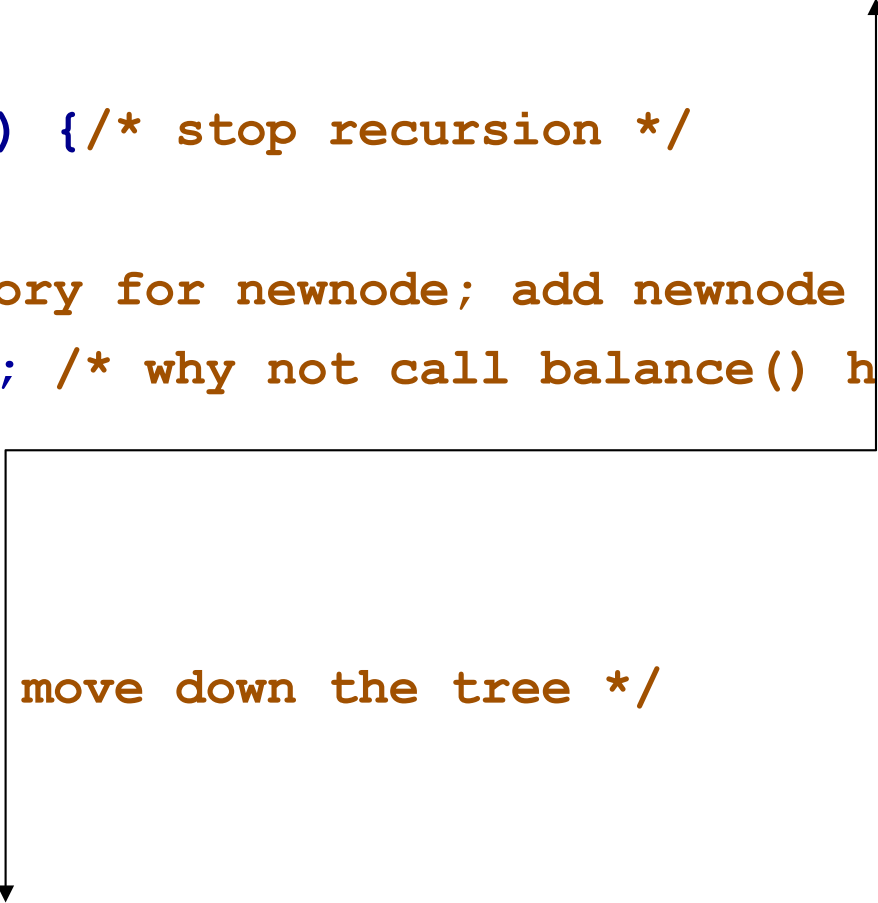
Add – Recursive Function

```
struct AVLNode *_addAVLNode(struct AVLNode* current, TYPE e) {
    ...
    if (current == 0) { /* stop recursion */
        ...
        /*allocate memory for newnode; add newnode to tree*/
        return newnode;
    }
    else {
        /* recursively move down the tree */
    }
    return /* ??? */;
}
```



Add – Recursive Function

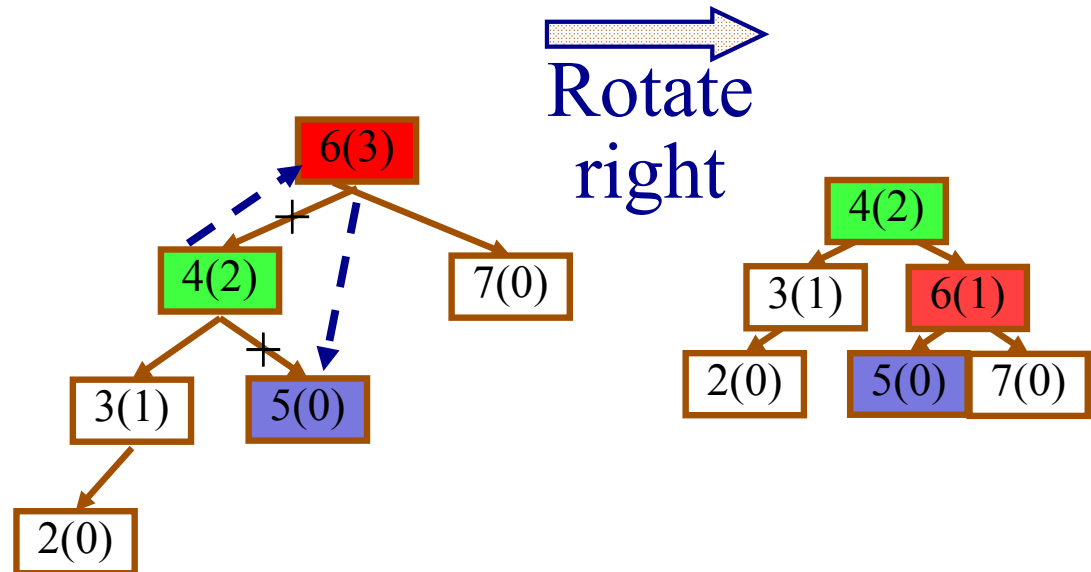
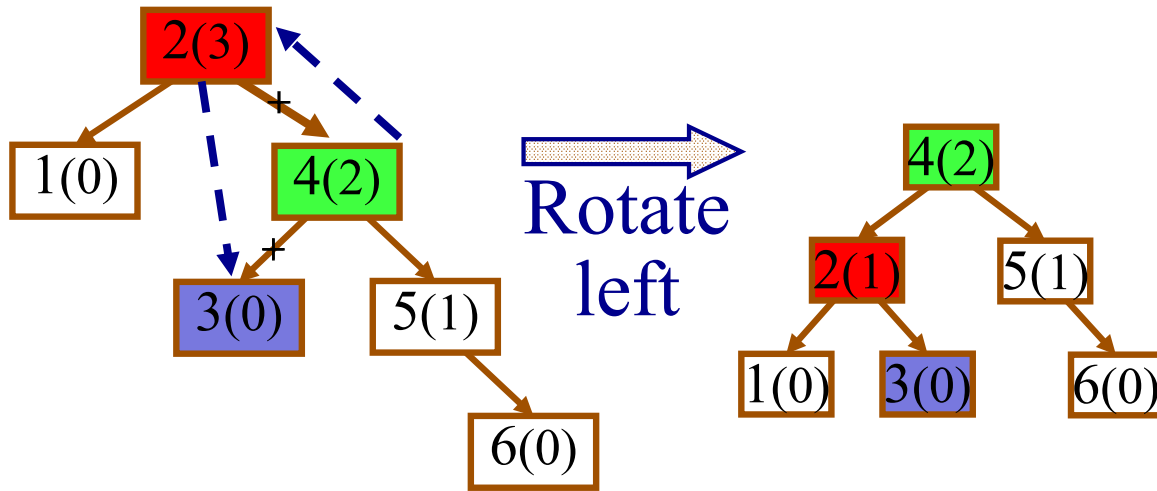
```
struct AVLNode *_addAVLNode(struct AVLNode* current, TYPE e) {  
    ...  
    if (current == 0) { /* stop recursion */  
        ...  
        /*allocate memory for newnode; add newnode to tree*/  
        return newnode; /* why not call balance() here? */  
    }  
    else {  
        /* recursively move down the tree */  
    }  
    return balance(current);  
}
```

A diagram consisting of two arrows. One arrow starts from the line 'return newnode;' and points upwards to the parameter 'current' in the function signature. The other arrow starts from the line 'return balance(current);' and points downwards to the parameter 'current' in the function signature. These arrows illustrate the recursive nature of the function, where the current node is passed to the next recursive call.

Add – Recursive Function

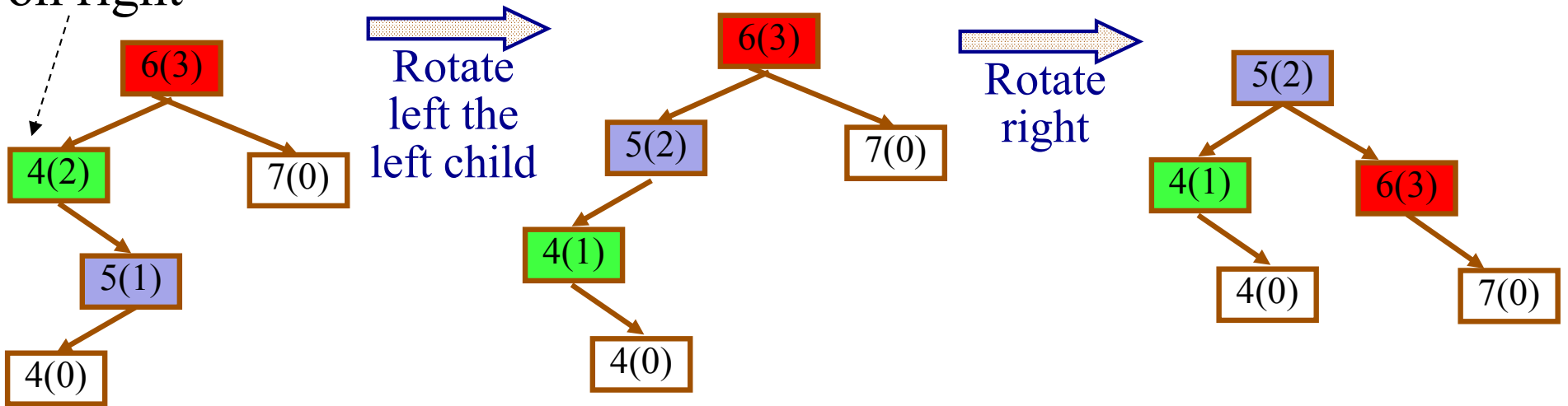
```
struct AVLNode *_addAVLNode(struct AVLNode* current, TYPE e) {
    ...
    if (current == 0) { /* stop recursion */
        ... /*allocate memory for newnode; add newnode to tree*/
        return newnode;
    }
    else { /* recursively move down the tree */
        if( LT(e, current->value) )
            current->left = _addAVLNode(current->left, e);
        else
            current->right = _addAVLNode(current->right, e);
    }
    return balance(current);
}
```

Rotation



Double Rotation

heavy
on right



Balance

```
struct AVLNode * balance (struct AVLNode * current) {  
    int rotation = _height(current->right)  
                  - _height(current->left);  
  
    if (rotation < -1) {  
        /* (double) rotation right */  
    } else if (rotation > 1) {  
        /* (double) rotation left */  
    }  
  
    _setHeight(current);  
  
    return current;  
}
```

Balance – (Double) Rotation Right

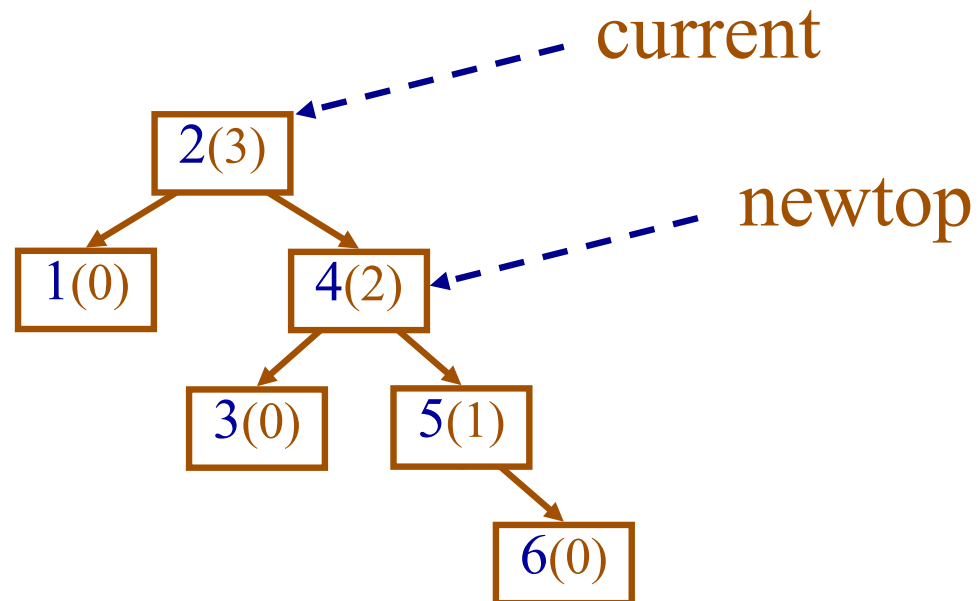
```
...  
  
if (rotation < -1) { /* (double) rotation right */  
    int drotation = _height(current->left->right)  
                  - _height(current->left->left);  
  
    if (drotation > 0) { /* double rotation */  
        /* left child is heavy on the right */  
        current->left = _rotateLeft(current->left);  
    }  
  
    return _rotateRight(current);  
}  
  
else { ...
```


Balance – (Double) Rotation Left

```
...
}else if (rotation > 1) { /* (double) rotation left */
    int drotation = _height(current->right->right)
                    - _height(current->right->left);
    if (drotation < 0) { /* double rotation */
        /* right child is heavy on the left */
        current->right = _rotateRight(current->right);
    }
    return _rotateLeft(current);
} ...
```

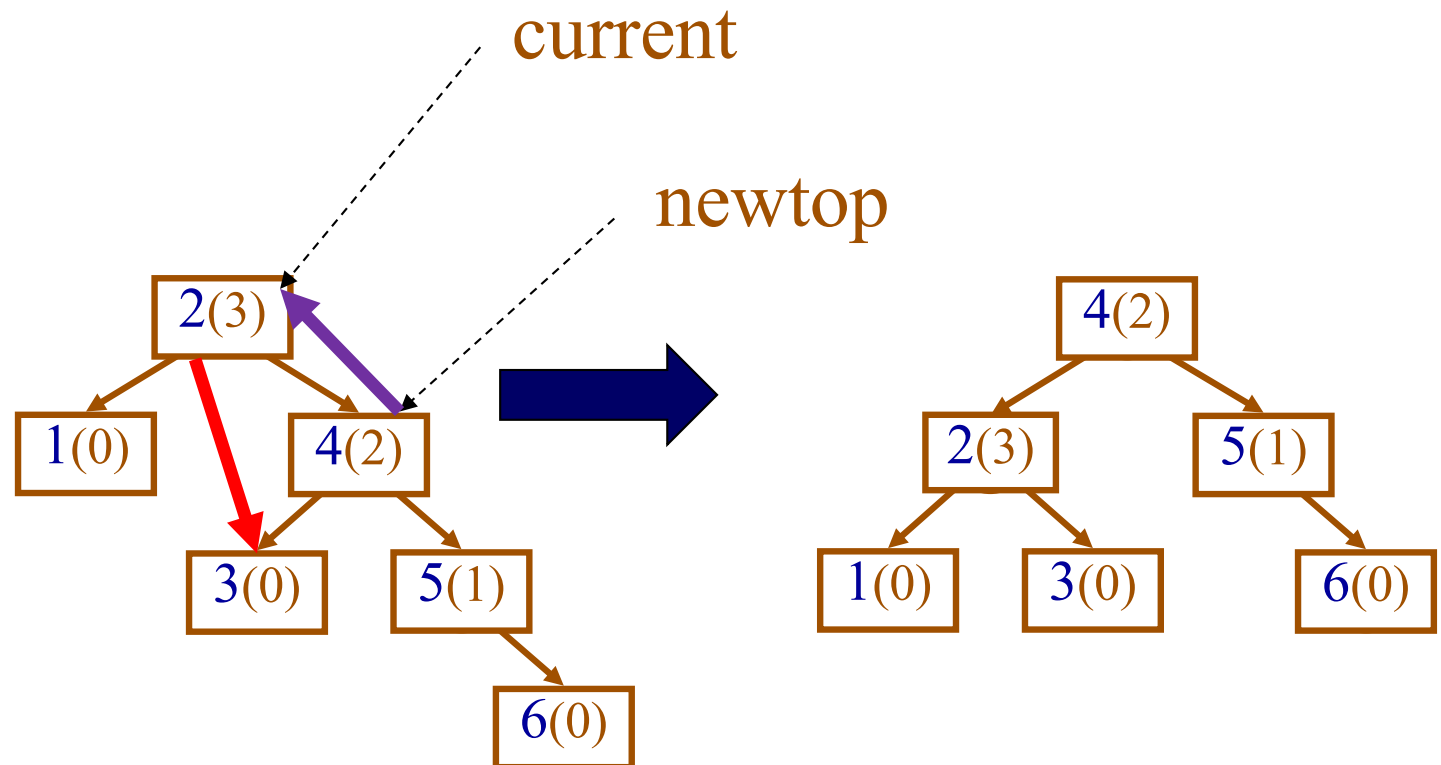
Rotation Left

```
struct AVLNode * _rotateLeft (struct AVLNode * current) {  
    struct AVLNode * newtop = current->right;  
    ...  
}
```



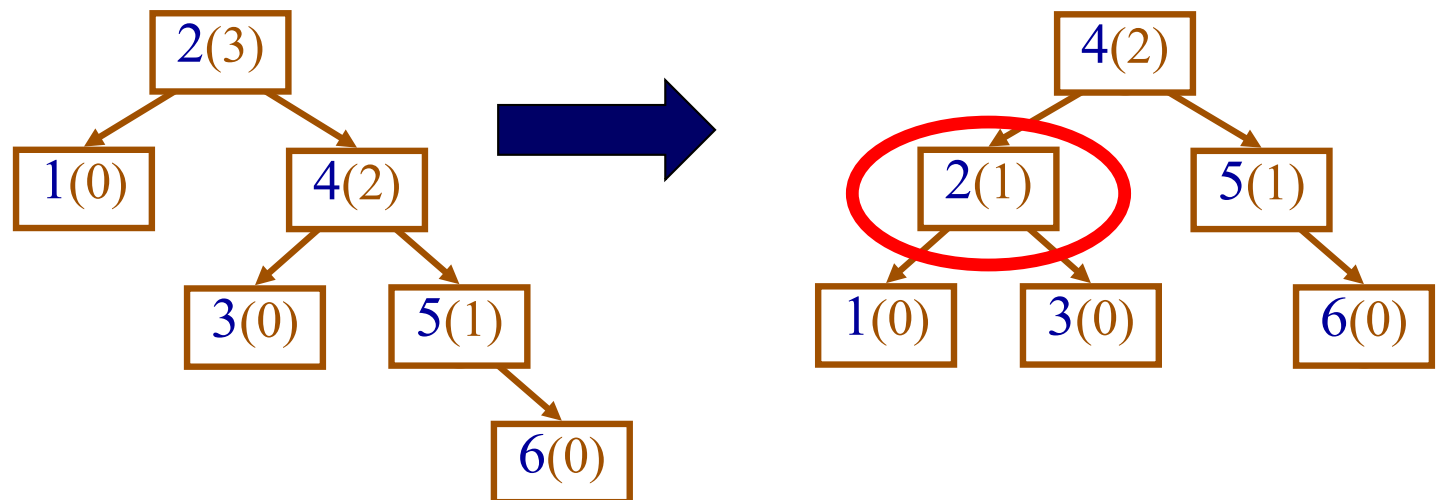
Rotation Left

```
struct AVLNode * _rotateLeft (struct AVLNode * current){  
    struct AVLNode * newtop = current->right;  
    current->right = newtop->left;  
    newtop->left = current;  
    ...  
}
```



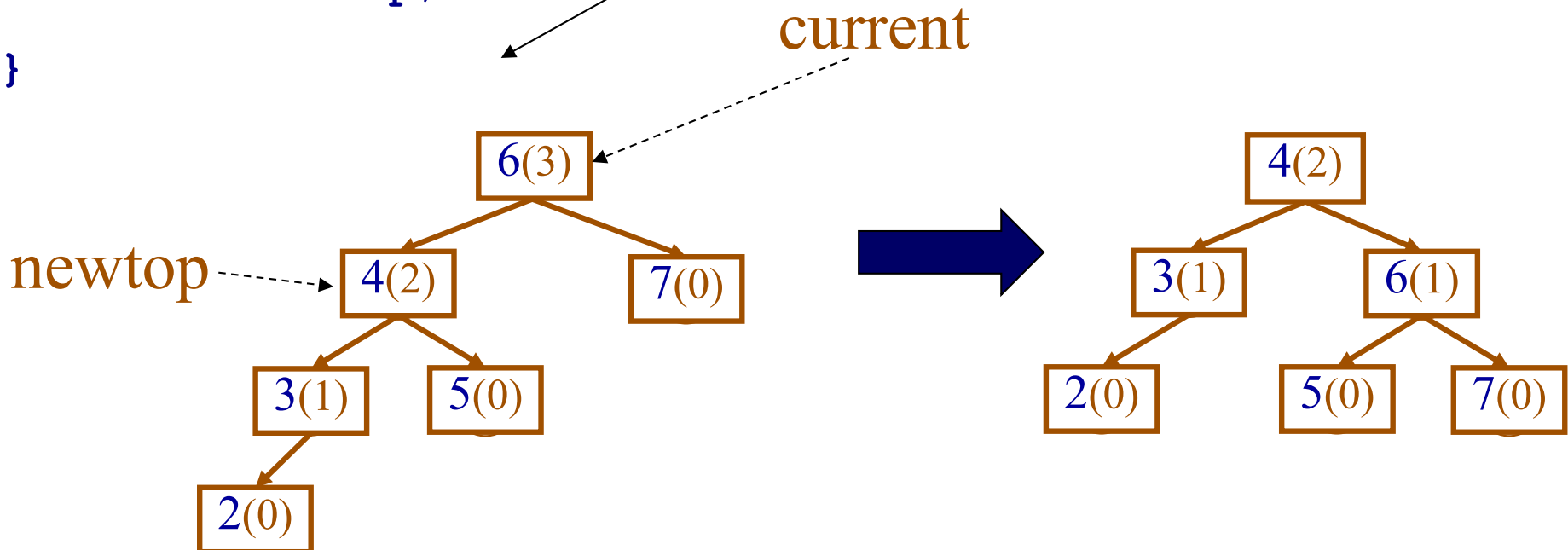
Rotation Left

```
struct AVLNode * _rotateLeft (struct AVLNode * current){  
    struct AVLNode * newtop = current->right;  
    current->right = newtop->left;  
    newtop->left = current;  
    _setHeight (current) ;  
    _setHeight (newtop) ;  
    return newtop;  
}
```



Rotation Right

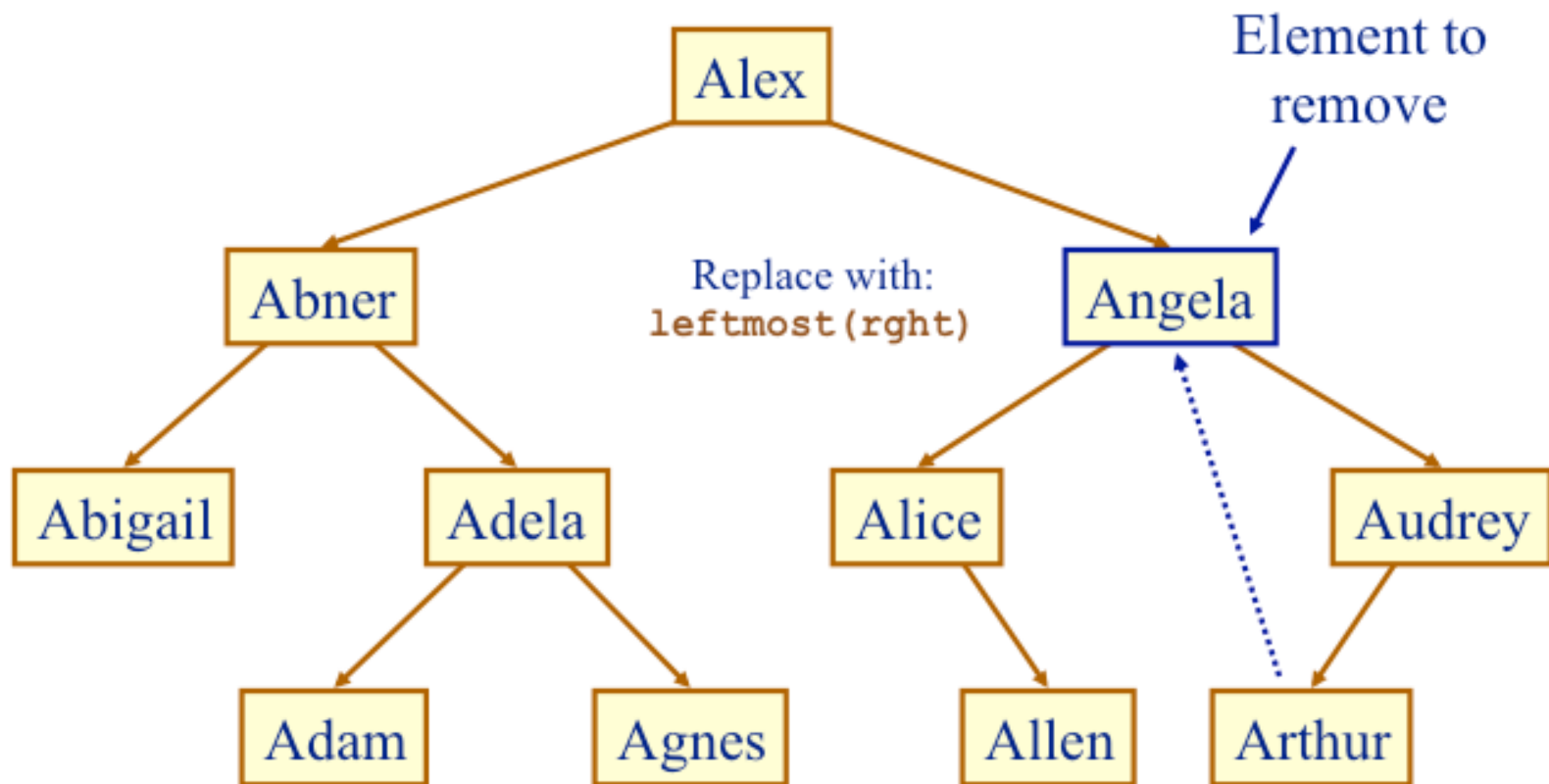
```
struct AVLNode * _rotateRight (struct AVLNode * current){  
    struct AVLNode * newtop = current->left;  
    current->left = newtop->right;  
    newtop->right = current;  
    _setHeight (current) ;  
    _setHeight (newtop) ;  
    return newtop;  
}
```



Remove: Who fills the hole in the tree?

Answer:

the leftmost child of the right child
(smallest element in right subtree)



Remove

```
void removeAVLTree(struct AVLTree *tree, TYPE val) {  
    if (containsAVLTree(tree, val)) {  
        tree->root = _removeNode(tree->root, val);  
        tree->cnt--;  
    }  
}
```

Remove

```
struct AVLNode *_removeNode(struct AVLNode *current, TYPE e)
{
    struct AVLNode *temp;
    assert(current);
    if(EQ(e, current->val)){
        /* replace current with the leftmost descendant
           of the right child */
    }
    else if(LT(e, current->val))
        current->left = _removeNode(current->left, e);
    else
        current->right = _removeNode(current->right, e);

    return balance(current);
}
```


AVL Trees: Sorting

- An AVL tree can sort a collection of values:
 1. Copy data into the AVL tree: $O(n \log n)$
 2. Copy them out using the $??$ traversal: $O(n)$

AVL Trees: Sorting

- An AVL tree can sort a collection of values:

Copy data into the AVL tree:

$$O(n \log_2 n)$$

Copy them out using the **in-order** traversal:

$$O(n)$$

AVL Trees: Sorting

- Execution time $\rightarrow O(n \log n)$:
 - Matches that of quick sort in benchmarks
 - Unlike quick sort, AVL trees don't have problems if data is already sorted or almost sorted (which degrades quick sort to $O(n^2)$)
- However, requires extra storage to maintain both the original data buffer (e.g., a **DynArr**) and the tree structure