ECE 468: Digital Image Processing

Lecture 2

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Outline

- Image interpolation
- MATLAB tutorial
- Review of image elements
- Affine transforms of images
- Spatial-domain filtering

Image Interpolation

- Bilinear N = 1
- Bicubic N = 3

$$f(x, y) = \sum_{i=0}^{N} \sum_{j=0}^{N} a_{ij} x^i y^j$$

MATLAB Image Processing Toolbox

Basic MATLAB Commands

- imread
- size
- whos
- imshow
- imwrite
- double, uint8
- im2uint8, im2bw
- plot(img(size(img,1)/2,:))
- zeros(m,n), ones(m,n), rand(m,n), randn(m,n)
- function [outputs] = name_func(inputs), return

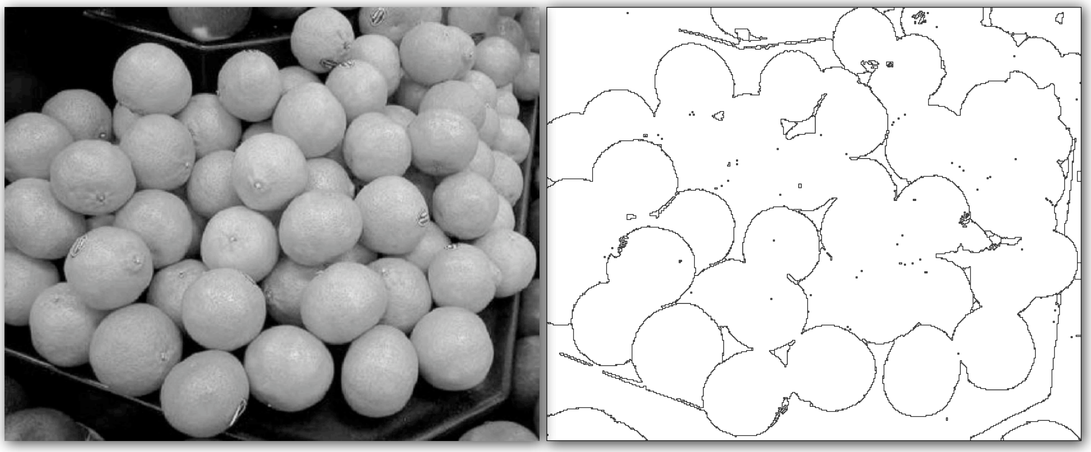
Basic MATLAB Commands

- ismember, isempty
- intersect, union
- for, while
- meshgrid
- imadjust
- imhist

Image Structure

Image Elements

- Pixels, 4-adjacency, 8-adjacency, m-adjacency
- Path -- directed, undirected, loop
- Region = Connected set of pixels
- Region boundary, inner and outer contour
- Foreground - background
- Edge = Connected pixels with high derivative values
- Interest points: T-junction, Y-junction
- Highlights or specularities
- Lambertian surface = isotropic reflectance
- Specular surface = zero reflectance except at an angle




Scott Alan Bacon
April 2006

Spatial Image Transformations

Affine transforms:

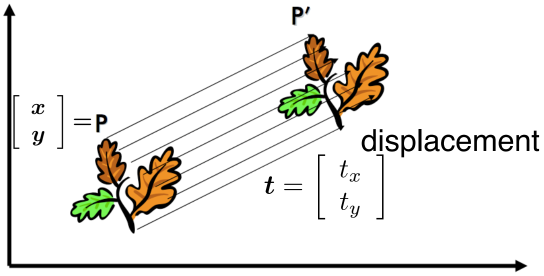
- Translation
- Scaling
- Rotation
- Shear

Example

$$(x', y') = T\{(x, y)\}$$

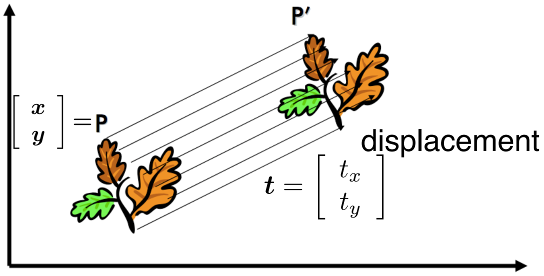
$$\left[\begin{array}{c} x' \\ y' \\ 1 \end{array}\right] = T \left[\begin{array}{c} x \\ y \\ 1 \end{array}\right]$$

2D Translation



source: S. Savarese

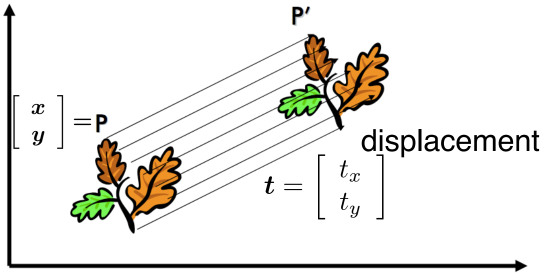
2D Translation



$$P' = P + \boldsymbol{t} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

source: S. Savarese

2D Translation

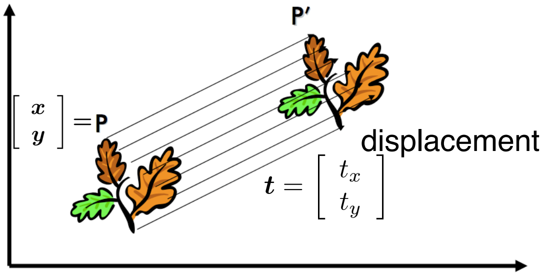


homogeneous coordinates

$$P' = P + \boldsymbol{t} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

source: S. Savarese

2D Translation



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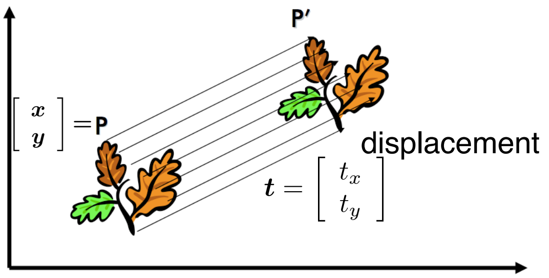
$$\Rightarrow \quad P' \rightarrow \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

source: S. Savarese

2D Translation



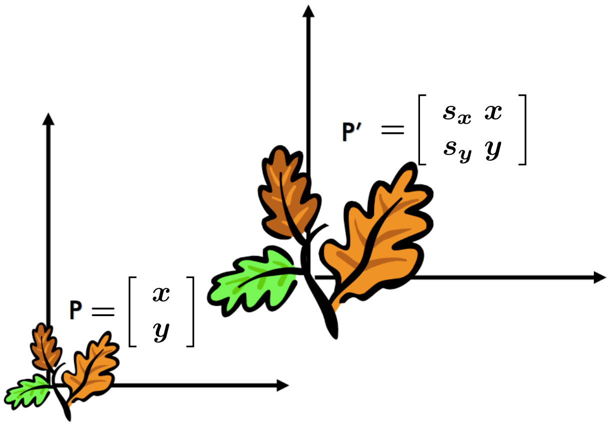
homogeneous coordinates

$$P' = P + \boldsymbol{t} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad P' \rightarrow \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translation matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

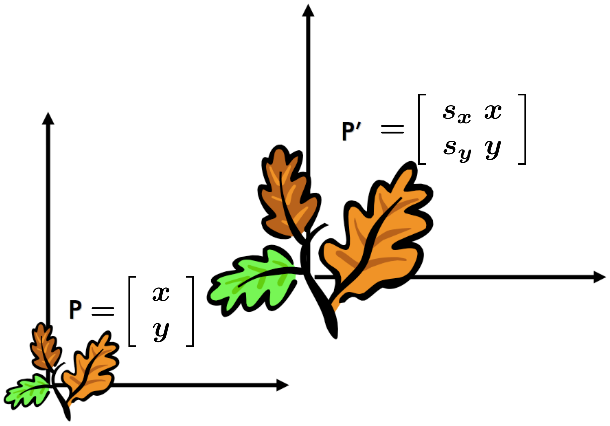
source: S. Savarese

2D Scaling



source: S. Savarese

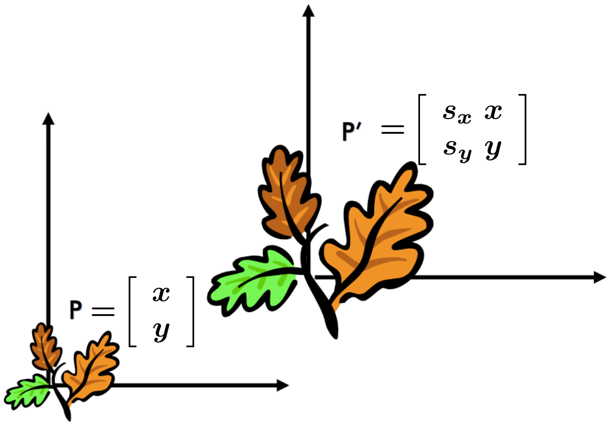
2D Scaling



$$\begin{bmatrix} s_x\, x \\ s_y\, y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

source: S. Savarese

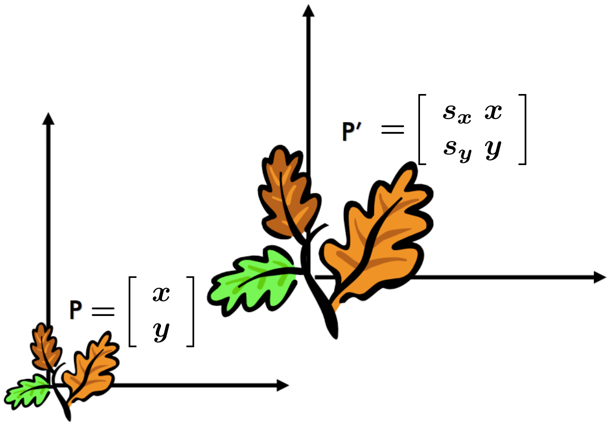
2D Scaling



$$\begin{bmatrix} s_x\, x \\ s_y\, y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{scaling matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

source: S. Savarese

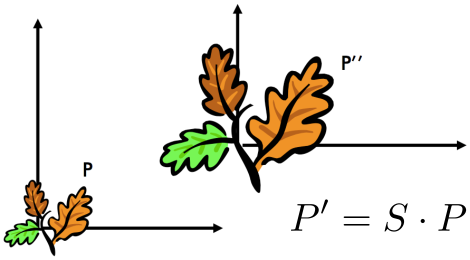
2D Scaling



$$\begin{bmatrix} s_x\,x \\ s_y\,y \\ 1 \end{bmatrix} = \underbrace{\overset{S}{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}}_{\text{scaling matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

source: S. Savarese

2D Scaling + Translation

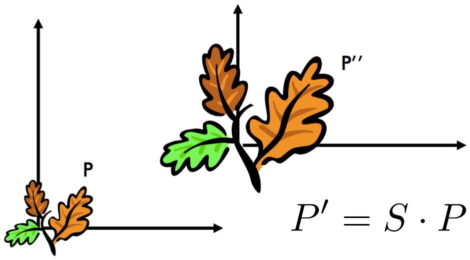


Is the ordering important?

$$P' = S \cdot P$$
$$P'' = T \cdot P'$$
$$\Rightarrow \quad P'' = (T \cdot S) \cdot P$$

source: S. Savarese

2D Scaling + Translation



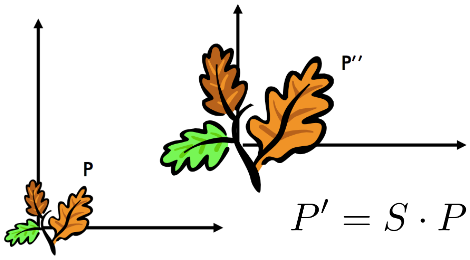
Is the ordering important?

$$P' = S \cdot P$$
$$P'' = T \cdot P'$$
$$\Rightarrow \quad P'' = (T \cdot S) \cdot P$$

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

source: S. Savarese

2D Scaling + Translation



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$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

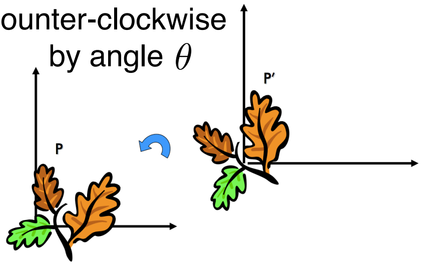
$$A = \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

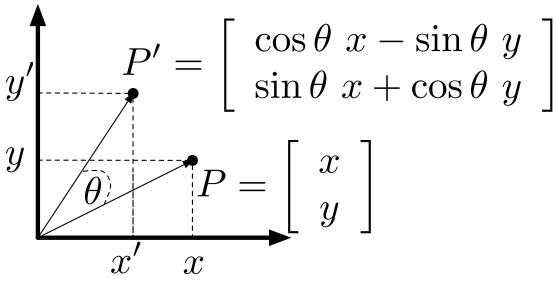
scaling + translation
matrix

source: S. Savarese

2D Rotation

counter-clockwise
by angle θ





$$P' = \begin{bmatrix} \cos\theta \; x - \sin\theta \; y \\ \sin\theta \; x + \cos\theta \; y \end{bmatrix}$$

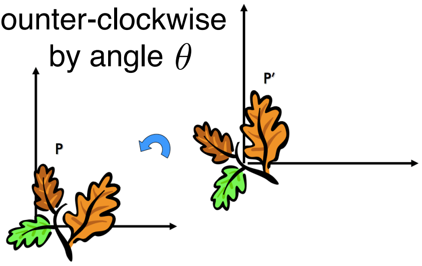
$$P = \begin{bmatrix} x \\ y \end{bmatrix}$$

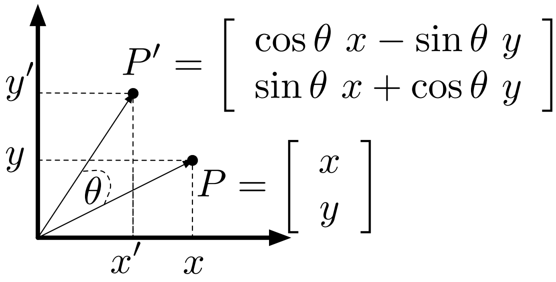
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

source: S. Savarese

2D Rotation

counter-clockwise
by angle θ





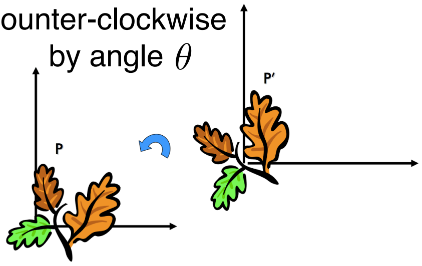
$$P' = \begin{bmatrix} \cos\theta \; x - \sin\theta \; y \\ \sin\theta \; x + \cos\theta \; y \end{bmatrix}$$

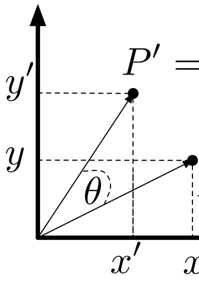
$$P = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{rotation matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

source: S. Savarese

2D Rotation


counter-clockwise by angle θ



$$P' = \begin{bmatrix} \cos\theta\ x - \sin\theta\ y \\ \sin\theta\ x + \cos\theta\ y \end{bmatrix}$$

$$P = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\overset{R}{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}}_{\text{rotation matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

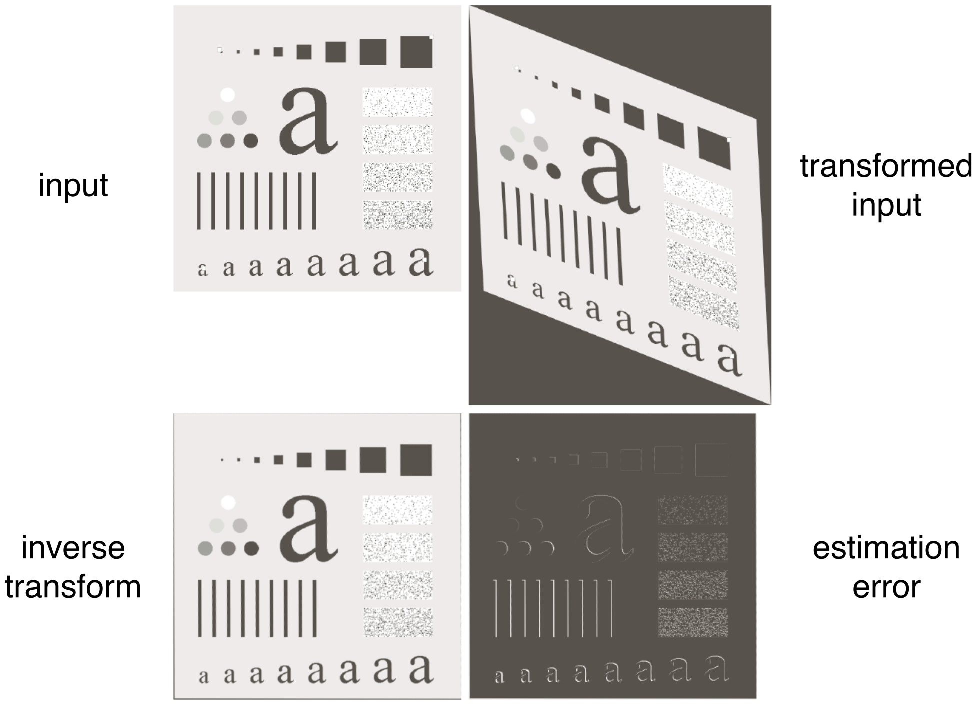
source: S. Savarese

2D Rotation + Scaling + Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\overset{R}{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}}_{\text{rotation matrix}} \underbrace{\overset{\boldsymbol{t}}{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}}_{\text{translation matrix}} \underbrace{\overset{S}{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}}_{\text{scaling matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R\ S & \boldsymbol{t} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Estimating the Spatial Transform



Estimating the Spatial Transform

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

What is the minimum number of point pairs to find matrix T?

$$AX = B$$

$$\text{if } \det(A) \neq 0 \Rightarrow X = A^{-1}B$$

or

$$\text{if } \det(A^TA) \neq 0 \Rightarrow X = (A^TA)^{-1}A^TB$$

Re-writing the Equation of Transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Re-writing the Equation of Transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x_i{\cdot}t_{11} \quad + \quad y_i{\cdot}t_{12} \quad + \quad 1{\cdot}t_{13} \quad + \quad 0{\cdot}t_{21} \quad + \quad 0{\cdot}t_{22} \quad + \quad 0{\cdot}t_{23} = x'_i$$

$$0{\cdot}t_{11} \quad + \quad 0{\cdot}t_{12} \quad + \quad 0{\cdot}t_{13} \quad + \quad x_i{\cdot}t_{21} \quad + \quad y_i{\cdot}t_{22} \quad + \quad 1{\cdot}t_{23} = y'_i$$

Re-writing the Equation of Transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

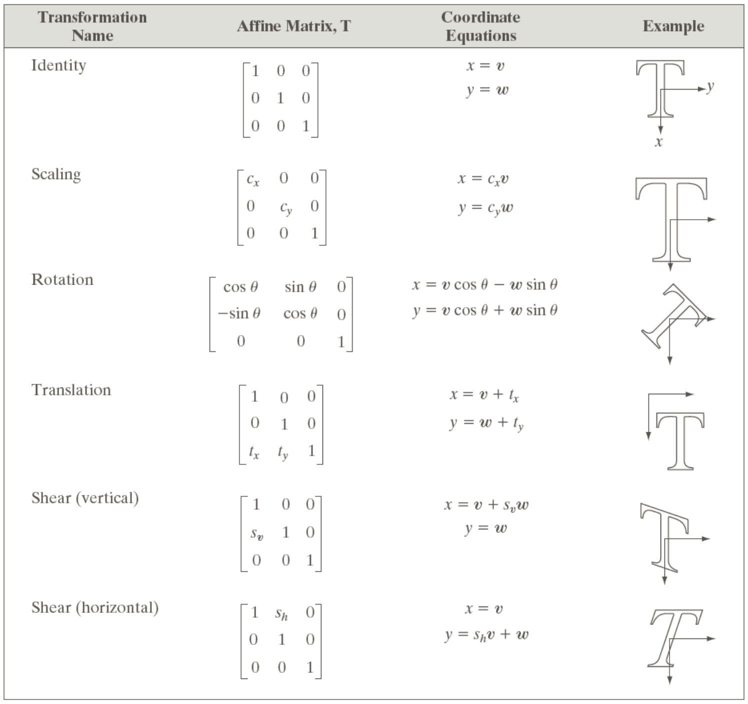
$$x_i{\cdot}t_{11} \quad + \quad y_i{\cdot}t_{12} \quad + \quad 1{\cdot}t_{13} \quad + \quad 0{\cdot}t_{21} \quad + \quad 0{\cdot}t_{22} \quad + \quad 0{\cdot}t_{23} = x'_i$$

$$0{\cdot}t_{11} \quad + \quad 0{\cdot}t_{12} \quad + \quad 0{\cdot}t_{13} \quad + \quad x_i{\cdot}t_{21} \quad + \quad y_i{\cdot}t_{22} \quad + \quad 1{\cdot}t_{23} = y'_i$$

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_i & y_i & 1 \end{bmatrix} \begin{bmatrix} t_{11} \\ t_{12} \\ t_{13} \\ t_{21} \\ t_{22} \\ t_{23} \end{bmatrix} = \begin{bmatrix} x'_i \\ y'_i \end{bmatrix}$$

Summary of Affine Transforms

TABLE 2.2
Affine transformations based on Eq. (2.6.–23).



Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Spatial-Domain Filtering of Images

Spatial-Domain Filtering of Images

$$g(x, y) = T\{f(x, y)\}$$

transformed input

input

Basic Operations on Images

- Addition
- Multiplication

Example: Averaging Noisy Measurements

$$g(x, y) = f(x, y) + \eta(x, y)$$

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x, y)$$

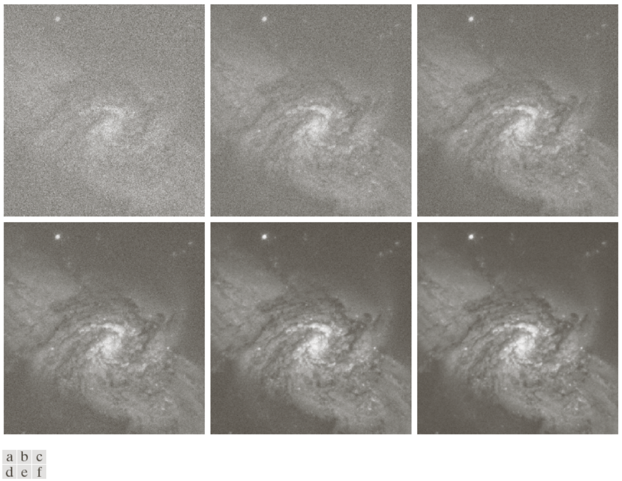


FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

Example: Shading Correction

$$g(x,y) = f(x,y)h(x,y)$$

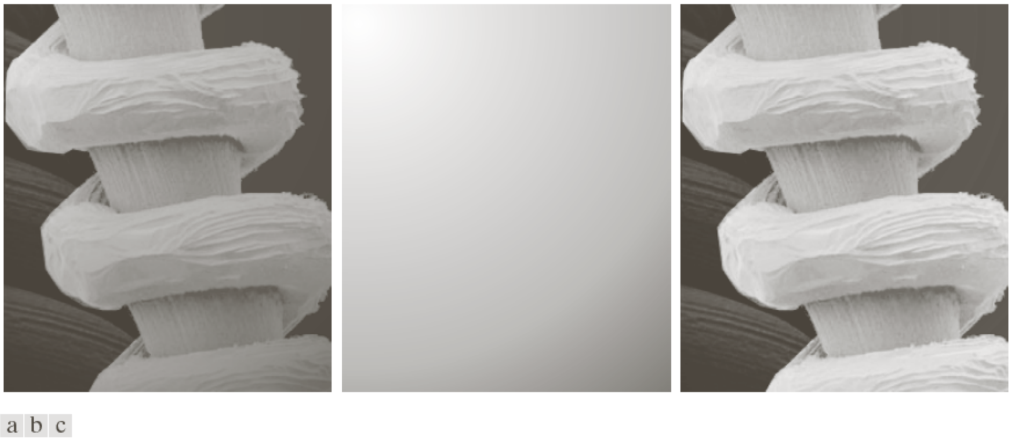


FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Example: Masking

$$g(x,y) = f(x,y)h(x,y)$$

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Example: Rescaling Intensity Values

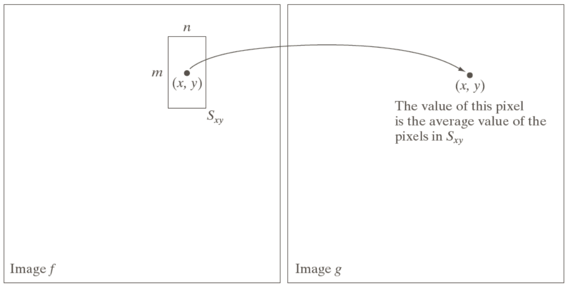
$$g(x,y) = K\frac{f(x,y) - \min[f(x,y)]}{\max[f(x,y)] - \min[f(x,y)]}$$

input

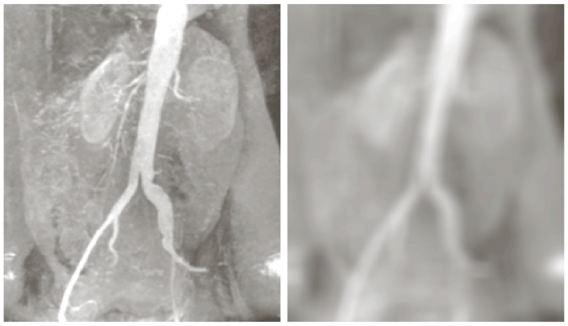
output

Example: Local Averaging - Smoothing

$$g(x,y) = \frac{1}{mn}\sum_{(x',y')\in N_{xy}} f(x',y')$$

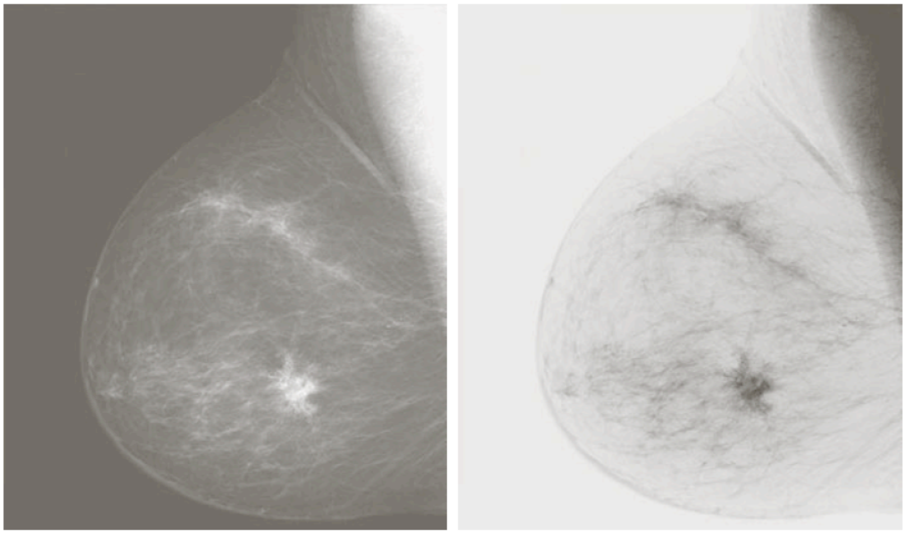


The value of this pixel is the average value of the pixels in S_{xy}



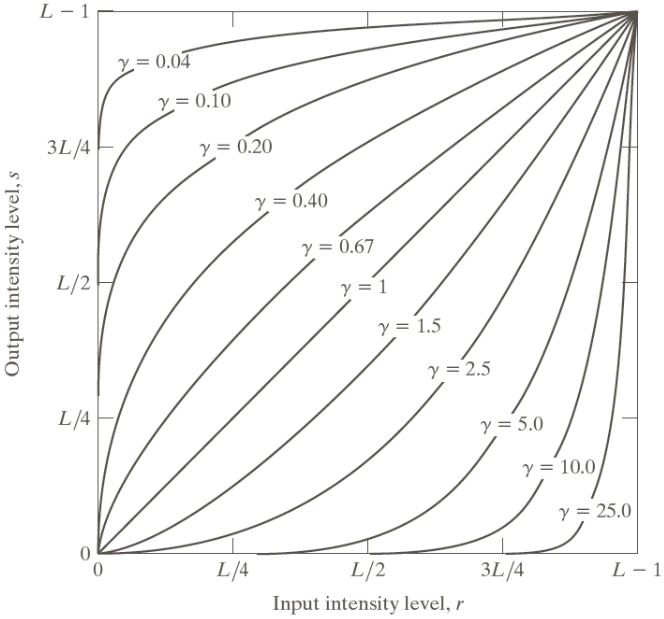
Example: Image Negatives

$$g(x,y) = L - 1 - f(x,y)$$

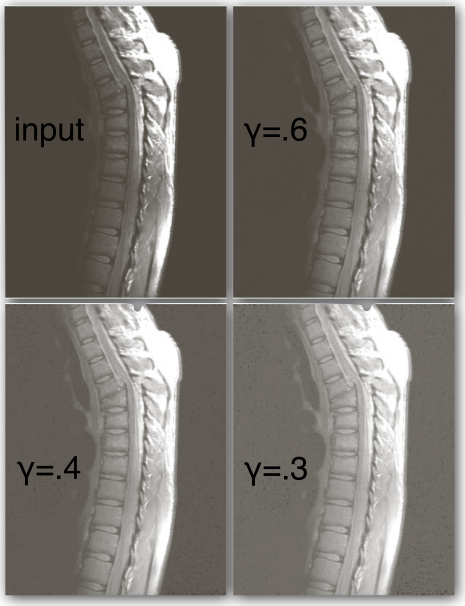


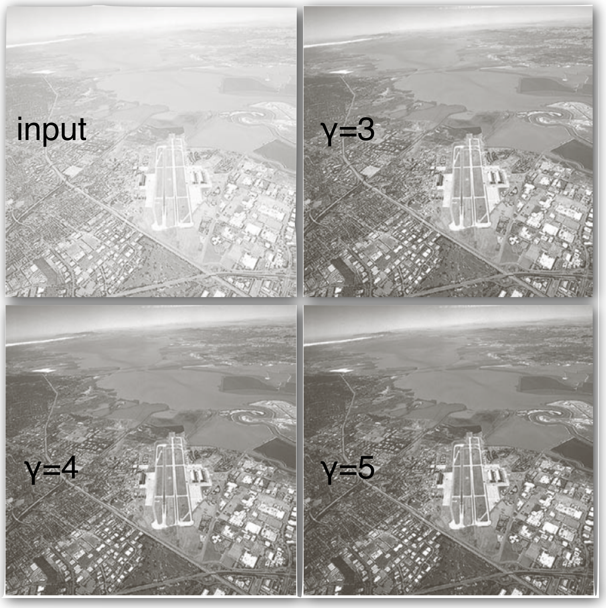
Example: Gamma Correction

$$s = cr^{\gamma}$$



Example: Gamma Correction





Next Class

- Homework 1 due
- Histogram equalization (Textbook: 3.3.1);
- Histogram specification (Textbook: 3.3.2);
- Spatial convolution and correlation (Textbook: 3.4.2);
- Smoothing and sharpening spatial filters (Textbook: 3.5);
- Homework 2