ECE 468: Digital Image Processing

Lecture 3

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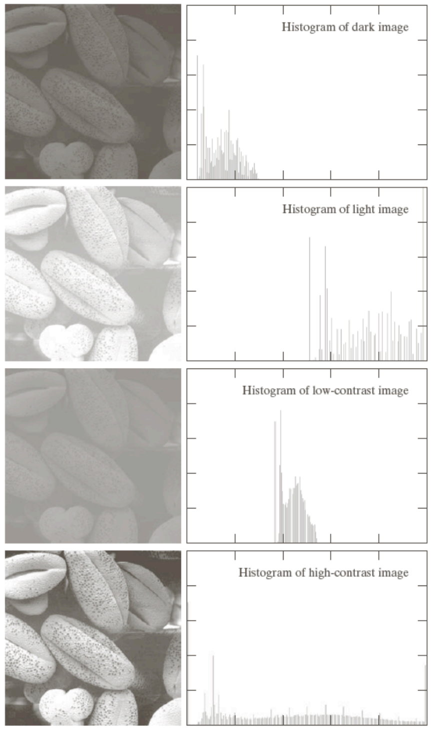
Next Class

- Homework 1 due
- Histogram equalization (Textbook: 3.3.1);
- Histogram specification (Textbook: 3.3.2);
- Spatial convolution and correlation (Textbook: 3.4.2);
- Smoothing and sharpening spatial filters (Textbook: 3.5);
- Homework 2

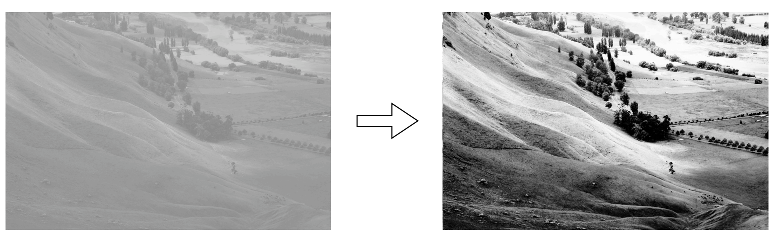
Histogram of Intensity Values



x axis: intensity values

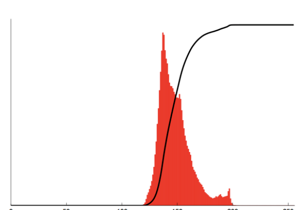
y axis: frequency

Histogram Equalization



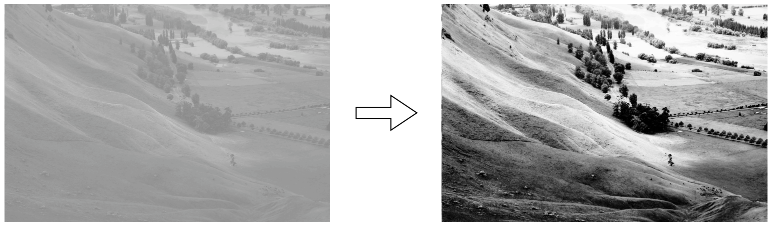
input output

- Increases local contrast by spreading out the intensity histogram
- Produces artifacts



intensity-level histogram (red)
cumulative histogram (black)

Histogram Equalization



input output

$$s = T(r)$$

output intensity → flat histogram of intensities → flat distribution of intensities: $p(s)$

input intensity → narrow histogram of intensities → narrow distribution of intensities: $p(r)$

Distribution of Intensity Values

$$p(r = i) = \frac{n_i}{n}$$

n_i: number of pixels with intensity *i*

n: total number of pixels

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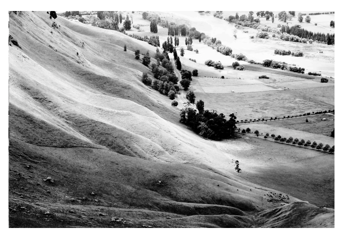
$$\Rightarrow \quad \frac{dT(r)}{dr} = p(r)$$

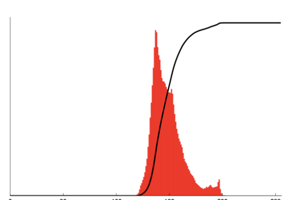
$$\Rightarrow \quad \boxed{T(r) = CDF(r)}$$

Histogram Equalization

input



output

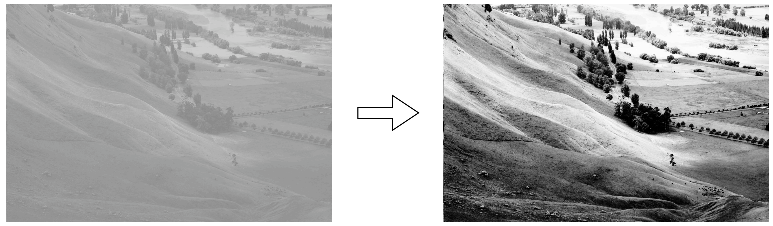


$$p(r = i) = \frac{n_i}{n} \quad \Rightarrow \quad CDF(r = i) = \sum_{j=0}^{i} p(r = j)$$

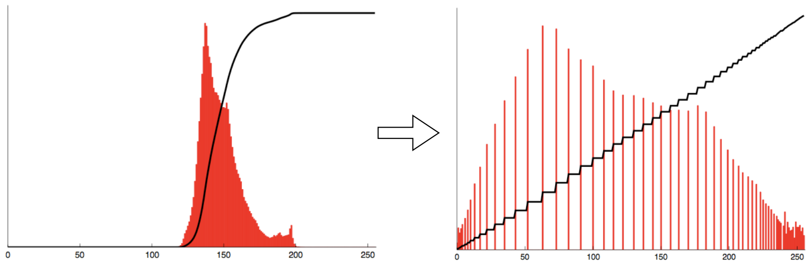
distribution

cumulative distribution

Histogram Equalization

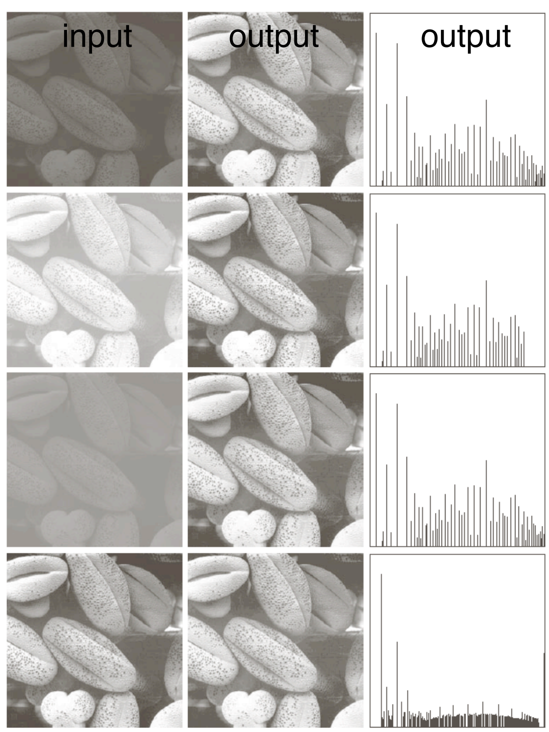


input output



intensity-level histogram (red); cumulative histogram (black)

Example: Histogram Equalization



Histogram Specification

$$s = T(r) \quad \Rightarrow \quad p_s(s) = p_r(r) \left(\frac{dT(r)}{dr} \right)^{-1}$$

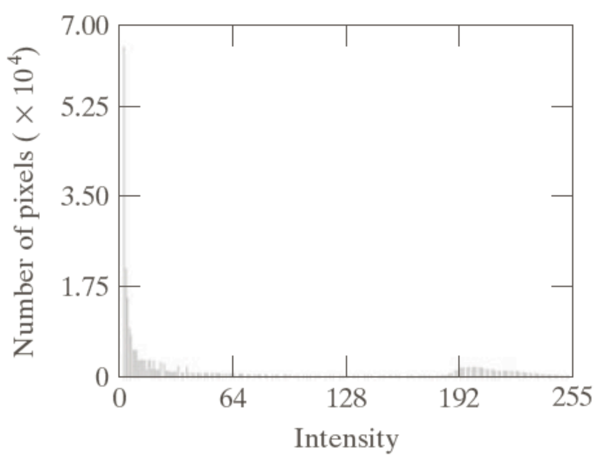
$$\Rightarrow \quad \frac{dT(r)}{dr} = \frac{p_r(r)}{p_s(s)}$$

$$s = T(r) = \left| \int_0^s \frac{p_r(w)}{p_s(w)} dw \right|$$

Example: Histogram Specification

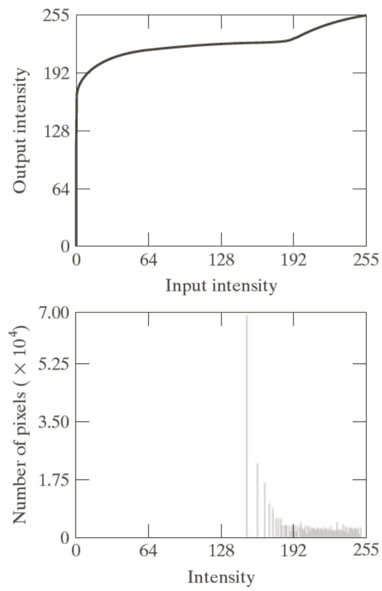
input image

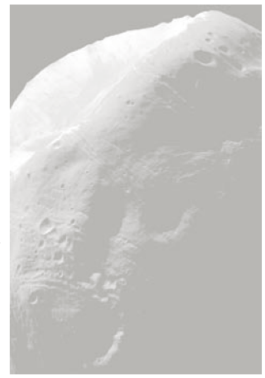
Histogram of Pixel Intensities



Example: Histogram Specification

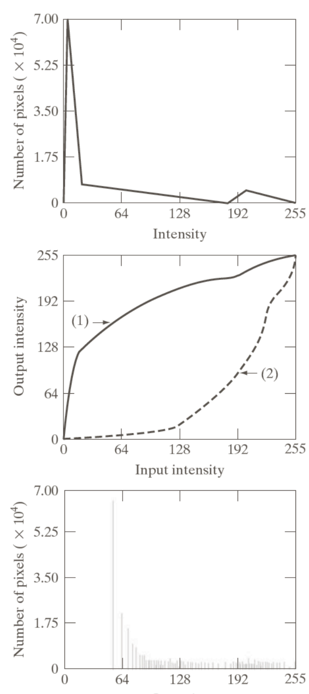
Histogram Equalization





output

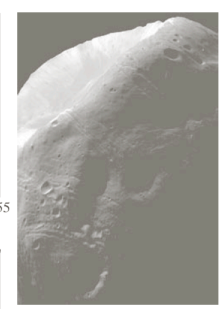
Example: Histogram Specification



specified histogram

estimated transform *T(r)*

output histogram



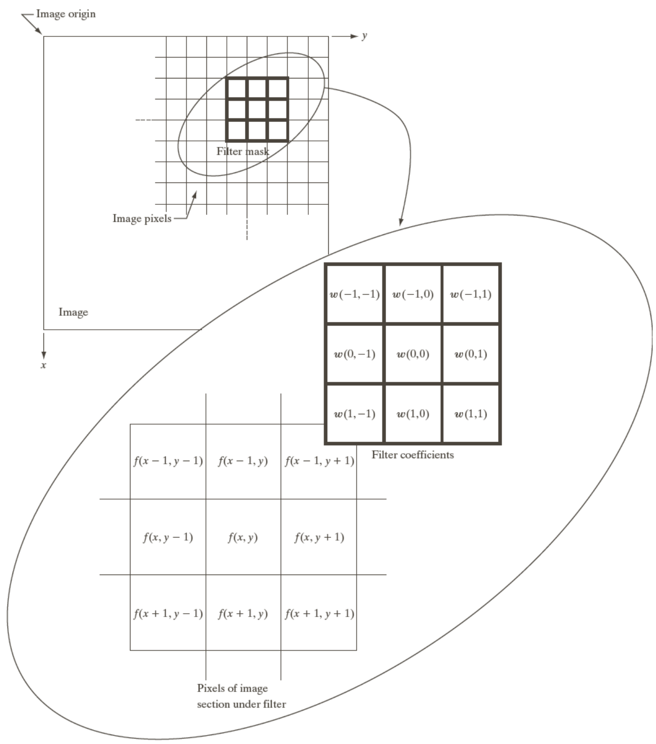
output

Linear Spatial Filtering of Images

Linear Spatial Filtering

$$g(x,y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} w(i,j) f(x+i, y+j)$$

Linear Spatial Filtering



Correlation vs. Convolution

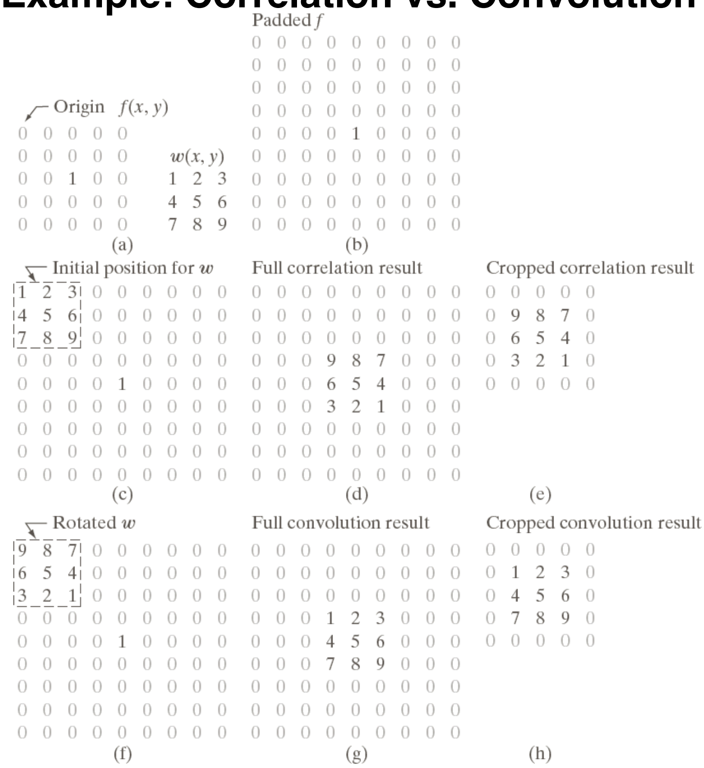
- Correlation:
  1. Move the filter mask to a location
  2. Compute the sum of products
  3. Go to 1.

$$w(x,y) \star f(x,y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} w(i,j) f(x+i, y+j)$$

- Convolution:
  1. Rotate the filter mask by 180 degrees
  2. Compute the sum of products
  3. Go to 2.

$$w(x,y) \ast f(x,y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} w(i,j) f(x-i, y-j)$$

Example: Correlation vs. Convolution



Gaussian Filter

$$N(x,y) = \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

3x3 filter:

$$w_1 = N(-1,-1),\ w_2 = N(-1,0),\ \ldots\ w_9 = N(1,1)$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Smoothing Filter -- Low Pass Filter

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

spatial averaging

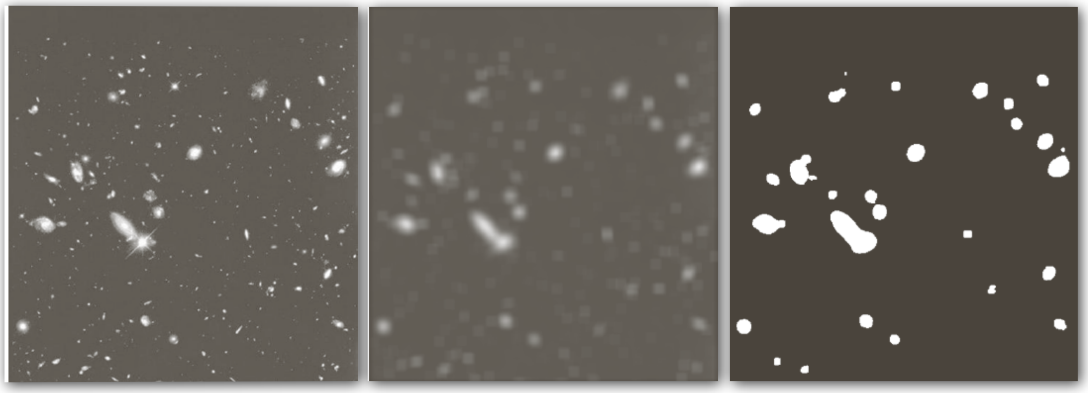
$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

weighted averaging

Example: Smoothing Filter

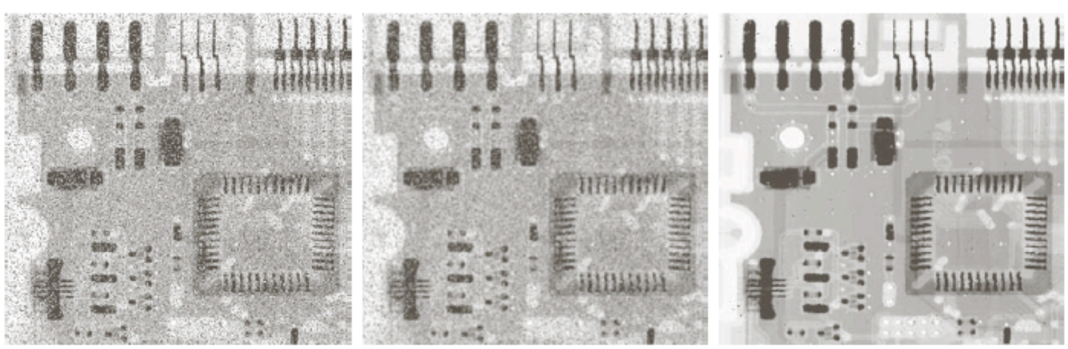
Application of Smoothing

an image from Hubble space telescope



input output thresholded

Application of Smoothing



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Sharpening Spatial Filters -- Review

$$\frac{\partial f(x,y)}{\partial x} \approx f(x+1,y) - f(x,y)$$

$$\frac{\partial f(x,y)}{\partial y} \approx f(x,y+1) - f(x,y)$$

Sharpening Spatial Filters -- Review

$$\frac{\partial f(x,y)}{\partial x} \approx f(x+1,y) - f(x,y)$$

$$\frac{\partial f(x,y)}{\partial y} \approx f(x,y+1) - f(x,y)$$

$$\frac{\partial^2 f(x,y)}{\partial^2 x} \approx f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f(x,y)}{\partial^2 y} \approx f(x,y+1) + f(x,y-1) - 2f(x,y)$$

Sharpening Spatial Filters -- Review

$$\frac{\partial f(x,y)}{\partial x} \approx f(x+1,y) - f(x,y)$$

$$\frac{\partial f(x,y)}{\partial y} \approx f(x,y+1) - f(x,y)$$

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$$\frac{\partial^2 f(x,y)}{\partial^2 y} \approx f(x,y+1) + f(x,y-1) - 2f(x,y)$$

Laplacian $\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial^2 x} + \frac{\partial^2 f(x,y)}{\partial^2 y}$

Sharpening Spatial Filters

Variants of the Laplacian filter

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

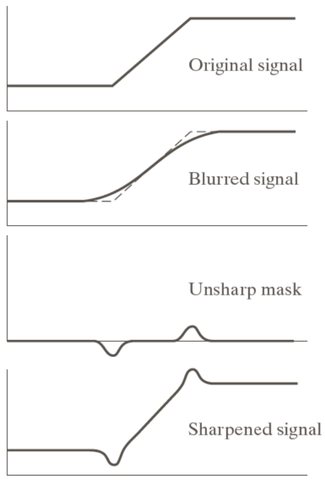
0	−1	0
−1	4	−1
0	−1	0

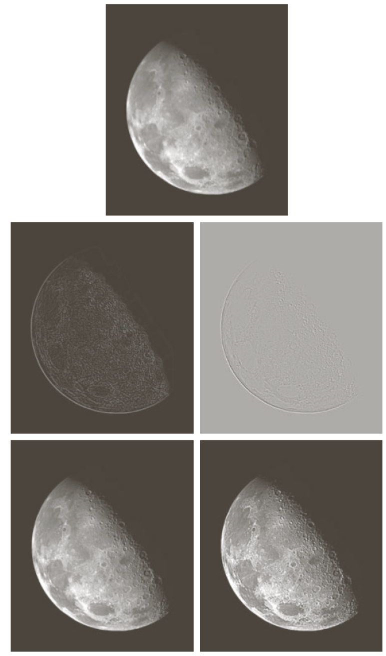
−1	−1	−1
−1	8	−1
−1	−1	−1

Image Sharpening

$$g(x,y) = f(x,y) \pm c \cdot \nabla^2 f(x,y), \quad c > 0$$

- Blur the original image
- Add the Laplacian





Homework 2

due 01/22

Next Class

- Review of
  - Continuous Fourier Transform
  - Discrete Fourier Transform (Textbook: 4.2-4.4)
- 2D Continuous Fourier Transform (Textbook: 4.5)