ECE 468: Digital Image Processing

Lecture 5

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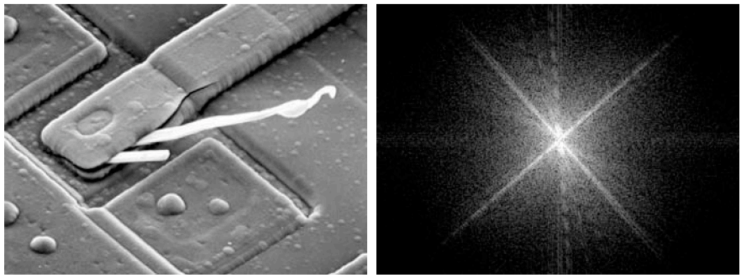
Disclaimer

The following slides are just excerpts from the textbook.

You should learn all material presented in chapter 4 in the textbook!

Filtering in the Frequency Domain

$f(x,y)$



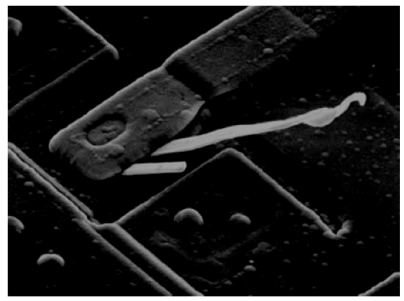
$|F(u,v)|$

a b

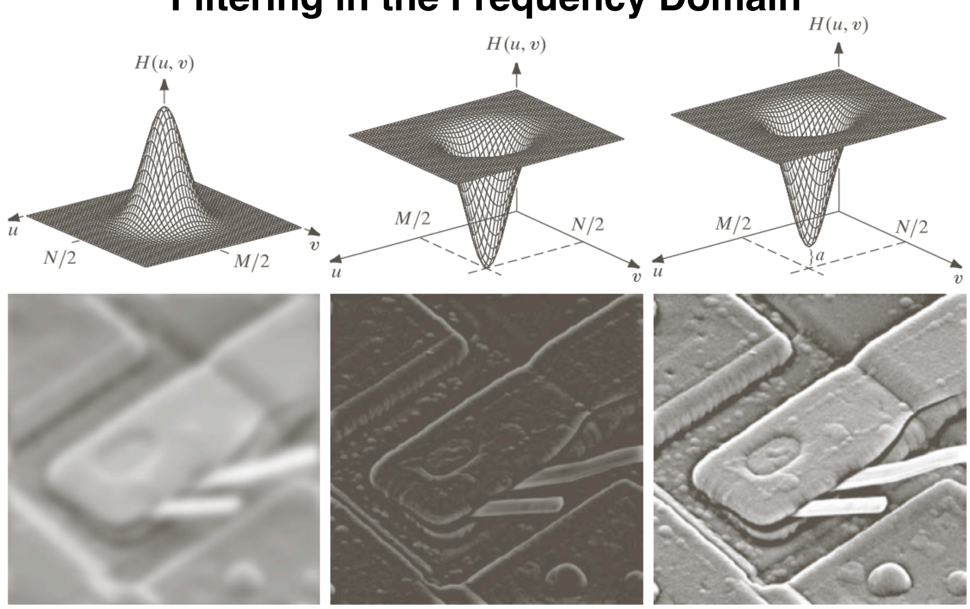
FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

$$H(u,v) = \begin{cases} 0 & , \quad u=0, v=0 \\ 1 & , \quad \text{otherwise} \end{cases}$$

$$\mathcal{F}^{-1}\{H(u,v)F(u,v)\}$$



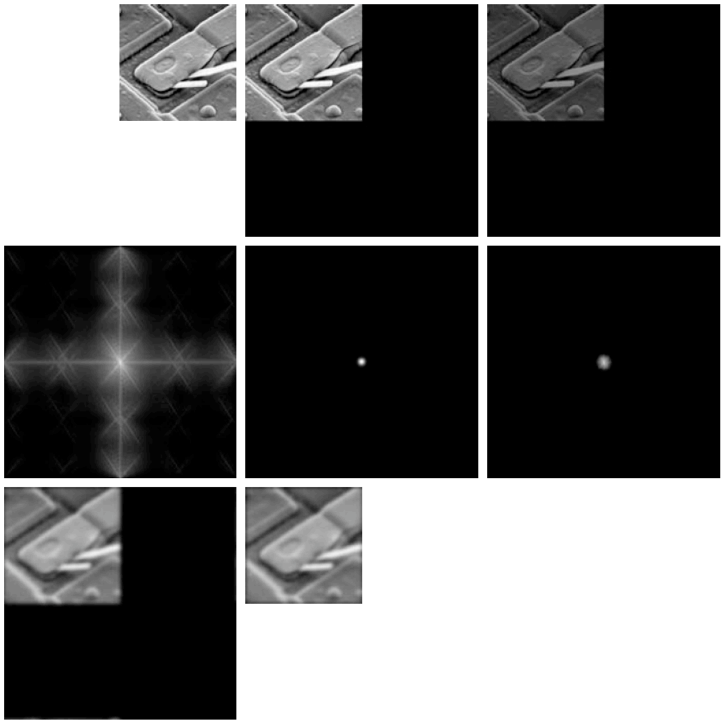
Filtering in the Frequency Domain



a b c
d e f

FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

Steps in Frequency Domain Filtering



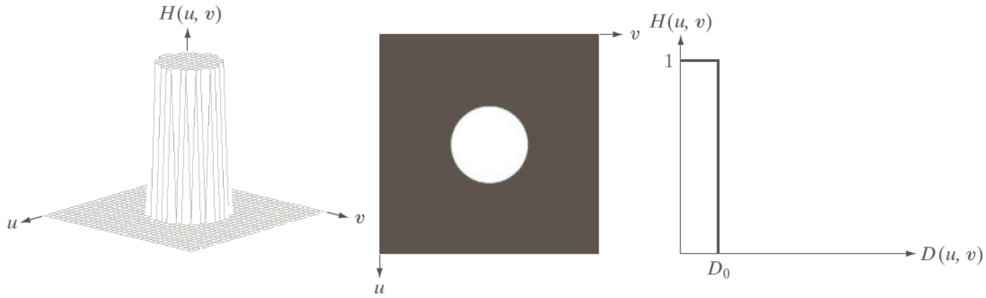
a b c
d e f
g h

FIGURE 4.36 (a) An $M \times N$ image, f. (b) Padded image, f_p of size $P \times Q$. (c) Result of multiplying f_p by $(-1)^{x+y}$. (d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$. (f) Spectrum of the product HF_p. (g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p. (h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

Steps in Frequency Domain Filtering

1. Input: $f(x,y)$ of size MxN
2. Compute padding $f_p(x,y)$ of size PxQ, where P = 2M, Q = 2N
3. Multiply $f_p(x,y)(-1)^{x+y}$ to center its DFT
4. Compute DFT of $f_p(x,y)(-1)^{x+y}$ --> F(u,v)
5. Use filter H(u,v) of size PxQ, with center at coordinates (P/2,Q/2)
6. Multiply element-wise G(u,v) = H(u,v)F(u,v)
7. Compute the real part of IDFT, $g_p(x,y)$ = real[IDFT(G(u,v))] $(-1)^{x+y}$
8. Crop the top left MxN region to get g(x,y)

Ideal Lowpass Filter



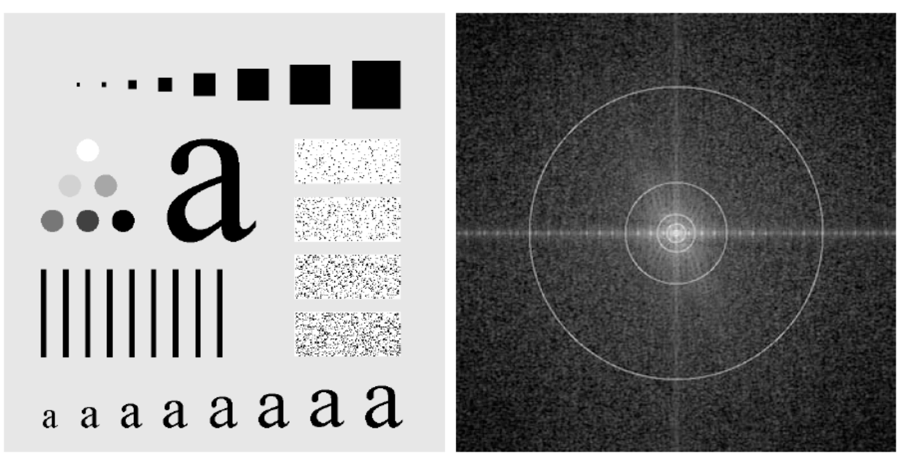
a b c

FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

$$H(u,v) = \begin{cases} 1 & , \quad D(u,v) \leq D_0 \\ 0 & , \quad D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \sqrt{\left(u - \frac{P}{2}\right)^2 + \left(v - \frac{Q}{2}\right)^2}$$

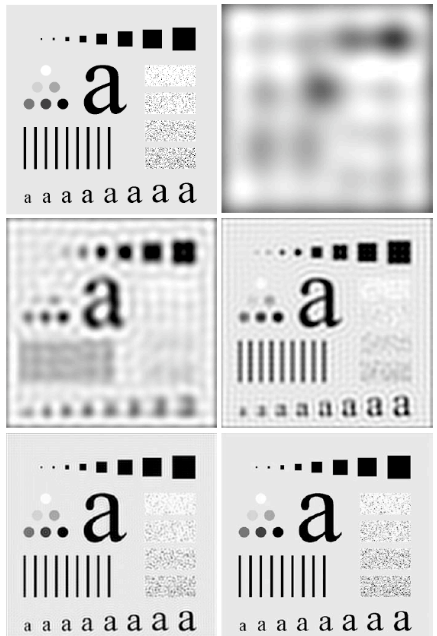
Example: Lowpass Filtering



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

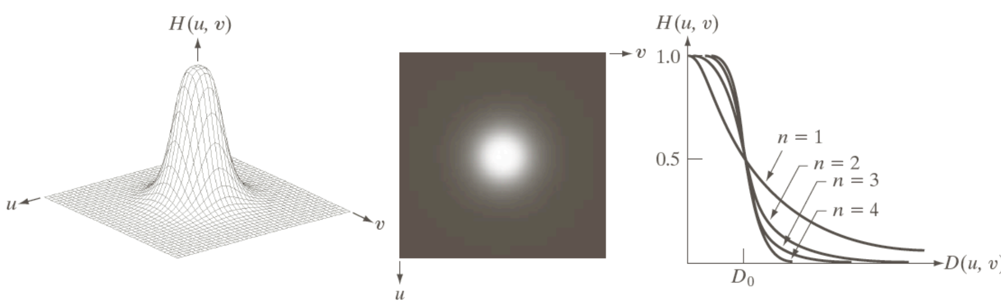
Example: Lowpass Filtering



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

Butterworth Lowpass Filter



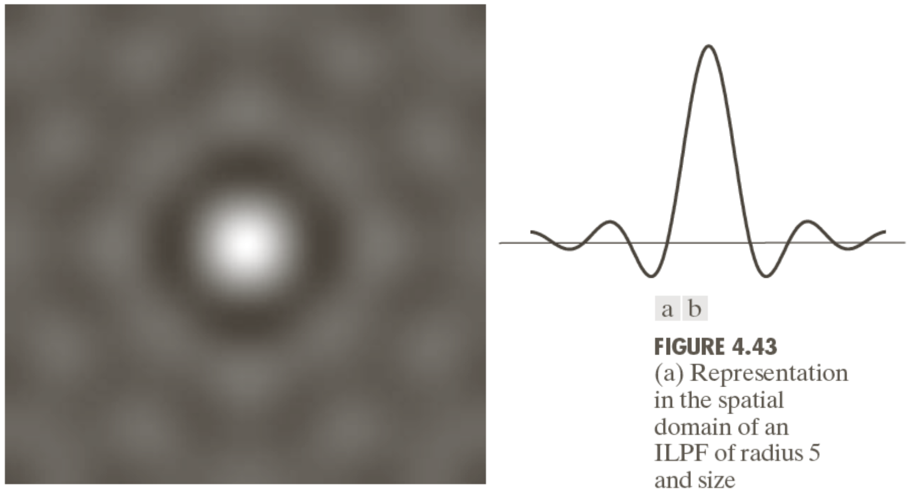
a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

$$H(u,v) = \frac{1}{1 + \left(\frac{D(u,v)}{D_0}\right)^{2n}}$$

$$D(u,v) = \sqrt{\left(u - \frac{P}{2}\right)^2 + \left(v - \frac{Q}{2}\right)^2}$$

Butterworth Lowpass Filter



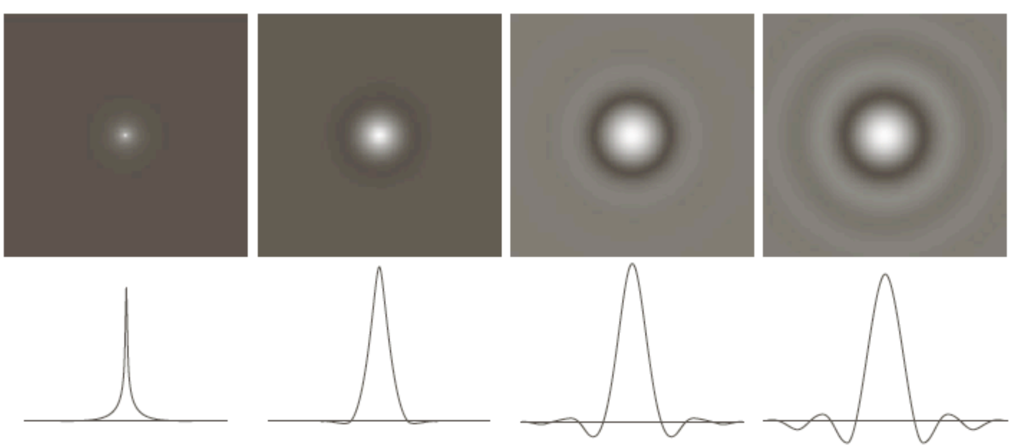
a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000. (b) Intensity profile of a horizontal line passing through the center of the image.

Ringing for large *n*

Butterworth Lowpass Filter

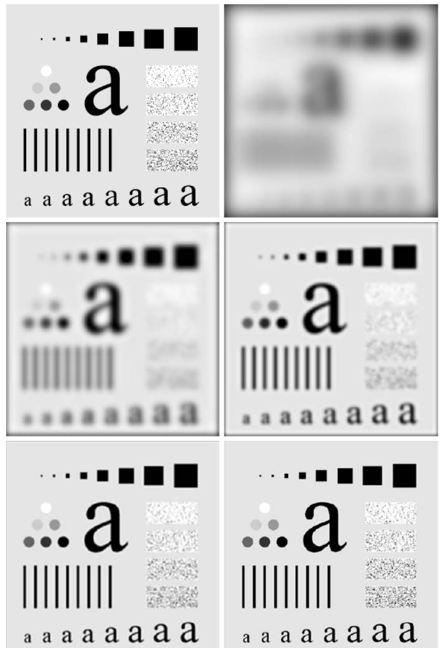
Ringing for large *n*



a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

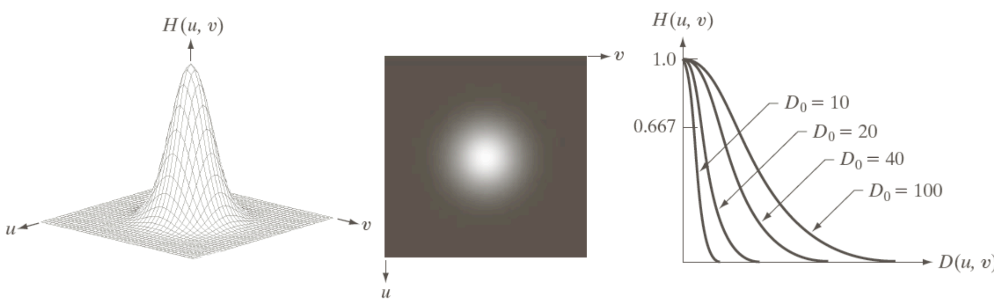
Example: Butterworth Lowpass Filter



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

Lowpass Gaussian Filter



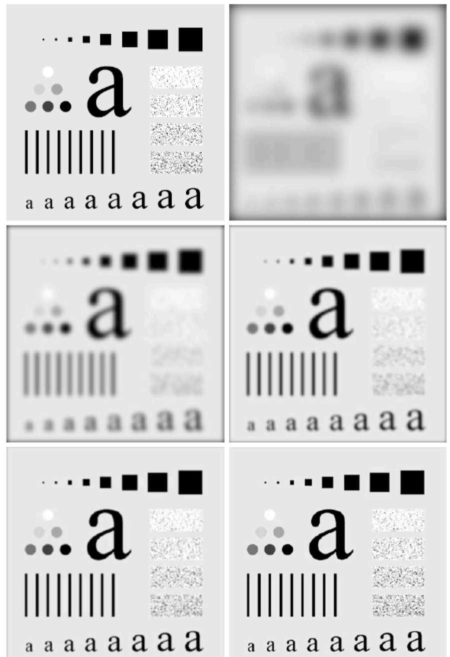
a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

$$H(u,v) = \exp\left(-\frac{D^2(u,v)}{2D_0^2}\right)$$

$$D(u,v) = \sqrt{\left(u - \frac{P}{2}\right)^2 + \left(v - \frac{Q}{2}\right)^2}$$

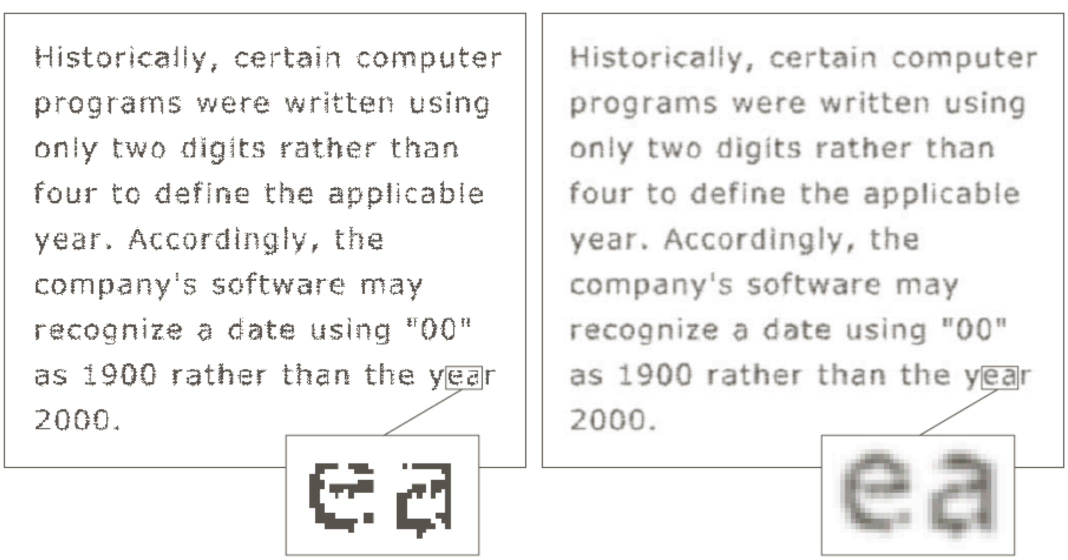
Example: Lowpass Gaussian Filter



a b
c d
e f

FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Example: Lowpass Gaussian Filter



Example: Lowpass Gaussian Filter

After this goes sharpening for Hollywood!!

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Highpass Filtering in the Frequency Domain

$$H_{\mathrm{HP}}(u, v) = 1 - H_{\mathrm{LP}}(u, v)$$

Highpass Filters

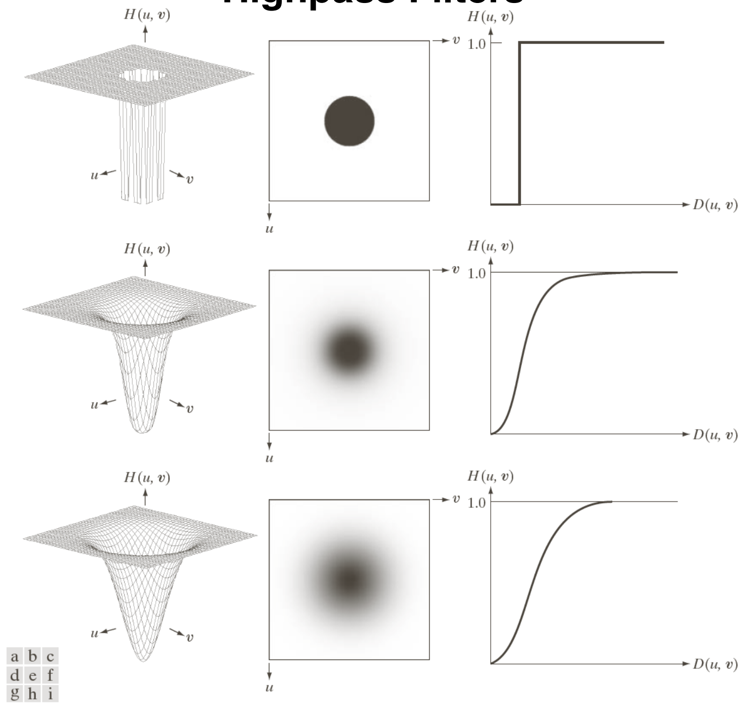


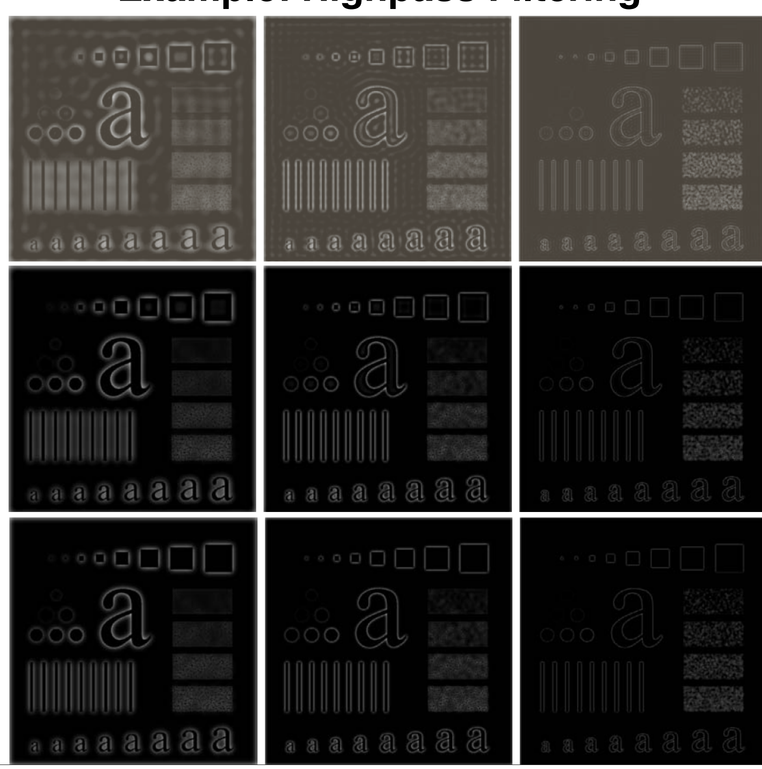
FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Example: Highpass Filtering

Ideal

Butterworth

Gaussian



Laplacian Filter (Homework 4)

$$H(u,v) = -4\pi^2(u^2+v^2)$$

a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Image Sharpening in the Frequency Domain

$$g(x,y) = f(x,y) \pm c\nabla^2 f(x,y)$$

$$G(u,v) = F(u,v) - H(u,v)F(u,v), \quad c=-1$$

needs scaling

$$g(x,y) = \mathcal{F}^{-1}\{[1+4\pi(u^2+v^2)]F(u,v)\}$$

Image Sharpening in the Frequency Domain

Unsharp Masking

$$g(x,y) = f(x,y) + c[f(x,y) - \bar{f}(x,y)]$$

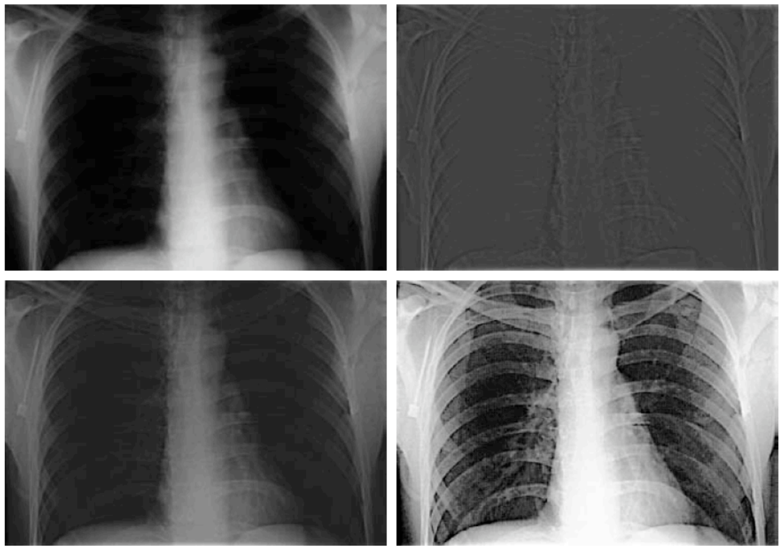
$$\bar{f}(x,y) = \mathcal{F}^{-1}\{H_{\mathrm{LP}}(u,v)F(u,v)\}$$

$$g(x,y) = \mathcal{F}^{-1}\{[1 + c(1 - H_{\mathrm{LP}}(u,v))]F(u,v)\}$$

$$g(x,y) = \mathcal{F}^{-1}\{[1 + cH_{\mathrm{HP}}(u,v)]F(u,v)\}$$

high frequency emphasis filter

Unsharp Masking in the Frequency Domain



a b
c d

FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)