ECE 468: Digital Image Processing

Lecture 6

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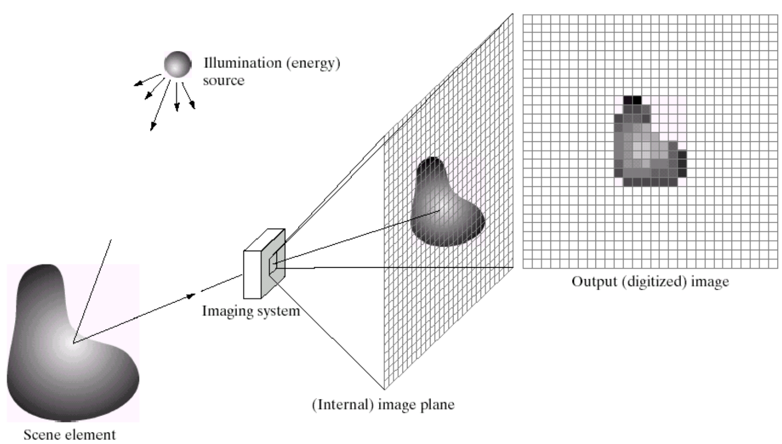
Image Formation



$$f(x, y) = i(x, y)r(x, y)$$

illumination $i(x, y) \in [0, \infty)$

reflectance $r(x, y) \in [0, 1]$

Homomorphic Filtering

=

Separation of intensity from reflectance

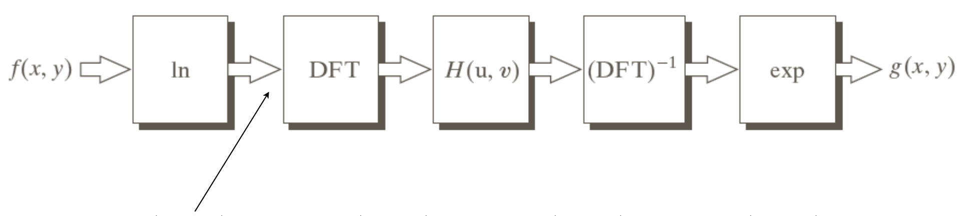
+

Intensity range compression

+

Contrast Enhancement

Homomorphic Filtering

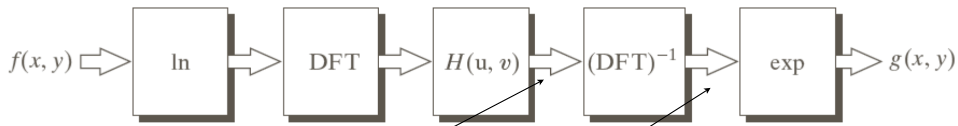


$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

$$i'(x,y) = \ln i(x,y) \;\leftrightarrow\; I'(u,v)$$

$$r'(x,y) = \ln r(x,y) \;\leftrightarrow\; R'(u,v)$$

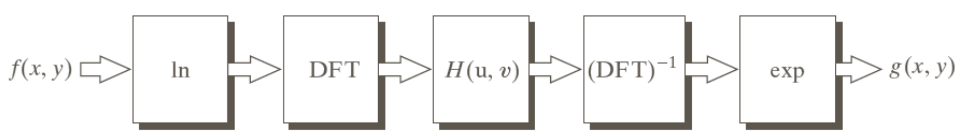
Homomorphic Filtering



$$S(u, v) = H(u, v)I'(u, v) + H(u, v)R'(u, v)$$

$$s(x, y) = \mathcal{F}^{-1}\{H(u, v)I'(u, v) + H(u, v)R'(u, v)\}$$

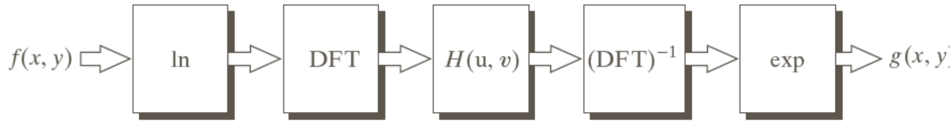
Homomorphic Filtering



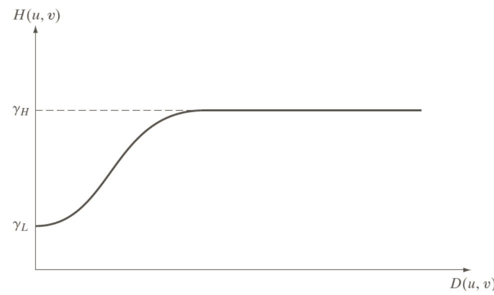
$$g(x, y) = \mathrm{e}^{(\mathcal{F}^{-1}\{H(u,v)I'(u,v)\})}\mathrm{e}^{(\mathcal{F}^{-1}\{H(u,v)R'(u,v)\})}$$

affects both low- and high-frequency components

Homomorphic Filtering



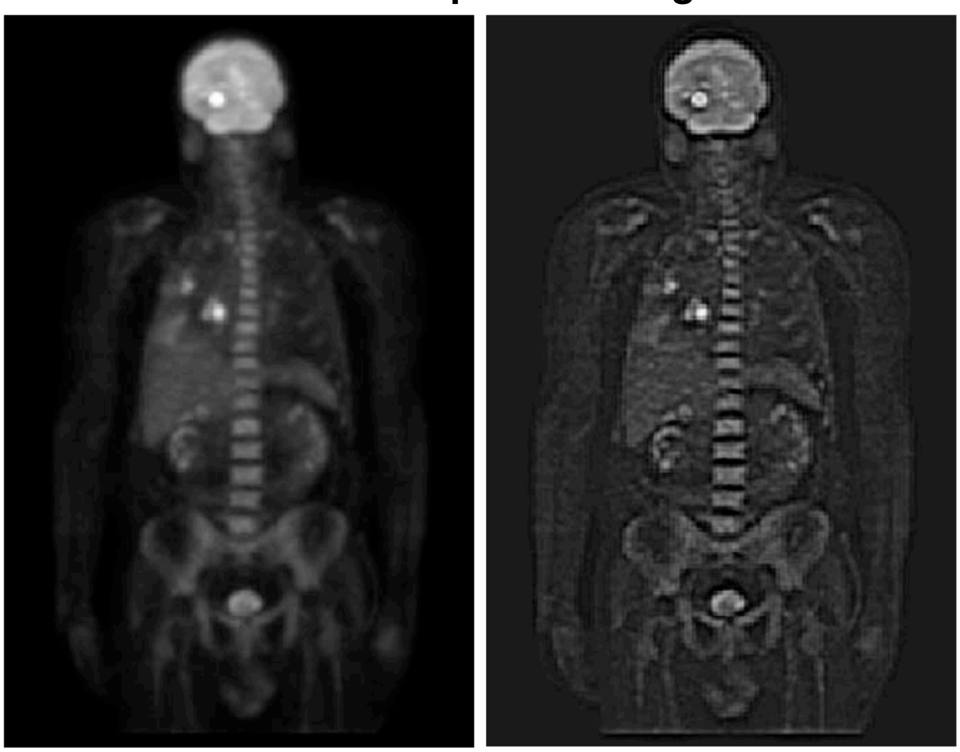
example:



affects both low- and high-frequency components

dynamic range compression and contrast enhancement

Homomorphic Filtering



Bandpass and Bandreject Filtering

$$H_{BP} = 1 - H_{BR}$$

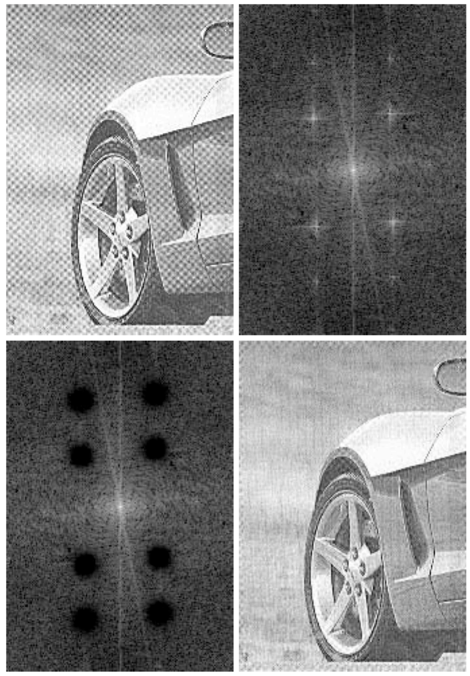
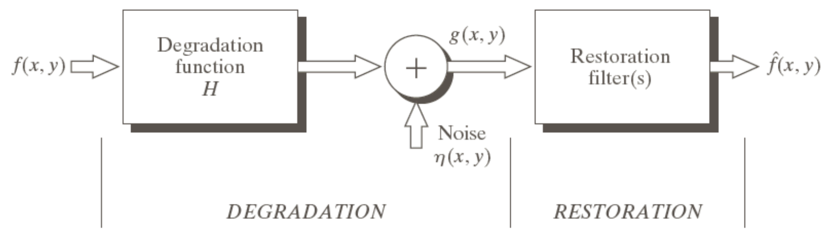


Image Restoration vs. Image Enhancement

- Unlike enhancement, improve an image in an objective sense
- Model the degradation and use the model for image restoration

Model of Image Degradation/Restoration

FIGURE 5.1 A model of the image degradation/restoration process.



$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Noise Models

Assumptions:

- Noise is independent of image coordinates
- Noise is not correlated with the image

Important Noise PDFs

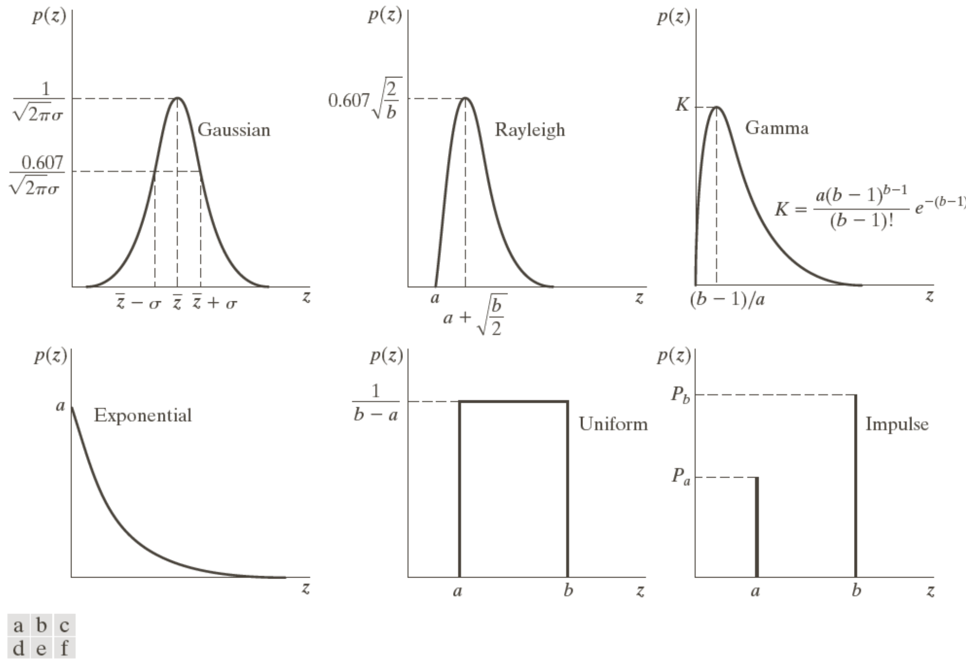
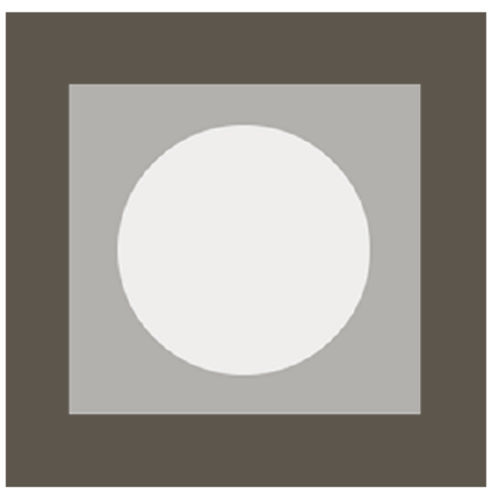


FIGURE 5.2 Some important probability density functions.

Noise in the Image



Noise in the Image

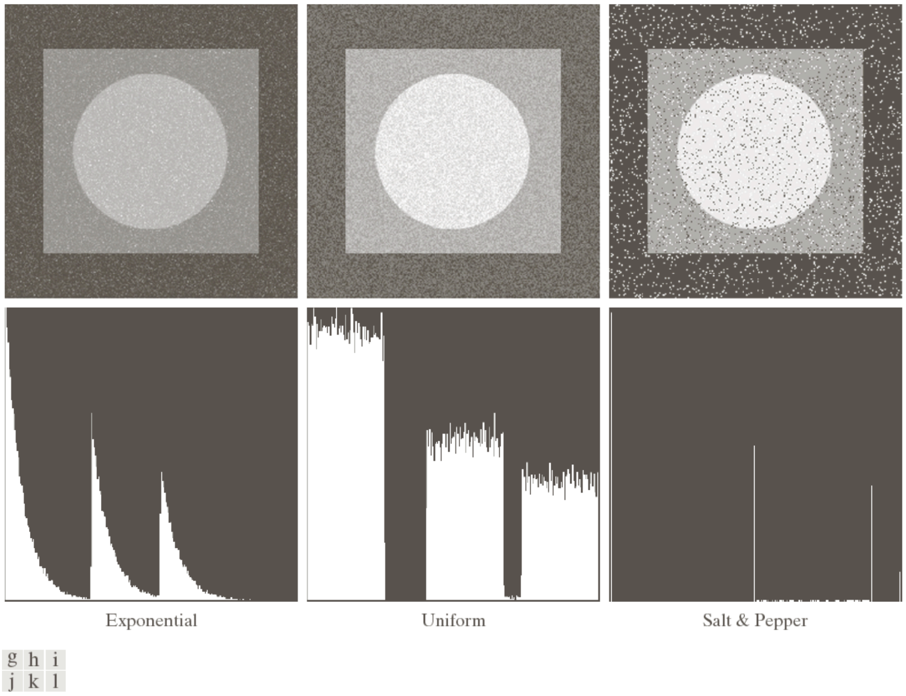


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Noise in the Image

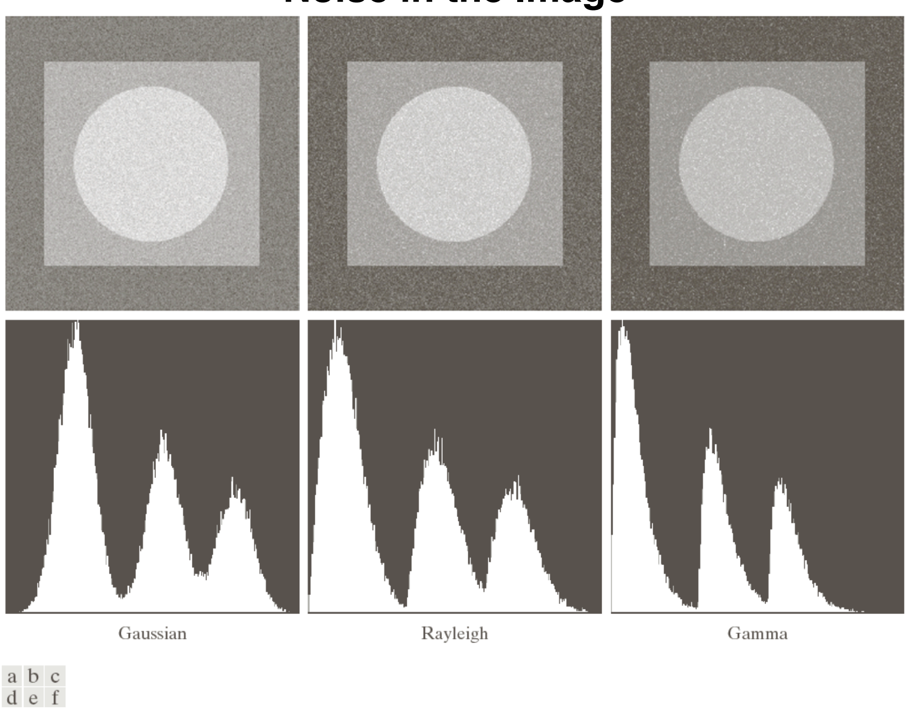
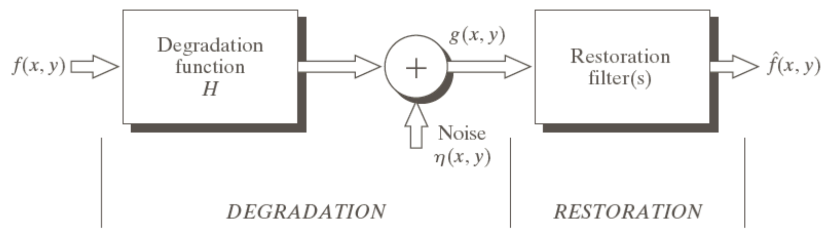


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Degradation Modeling

FIGURE 5.1 A model of the image degradation/restoration process.

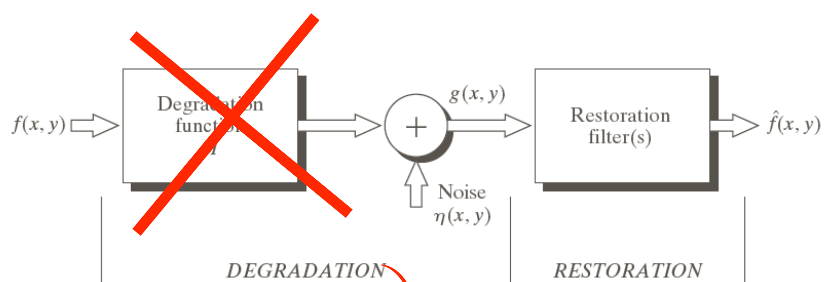


$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Degradation Modeling

FIGURE 5.1 A model of the image degradation/restoration process.



$$g(x,y) = \overset{=\delta(x,y)}{h(x,y)} \star f(x,y) + \eta(x,y)$$

$$G(u,v) = \overset{=1}{H(u,v)}F(u,v) + N(u,v)$$

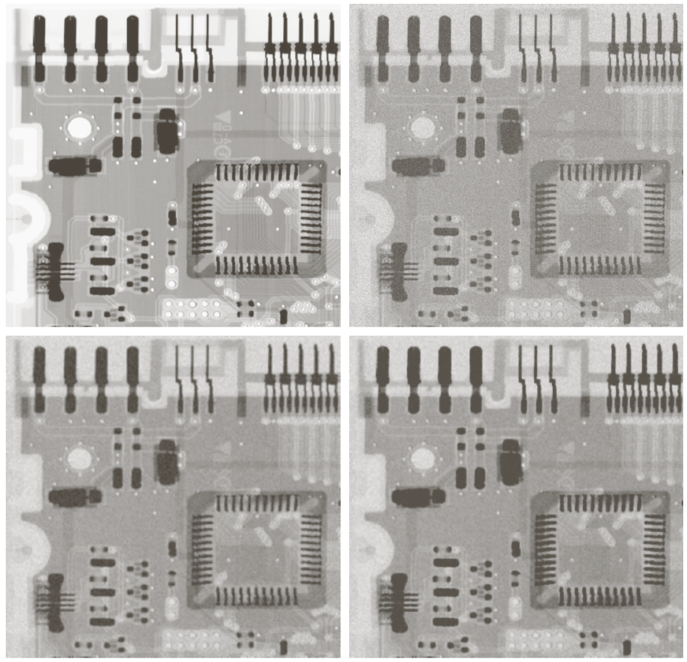
Gaussian Noise + Arithmetic vs. Geometric Mean Filter

S_{xy}

filter window

$$g(x, y) = f(x, y) + \eta(x, y)$$

output input



arithmetic mean geometric mean

Gaussian Noise + Arithmetic vs. Geometric Mean Filter

S_{xy}

filter window

$$g(x, y) = f(x, y) + \eta(x, y)$$

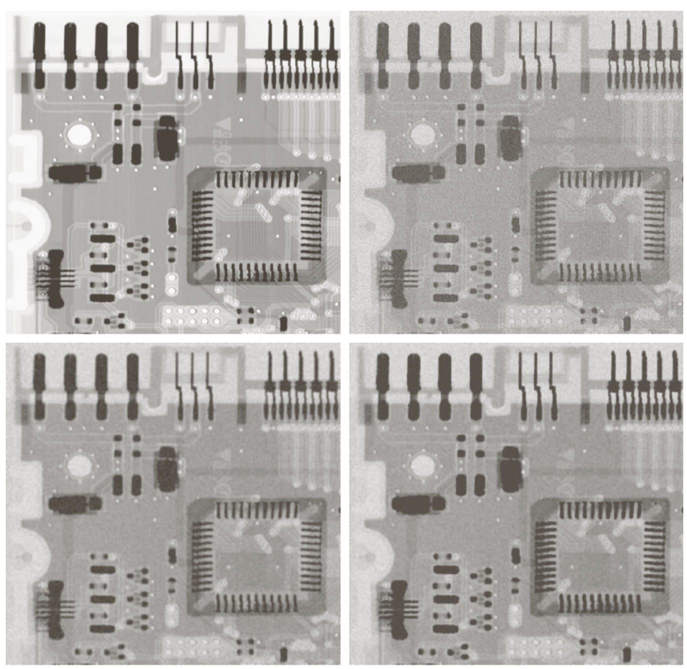
output input

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

arithmetic mean filtering

$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

geometric mean filtering



arithmetic mean geometric mean

Salt-and-Pepper Noise + Median Filter

S_{xy}

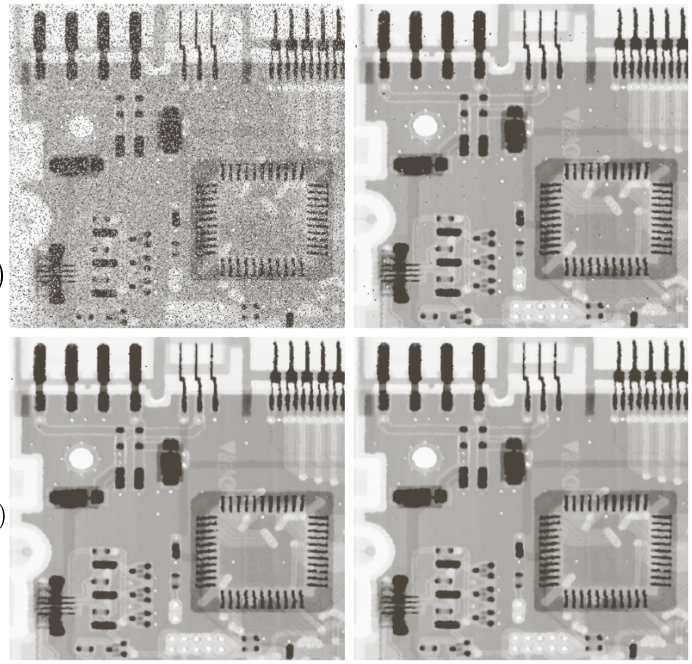
filter window

$$g(x,y) = f(x,y) + \eta(x,y)$$

output input

$$\hat{f}(x,y) = \mathrm{median}_{(s,t)\in S_{xy}} \quad g(s,t)$$

median filtering



repeated application of median filter

Gaussian Noise + Adaptive Filter

arithmetic mean
$$m_{S_{xy}} = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

arithmetic variance
$$\sigma^2_{S_{xy}} = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} (g(s,t) - m_{S_{xy}})^2$$

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma^2_\eta}{\sigma^2_{S_{xy}}} \left[g(x,y) - m_{S_{xy}} \right]$$

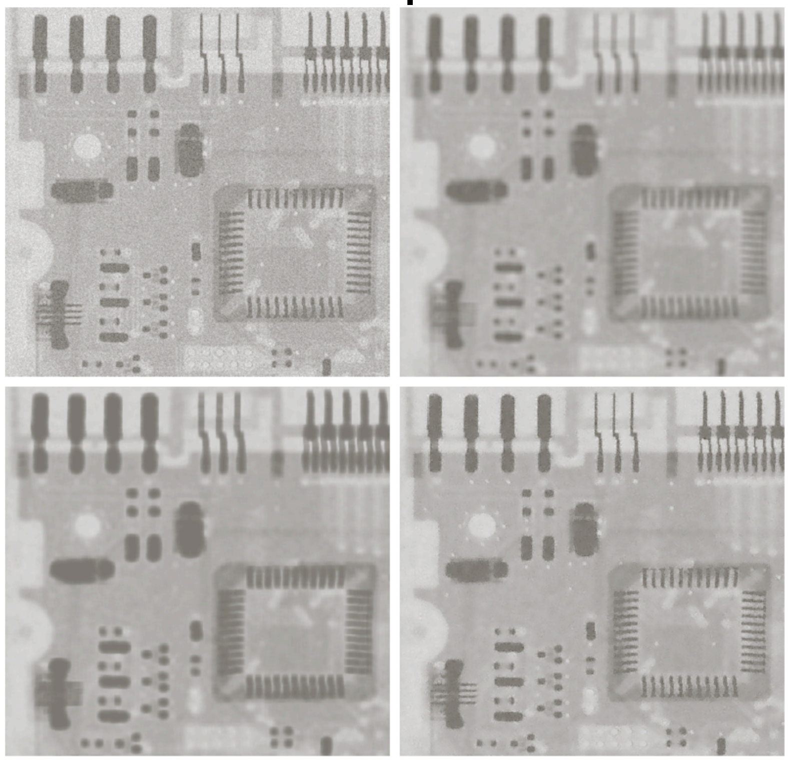
Properties:

- Zero-noise $\quad \sigma^2_\eta = 0 \quad \Rightarrow \quad \hat{f}(x,y) = g(x,y)$
- On edges $\quad \sigma^2_\eta \ll \sigma^2_{S_{xy}} \quad \Rightarrow \quad \hat{f}(x,y) = g(x,y)$

Gaussian Noise + Adaptive Filter

a b
c d

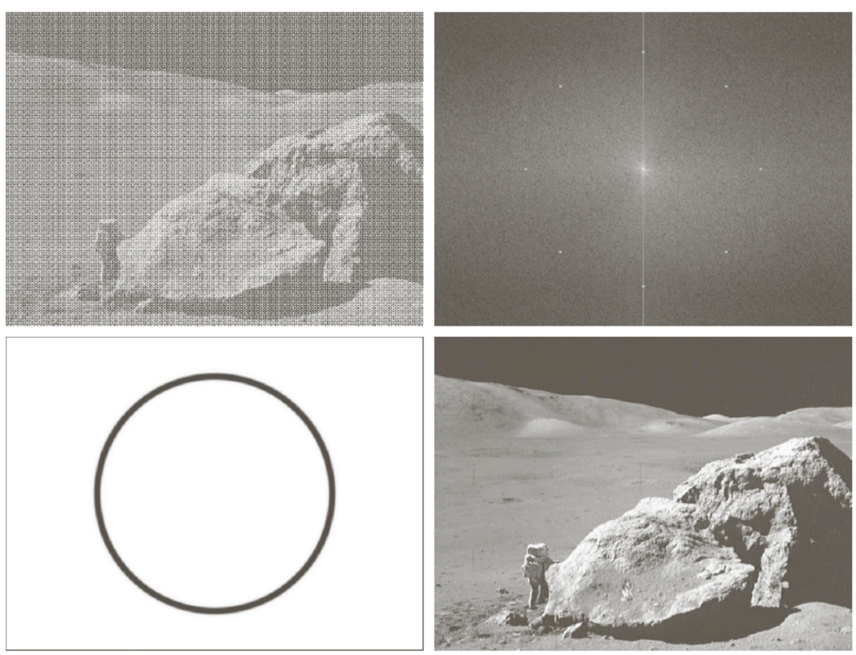
FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



Periodic Noise = 2D Sinusoid

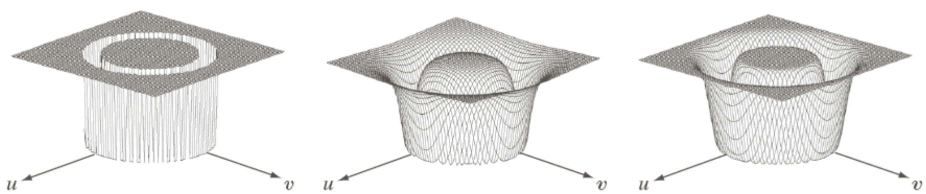
a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)



filtering in the frequency domain

Bandreject Filters in the Frequency Domain

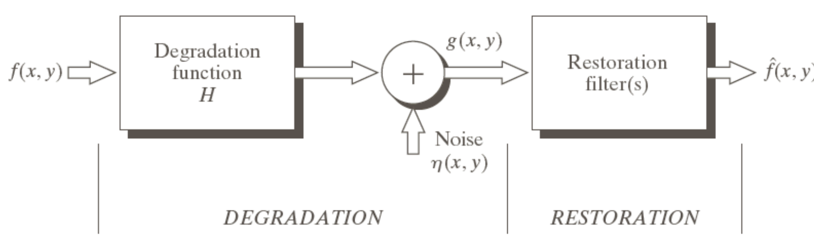


a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Degradation Modeling

FIGURE 5.1
A model of the image degradation/restoration process.



$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Modeling Degradation Due to Atmospheric Turbulence

$$H(u,v) = \mathrm{e}^{-k(u^2+v^2)^{5/6}}$$

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Modeling Uniform Linear Motion Blur

T - duration of the exposure

blurred output:

$$g(x,y) = \int_0^T f(x - x_0(t), y - y_0(t))dt$$

$$G(u,v) = F(u,v)\int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt$$

$$\Rightarrow \ H(u,v) = \int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt$$

Modeling Uniform Linear Motion Blur

$$x_0(t) = \frac{at}{T} \qquad y_0(t) = \frac{bt}{T}$$

$$\begin{aligned} H(u,v) &= \int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt \\ &= \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)]\mathrm{e}^{-j\pi(ua+ub)} \end{aligned}$$

Modeling Uniform Linear Motion Blur

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.