ECE 468: Digital Image Processing

Lecture 7

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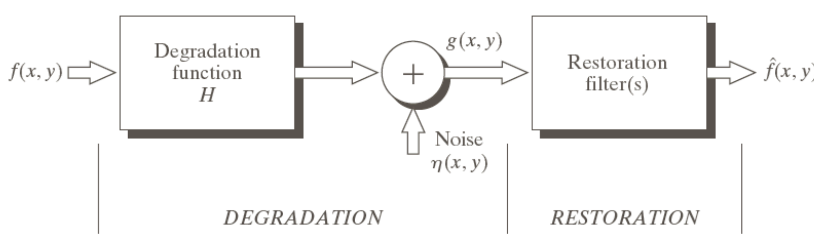
Model of Image Degradation/Restoration

FIGURE 5.1 A model of the image degradation/restoration process.



$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Image Restoration by Inverse Filtering

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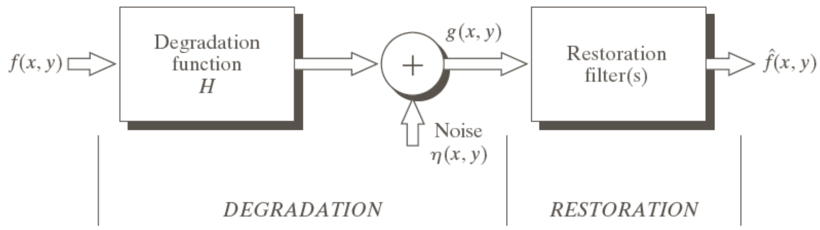

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)}$$

Image Restoration by Inverse Filtering

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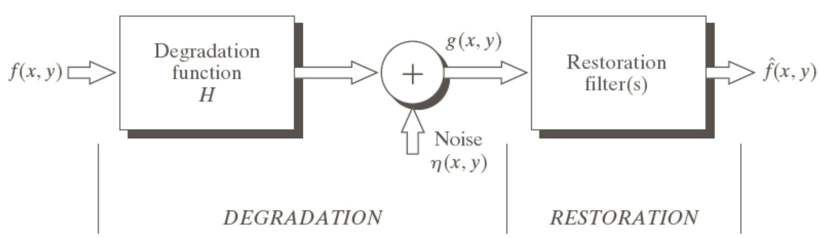

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Image Restoration by Inverse Filtering

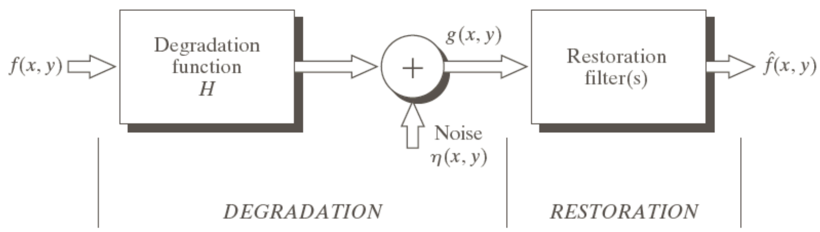

FIGURE 5.1 A model of the image degradation/restoration process.

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Inverse Filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Bad news:

- Even when H(u,v) is known, there is always unknown noise
- Often H(u,v) has values close to zero

Example: Inverse Filtering

Atmospheric turbulence effect

$$H(u,v) = \exp\left\{-k\left[(u-M/2)^2 + (v-N/2)^2\right]^{5/6}\right\}$$

Example: Inverse Filtering

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

$$\frac{G(u,v)}{H(u,v)}$$

Wiener Filtering = Mean Square Error Filtering

- Incorporates both:
  - Degradation function
  - Statistical characteristics of noise

- Assumption: noise and the image are uncorrelated

- Optimizes the filter so that MSE is minimized

$$e^2 = E\left\{[f(x,y) - \hat{f}(x,y)]^2\right\}$$

Wiener Filtering

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

degraded output input

$$\hat{F}(u,v) = H_W(u,v)G(u,v)$$

restored

$$H_W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}$$

Signal to Noise Ratio

$$H_W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}$$

Signal to Noise Ratio

$$H_W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}$$

inverse filter

Signal to Noise Ratio

$$H_W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}$$

inverse filter

unknown

Signal to Noise Ratio

$$H_W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}$$

inverse filter

unknown

$$\mathrm{SNR} = \frac{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |F(u,v)|^2}{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |N(u,v)|^2}$$

Approximation

$$H_W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}} \approx \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{1}{\text{SNR}}}$$

Example: Wiener Filtering

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Example: Wiener Filtering

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Midterm Exam Review

- Exam:
  - Average: 66.8
  - Median: 70.5
  - STD: 14.2
- Projected Total:
  - Average: 85.5
  - Median: 87.1
  - STD: 6.1