ECE 468: Digital Image Processing

Lecture 9

Prof. Sinisa Todorovic

sinisa@eecs.oregonstate.edu

OSU Oregon State University

Multiresolution Image Processing

- Informal motivation:
- Images may show both very large and very small objects.
- It may be useful to process the images at different resolutions.

Multiresolution Image Processing

- A more formal motivation:
- An image is a 2D random process with locally varying statistics of pixel intensities
- Analysis of statistical properties of pixel neighborhoods of varying sizes may be useful

Histogram of Small Pixel Neighborhoods

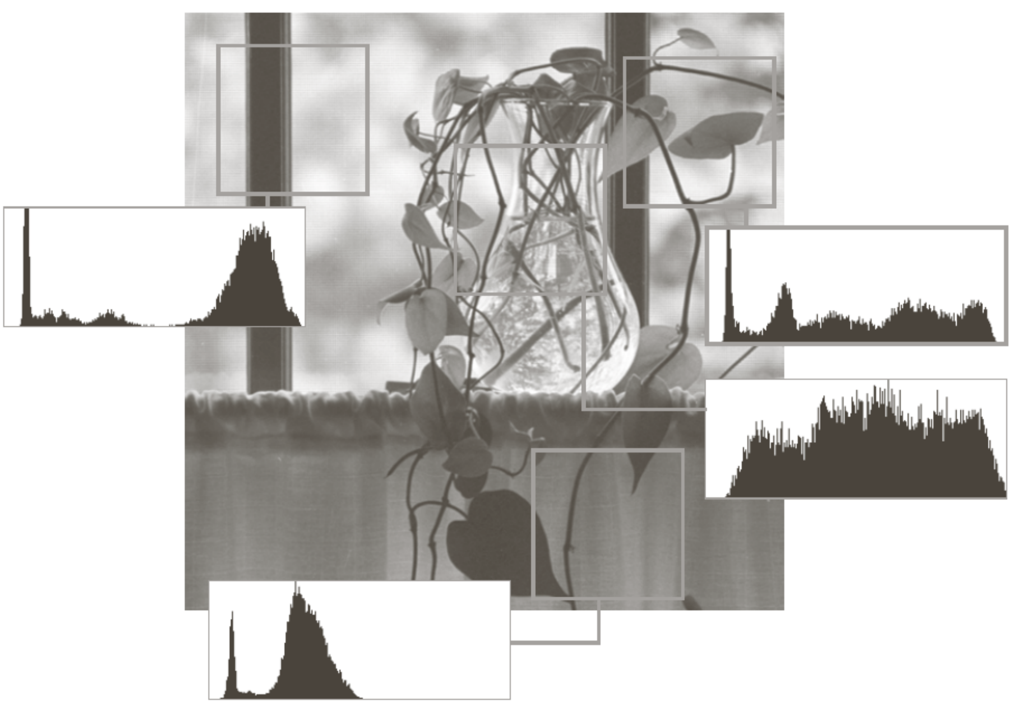
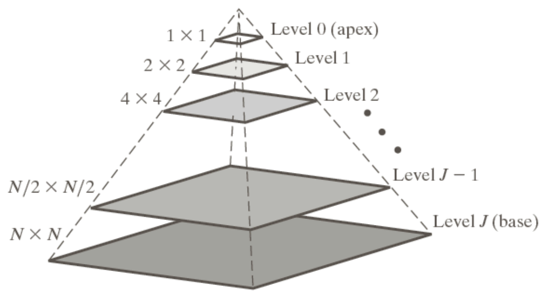
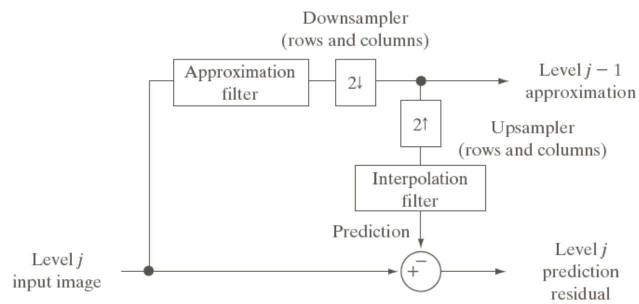


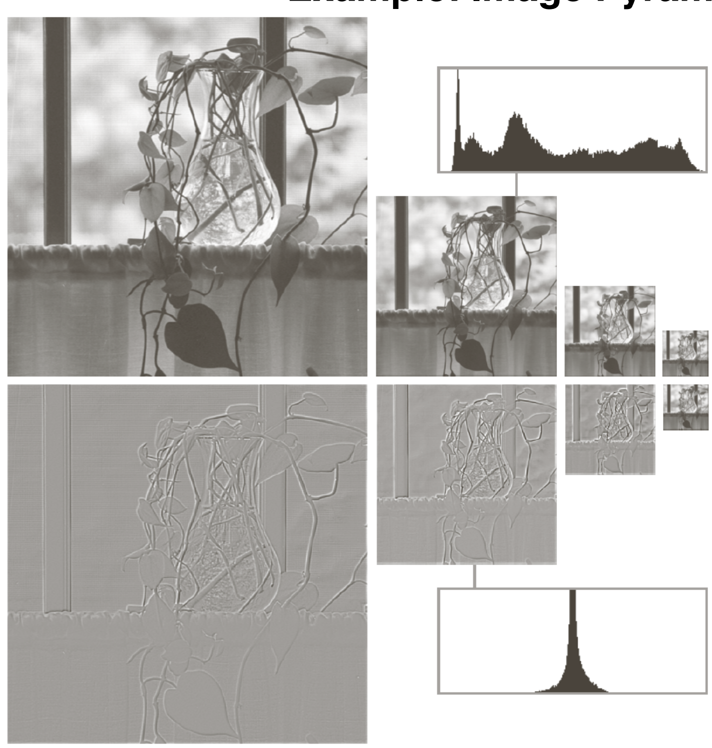
Image Pyramids

- A representation of the image that allows its multiresolution analysis





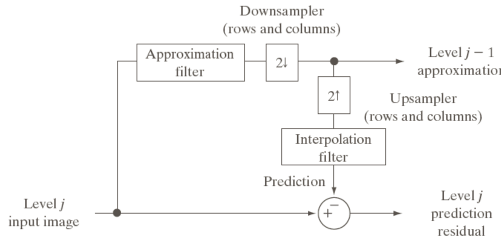
Example: Image Pyramids



a
b

FIGURE 7.3 Two image pyramids and their histograms: (a) an approximation pyramid; (b) a prediction residual pyramid.

Steps to Construct the Image Pyramid



1. Given an image at level j
2. Filter the input and and downsample the filtered result by a factor of 2; This gives the image at level j-1
3. Goto 1
4. Upsample and filter the image at level j-1; this gives an approximation of the image at level j
5. Subtract this result from the image at level j; this give the prediction residual at level j
6. Goto 1

Typical Filters

- For the multiresolution pyramid, we use spatial filters:
  - Neighborhood averaging
  - Lowpass Gaussian filter
- For the residual pyramid, we use interpolation filters:
  - bilinear
  - bicubic

Upsampling/Downsampling

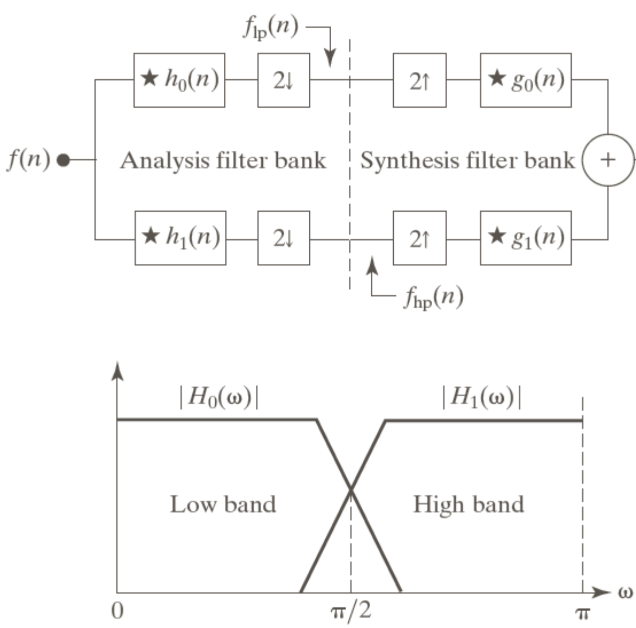
- Upsampling = Inserting zeros

$$f_{2\uparrow}(x,y) = \begin{cases} f(x/2, y/2) & , \quad x, y \text{ are even} \\ 0 & , \quad \text{o.w.} \end{cases}$$

- Downsampling = Discarding pixels

$$f_{2\downarrow}(x,y) = f(2x, 2y)$$

Subband Image Coding



a
b

FIGURE 7.6 (a) A two-band subband coding and decoding system, and (b) its spectrum splitting properties.

Example: Analysis Filter Bank

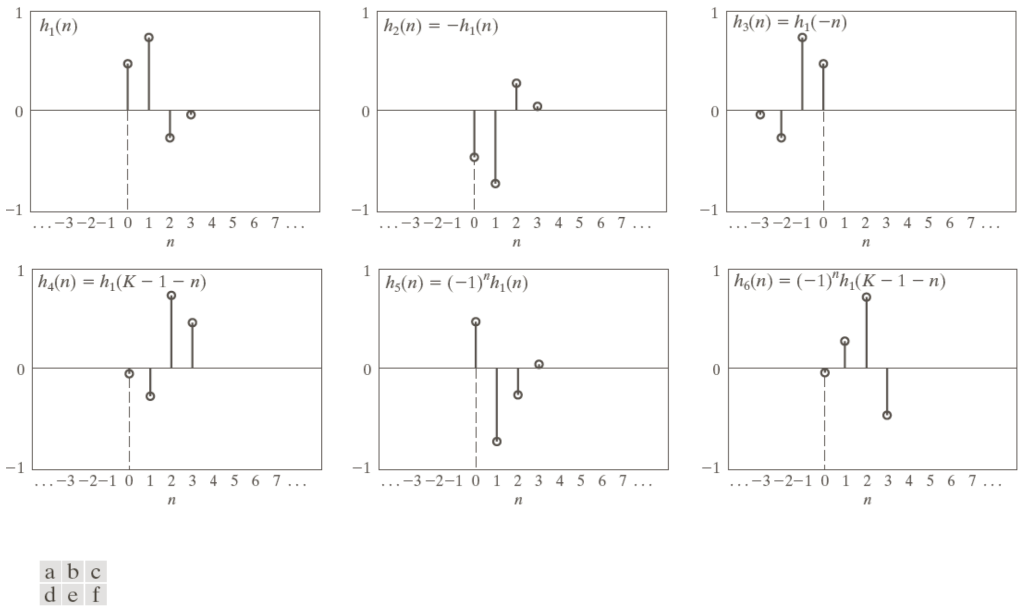
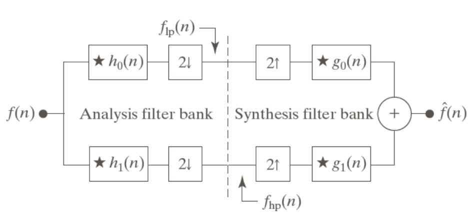
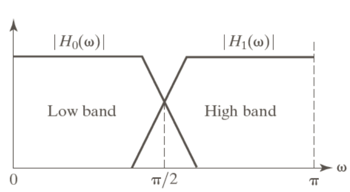


FIGURE 7.5 Six functionally related filter impulse responses: (a) reference response; (b) sign reversal; (c) and (d) order reversal (differing by the delay introduced); (e) modulation; and (f) order reversal and modulation.

Subband Image Coding





for perfect reconstruction:

$$f(n) = \hat{f}(n)$$

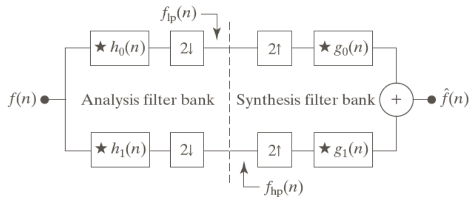
$$\Downarrow$$

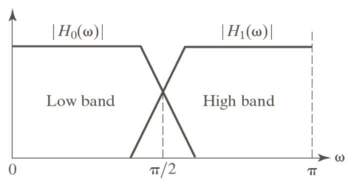
$$g_0(n) = (-1)^n h_1(n)$$

$$g_1(n) = (-1)^{n+1} h_0(n)$$

h_0, h_1, g_0, g_1 -- cross-modulated filters

Subband Image Coding



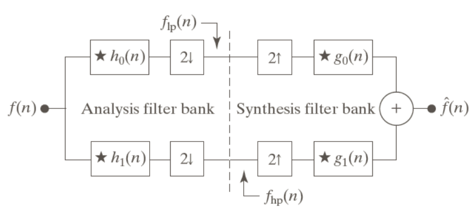


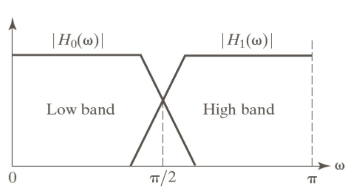
for perfect reconstruction:

$$g_0(n) = (-1)^n h_1(n)$$

$$g_1(n) = (-1)^{n+1} h_0(n)$$

Subband Image Coding





for perfect reconstruction:

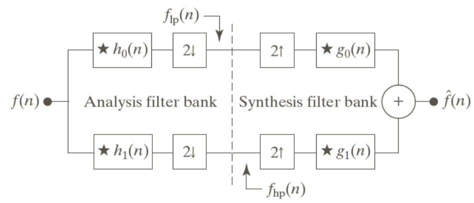
$$g_0(n) = (-1)^n h_1(n)$$

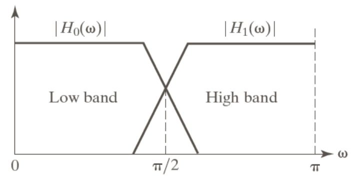
$$g_1(n) = (-1)^{n+1} h_0(n)$$

$$\hat{f}_{lp}(n) = \begin{cases} f(2n) \star h_0(2n) & , \quad 2n \\ 0 & , \quad 2n+1 \end{cases}$$

$$\hat{f}_{hp}(n) = \begin{cases} f(2n+1) \star h_1(2n+1) & , \quad 2n+1 \\ 0 & , \quad 2n \end{cases}$$

Subband Image Coding





for perfect reconstruction:

$$g_0(n) = (-1)^n h_1(n)$$

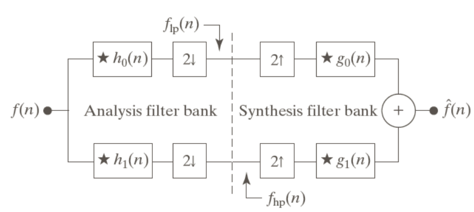
$$g_1(n) = (-1)^{n+1} h_0(n)$$

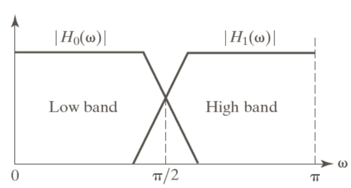
$$\hat{f}_{lp}(n) = \begin{cases} f(2n) \star h_0(2n) & , \quad 2n \\ 0 & , \quad 2n+1 \end{cases}$$

$$\hat{f}_{hp}(n) = \begin{cases} f(2n+1) \star h_1(2n+1) & , \quad 2n+1 \\ 0 & , \quad 2n \end{cases}$$

$$\hat{f} = f(2n) \star h_0(2n) \star g_0(2n) + f(2n+1) \star h_1(2n+1) \star g_1(2n+1)$$

Subband Image Coding





for perfect reconstruction:

$$g_0(n) = (-1)^n h_1(n)$$

$$g_1(n) = (-1)^{n+1} h_0(n)$$

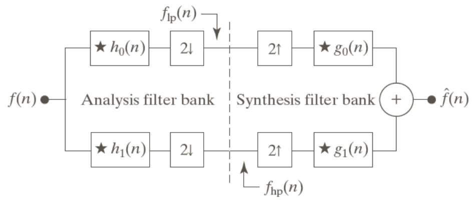
$$\hat{f}_{lp}(n) = \begin{cases} f(2n) \star h_0(2n) & , \quad 2n \\ 0 & , \quad 2n+1 \end{cases}$$

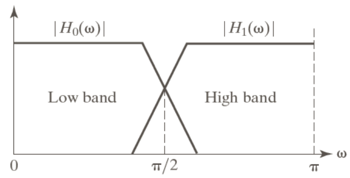
$$\hat{f}_{hp}(n) = \begin{cases} f(2n+1) \star h_1(2n+1) & , \quad 2n+1 \\ 0 & , \quad 2n \end{cases}$$

$$\hat{f} = f(2n) \star h_0(2n) \star g_0(2n) + f(2n+1) \star h_1(2n+1) \star g_1(2n+1)$$

$$\hat{f} = f(2n) \star h_0(2n) \star h_1(2n) + f(2n+1) \star h_1(2n+1) \star h_0(2n+1)$$

Subband Image Coding





for perfect reconstruction:

$$g_0(n) = (-1)^n h_1(n)$$

$$g_1(n) = (-1)^{n+1} h_0(n)$$

$$\hat{f}_{lp}(n) = \begin{cases} f(2n) \star h_0(2n) & , \quad 2n \\ 0 & , \quad 2n+1 \end{cases}$$

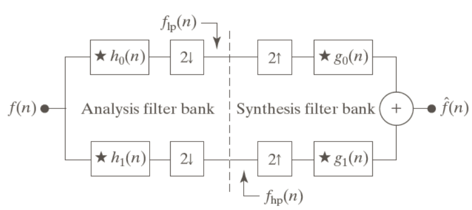
$$\hat{f}_{hp}(n) = \begin{cases} f(2n+1) \star h_1(2n+1) & , \quad 2n+1 \\ 0 & , \quad 2n \end{cases}$$

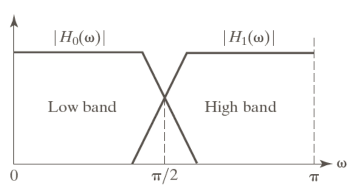
$$\hat{f} = f(2n) \star h_0(2n) \star g_0(2n) + f(2n+1) \star h_1(2n+1) \star g_1(2n+1)$$

$$\hat{f} = f(2n) \star h_0(2n) \star h_1(2n) + f(2n+1) \star h_1(2n+1) \star h_0(2n+1)$$

$$\hat{f} = f(n) \star [h_0(n) \star h_1(n)]$$

Subband Image Coding





for perfect reconstruction:

$$g_0(n) = (-1)^n h_1(n)$$

$$g_1(n) = (-1)^{n+1} h_0(n)$$

$$\hat{f}_{lp}(n) = \begin{cases} f(2n) \star h_0(2n) & , \quad 2n \\ 0 & , \quad 2n+1 \end{cases}$$

$$\hat{f}_{hp}(n) = \begin{cases} f(2n+1) \star h_1(2n+1) & , \quad 2n+1 \\ 0 & , \quad 2n \end{cases}$$

$$\hat{f} = f(2n) \star h_0(2n) \star g_0(2n) + f(2n+1) \star h_1(2n+1) \star g_1(2n+1)$$

$$\hat{f} = f(2n) \star h_0(2n) \star h_1(2n) + f(2n+1) \star h_1(2n+1) \star h_0(2n+1)$$

$$\hat{f} = f(n) \star [h_0(n) \star h_1(n)]$$

$$\hat{f} = f(n)$$

Vector Inner Product

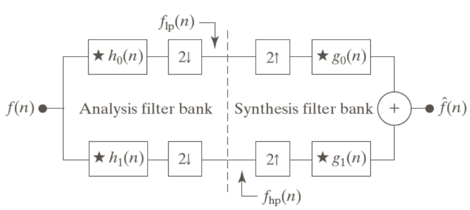
Given sequences

$$f_1(n), \quad f_2(n)$$

$$\Downarrow$$

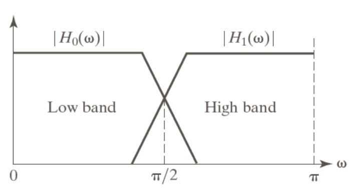
$$\langle f_1, f_2 \rangle = \sum_n f_1^*(n) f_2(n)$$

Subband Image Coding



$$g_0(n) = (-1)^n h_1(n)$$

$$g_1(n) = (-1)^{n+1} h_0(n)$$

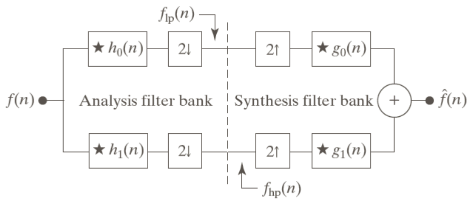


h_0, h_1, g_0, g_1 are biorthogonal

$$\Downarrow$$

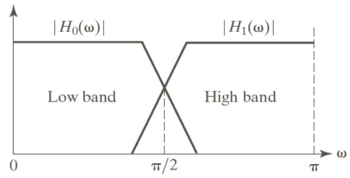
$$\langle h_i(2n-k), g_j(k) \rangle = \delta(i-j)\delta(n), \quad i,j = \{0,1\}$$

Subband Image Coding



$$g_0(n) = (-1)^n h_1(n)$$

$$g_1(n) = (-1)^{n+1} h_0(n)$$



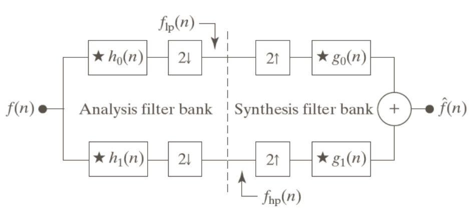
h_0, h_1, g_0, g_1 are biorthogonal

$$\Downarrow$$

$$\langle h_i(2n-k), g_j(k) \rangle = \delta(i-j)\delta(n), \quad i,j = \{0,1\}$$

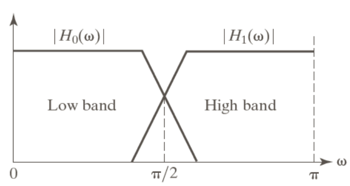
Example: $\langle h_0(2n-k), g_1(k) \rangle = 0$

Subband Image Coding



$$g_0(n) = (-1)^n h_1(n)$$

$$g_1(n) = (-1)^{n+1} h_0(n)$$



h_0, h_1, g_0, g_1 are orthonormal

$$\Downarrow$$

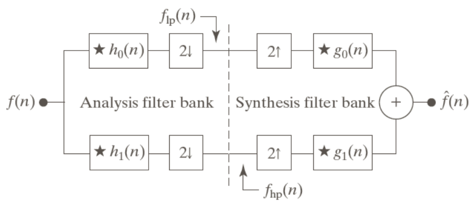
$$\langle h_i(2n-k), g_j(k) \rangle = \delta(i-j)\delta(n), \quad i,j = \{0,1\}$$

+

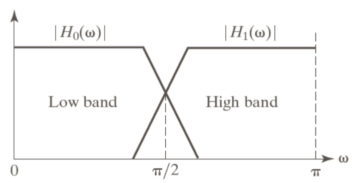
$$\langle g_i(n), g_j(n+2m) \rangle = \delta(i-j)\delta(m), \quad i,j = \{0,1\}$$

Orthonormal Filter Bank

K_{even} is the number of filter coefficients that must be even



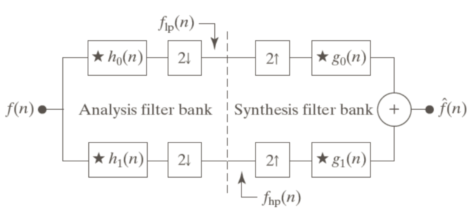
$$g_1(n) = (-1)^n g_0(K_{\text{even}} - 1 - n)$$



$$h_i(n) = g_i(K_{\text{even}} - 1 - n), \quad i = \{0, 1\}$$

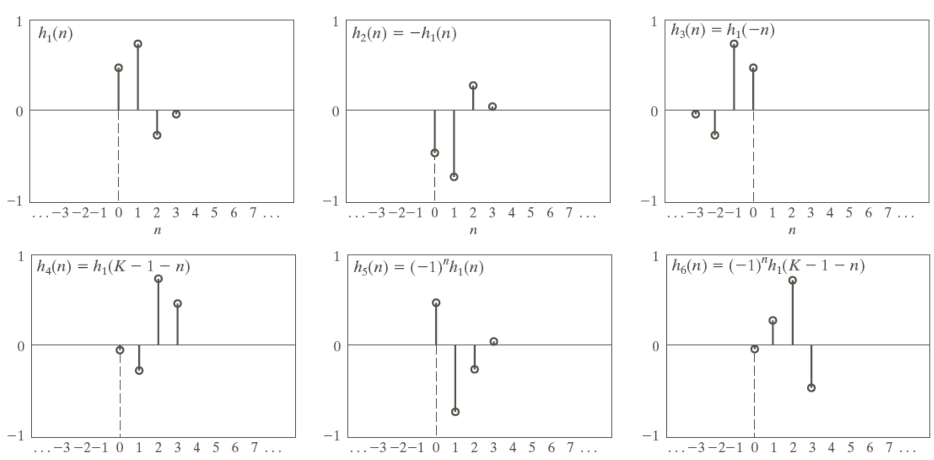
Orthonormal filter bank can be obtained from a single filter -- prototype

Example: Orthonormal Filter Bank

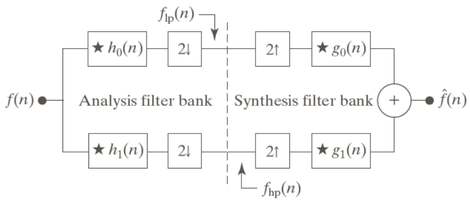


$$g_1(n) = (-1)^n g_0(K_{\text{even}} - 1 - n)$$

$$h_i(n) = g_i(K_{\text{even}} - 1 - n), \quad i = \{0, 1\}$$

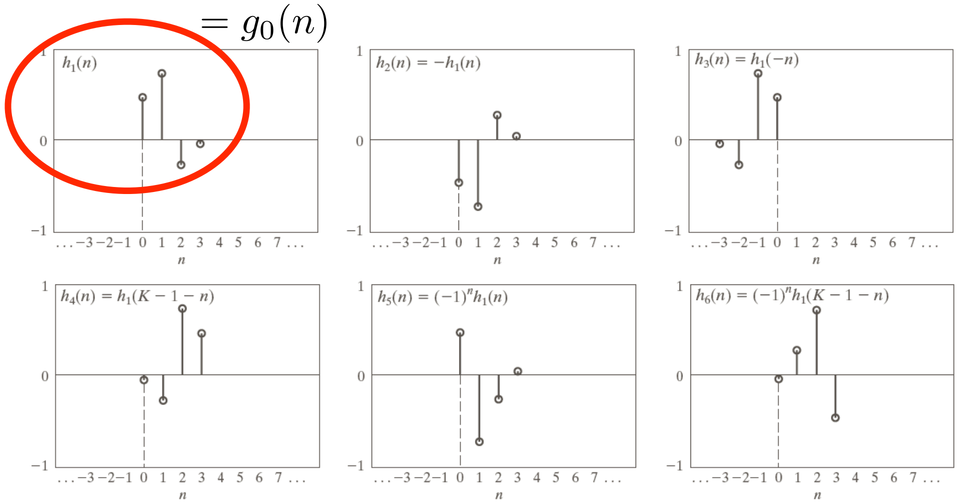


Example: Orthonormal Filter Bank

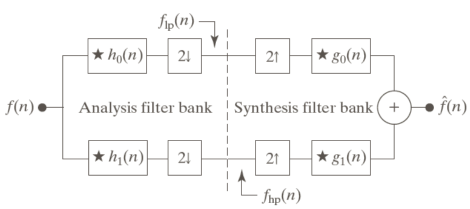


$$g_1(n) = (-1)^n g_0(K_{\text{even}} - 1 - n)$$

$$h_i(n) = g_i(K_{\text{even}} - 1 - n), \quad i = \{0, 1\}$$

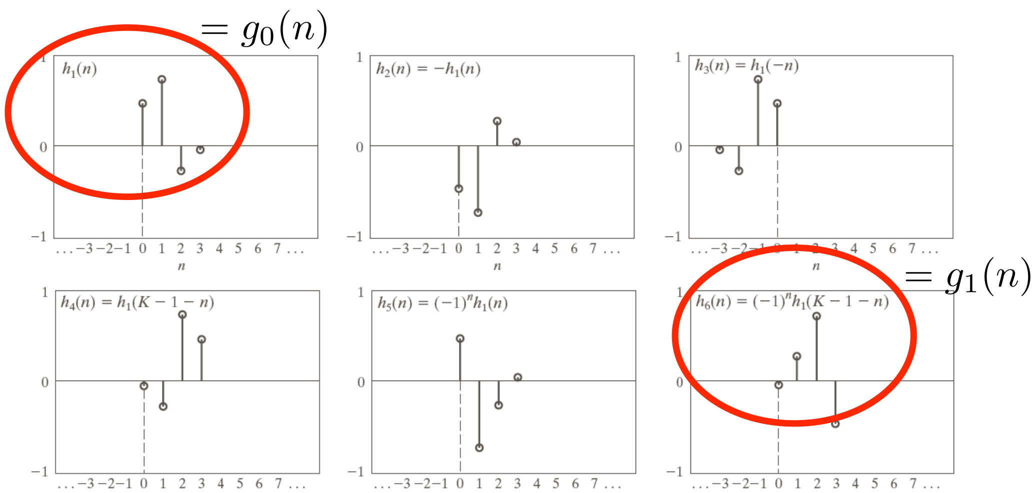


Example: Orthonormal Filter Bank

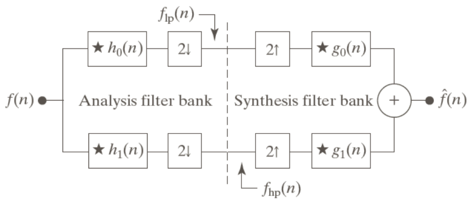


$$g_1(n) = (-1)^n g_0(K_{\text{even}} - 1 - n)$$

$$h_i(n) = g_i(K_{\text{even}} - 1 - n), \quad i = \{0, 1\}$$

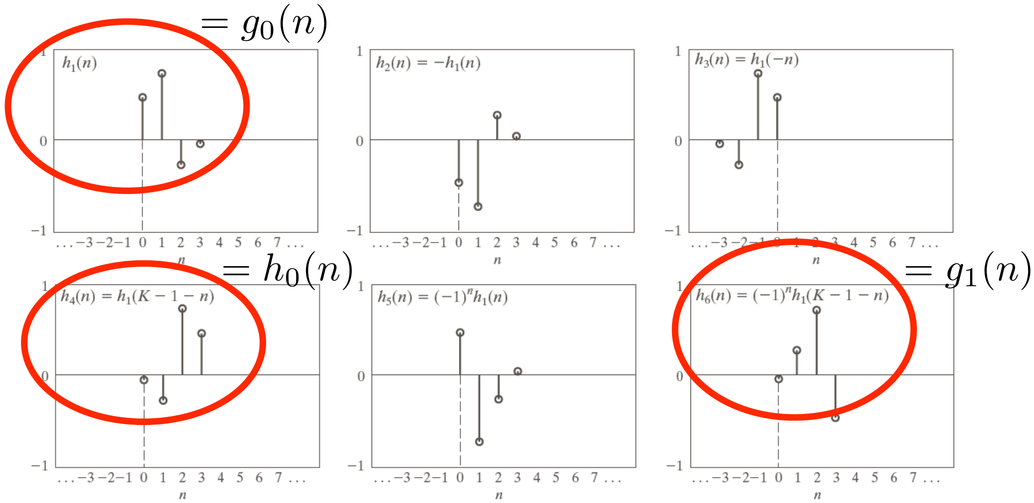


Example: Orthonormal Filter Bank

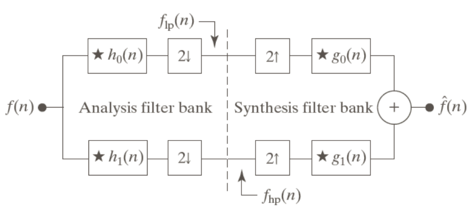


$$g_1(n) = (-1)^n g_0(K_{\text{even}} - 1 - n)$$

$$h_i(n) = g_i(K_{\text{even}} - 1 - n), \quad i = \{0, 1\}$$



Example: Orthonormal Filter Bank



$$g_1(n) = (-1)^n g_0(K_{\text{even}} - 1 - n)$$

$$h_i(n) = g_i(K_{\text{even}} - 1 - n), \quad i = \{0, 1\}$$