ECE 468: Digital Image Processing

Lecture 13

Prof. Sinisa Todorovic

sinisa@eecs.oregonstate.edu

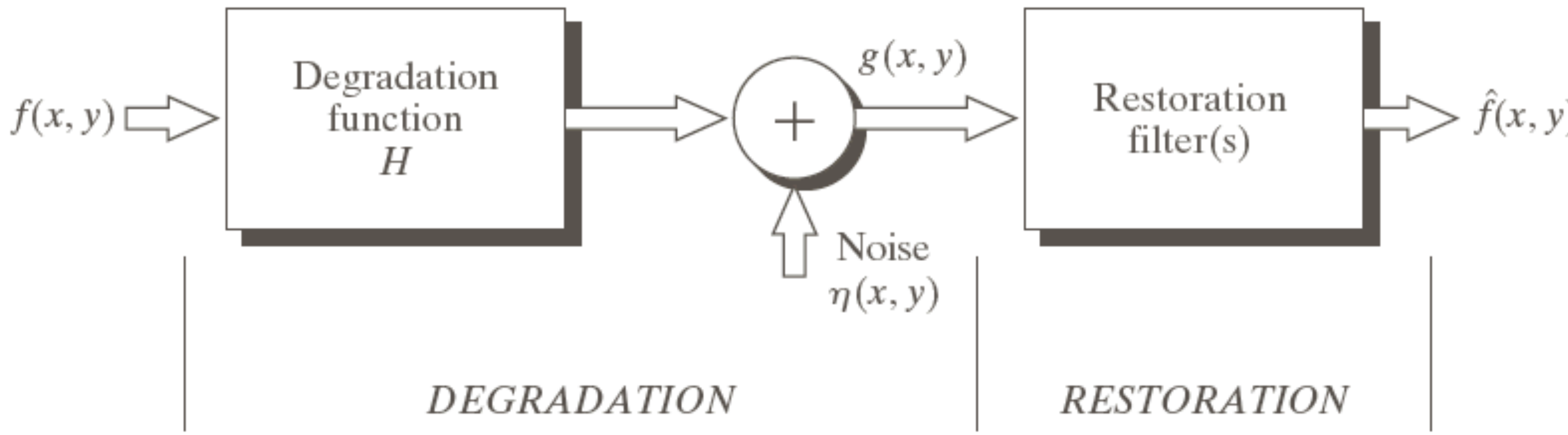
Oregon State University

Outline

- Image Restoration by Filtering (Textbook 5.3)

Image Restoration in the Frequency Domain

FIGURE 5.1
A model of the image degradation/restoration process.



$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

$$\hat{F}(u, v) = G(u, v)H_R(u, v)$$

Review

X random variable $\quad$ c deterministic constant

Expected value

$$E[c + X] = c + E[X]$$

$$E[cX] = cE[X]$$

Variance

$$\text{Var}[cX] = c^2\text{Var}[X]$$

$$\text{Var}[c + X] = \text{Var}[X]$$

Review

corrupted image $$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

Expected value

$$E[g(x,y)] = f(x,y) * h(x,y) + E[\eta(x,y)]$$

Variance

$$\mathrm{Var}[g(x,y)] = E[g(x,y)^2] - E^2[g(x,y)]$$

$$\mathrm{Var}[g(x,y)] = \mathrm{Var}[\eta(x,y)]$$

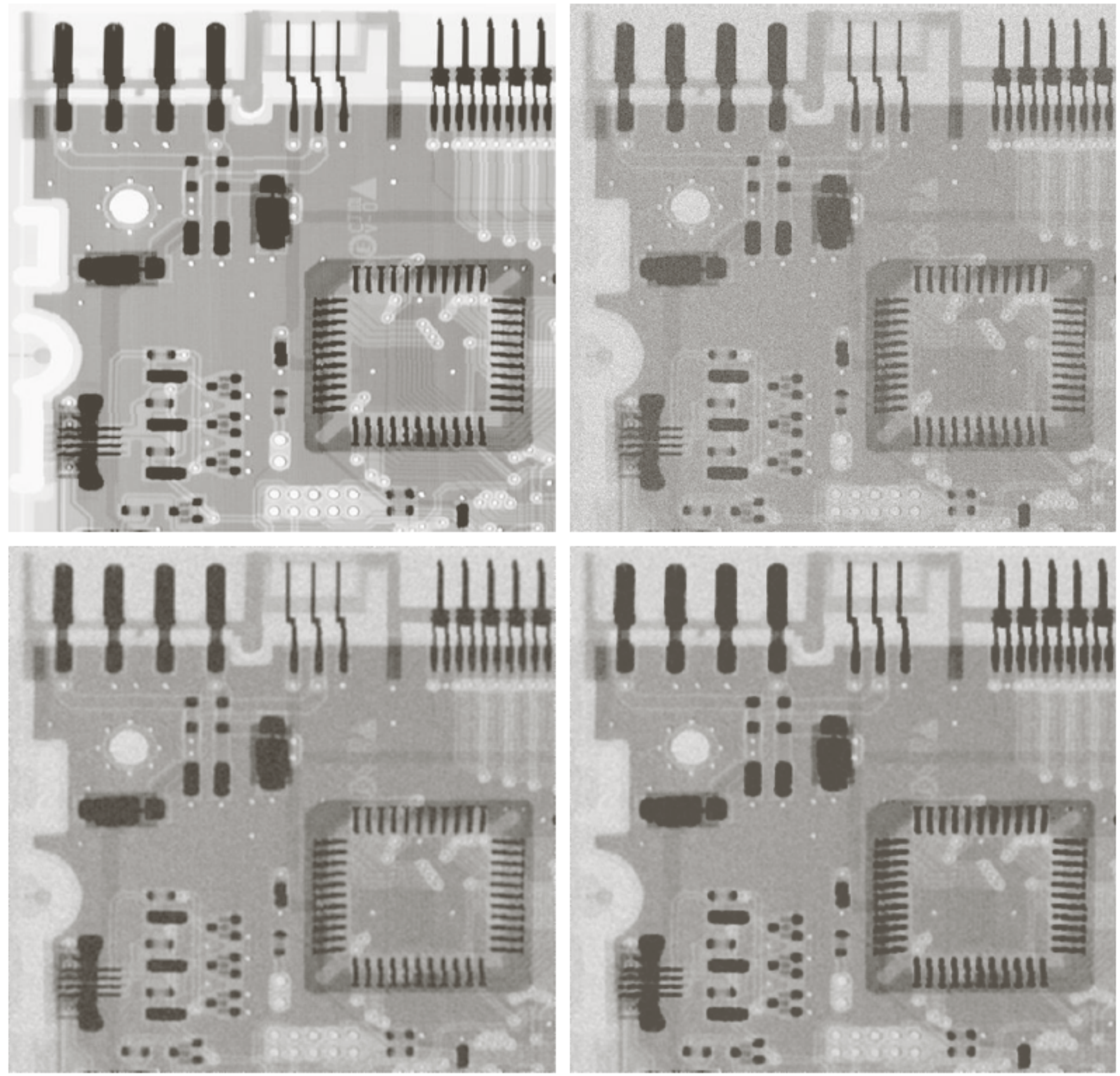
Gaussian Noise + Arithmetic vs. Geometric Mean Filter

$$S_{xy}$$

filter window

$$g(x,y) = f(x,y) + \eta(x,y)$$

output input



arithmetic mean geometric mean

Gaussian Noise + Arithmetic vs. Geometric Mean Filter

$$S_{xy}$$

filter window

$$g(x,y) = f(x,y) + \eta(x,y)$$

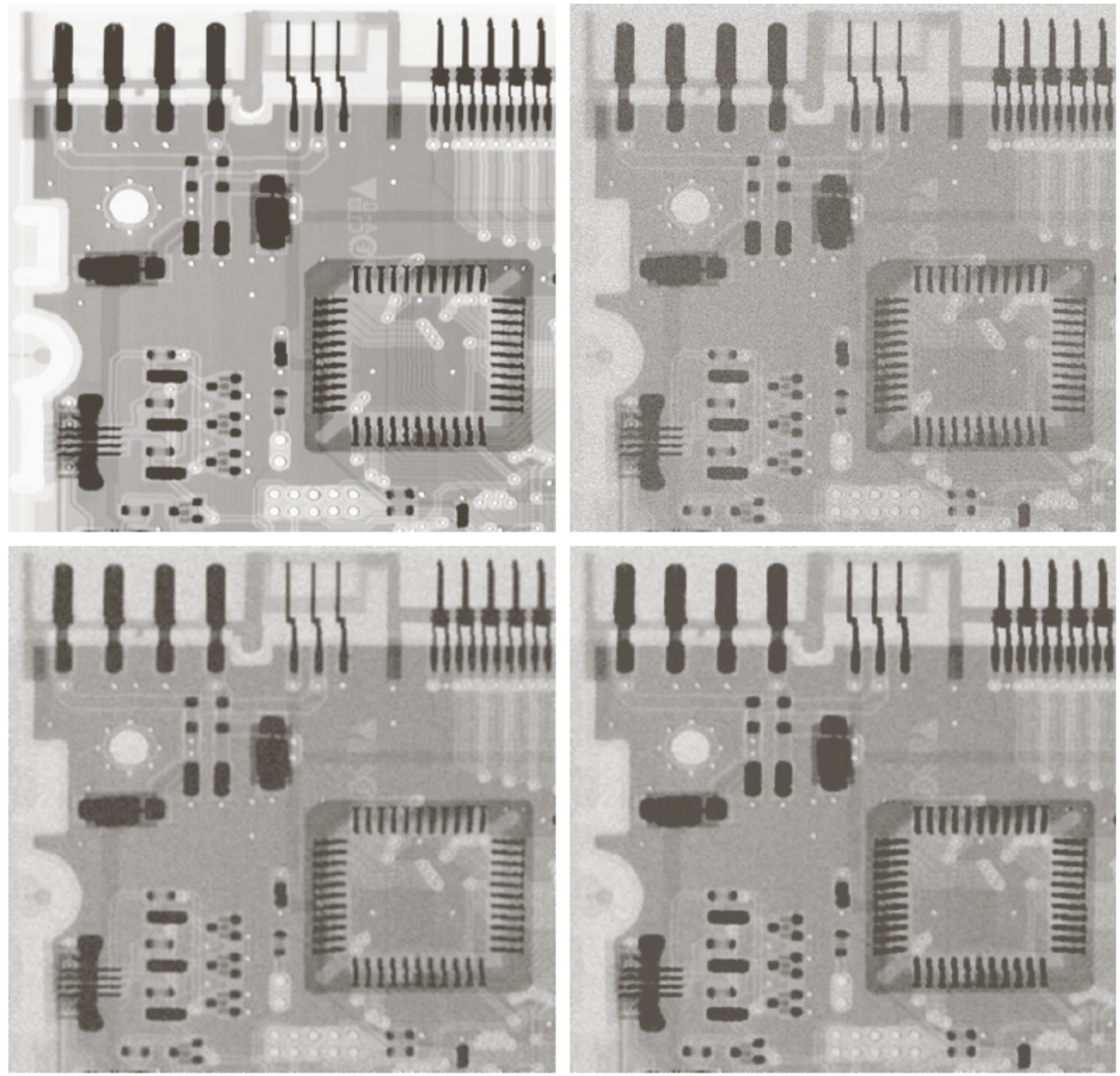
output input

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

arithmetic mean filtering

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

geometric mean filtering



arithmetic mean geometric mean

Salt-and-Pepper Noise + Median Filter

$$S_{xy}$$

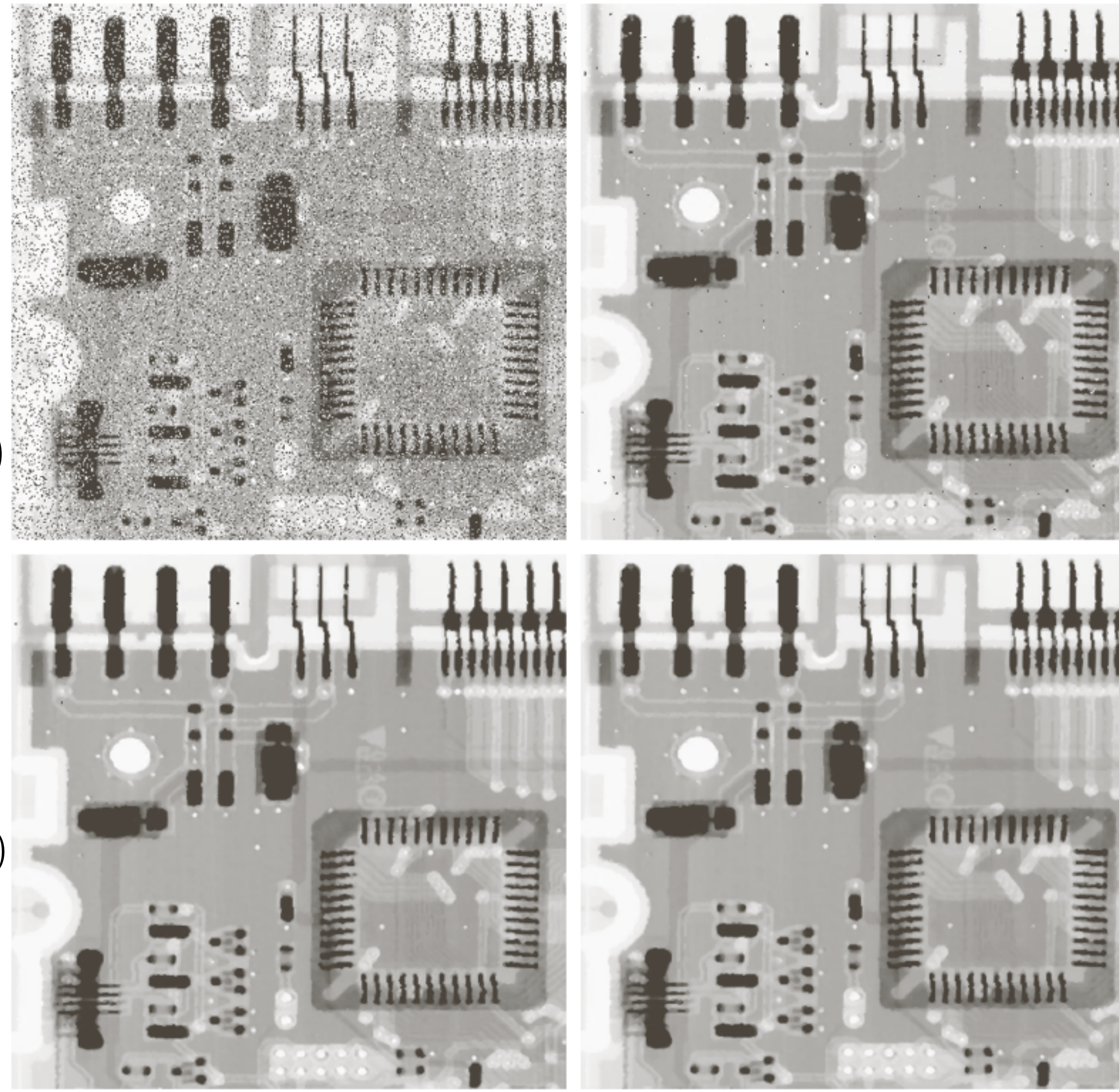
filter window

$$g(x,y) = f(x,y) + \eta(x,y)$$

output input

$$\hat{f}(x,y) = \text{median}_{(s,t)\in S_{xy}} \; g(s,t)$$

median filtering



repeated application of median filter

Adaptive Filter

arithmetic mean

$$m_{S_{xy}} = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

arithmetic variance

$$\sigma^2_{S_{xy}} = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} (g(s,t) - m_{S_{xy}})^2$$

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma^2_\eta}{\sigma^2_{S_{xy}}} \left[g(x,y) - m_{S_{xy}} \right]$$

output of the filter

Gaussian Noise + Adaptive Filter

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_{S_{xy}}^2}\left[g(x,y) - m_{S_{xy}}\right]$$

Properties:

- Zero-noise $\sigma_\eta^2 = 0 \quad \Rightarrow \quad \hat{f}(x,y) = g(x,y)$

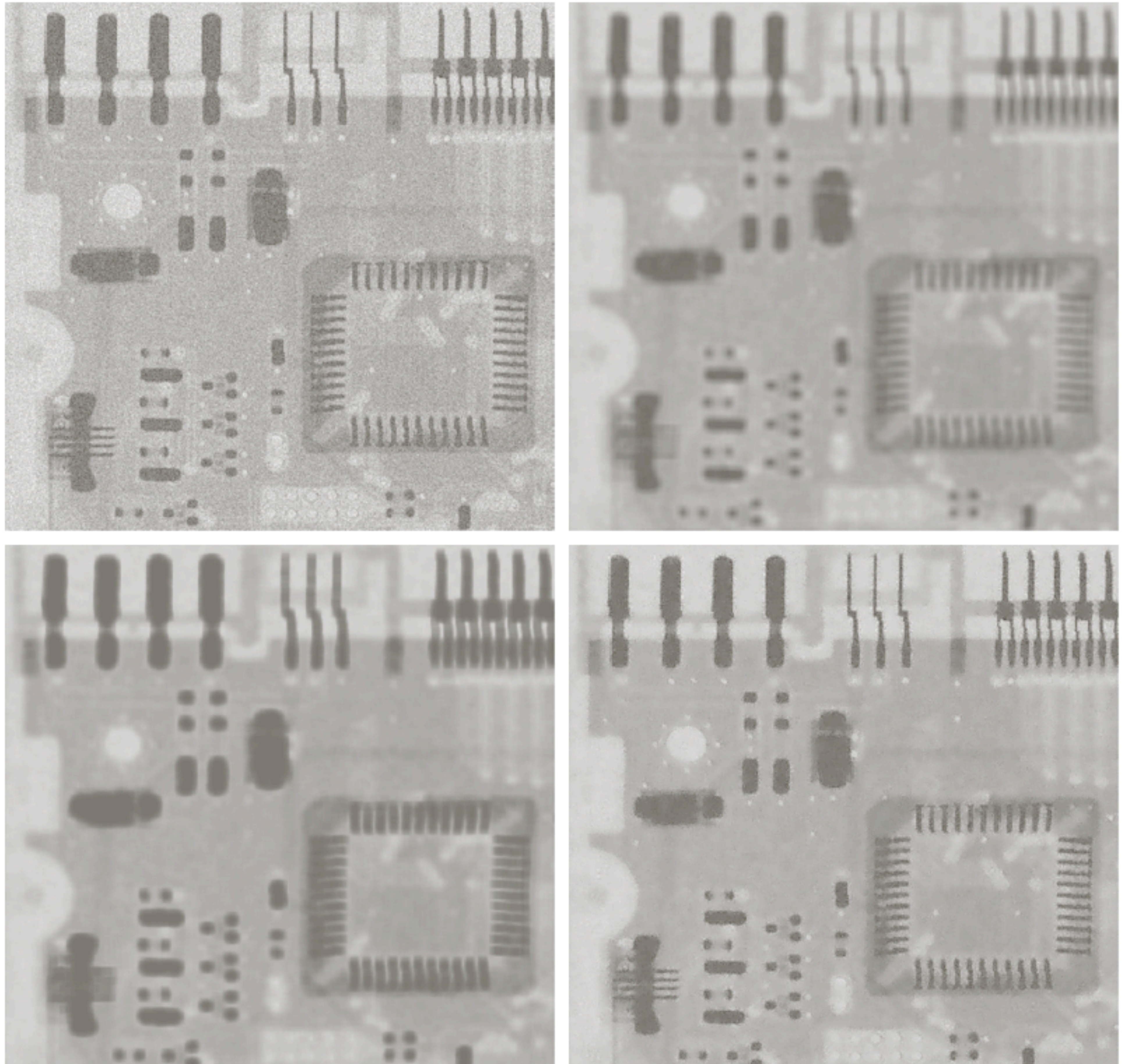
- On edges $\sigma_\eta^2 \ll \sigma_{S_{xy}}^2 \Rightarrow \quad \hat{f}(x,y) = g(x,y)$

Adaptive Filter

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_{S_{xy}}^2}\left[g(x,y) - m_{S_{xy}}\right]$$

$$= g(x,y) * \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{\sigma_\eta^2}{\sigma_{S_{xy}}^2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{mn} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)$$

Gaussian Noise + Adaptive Filter



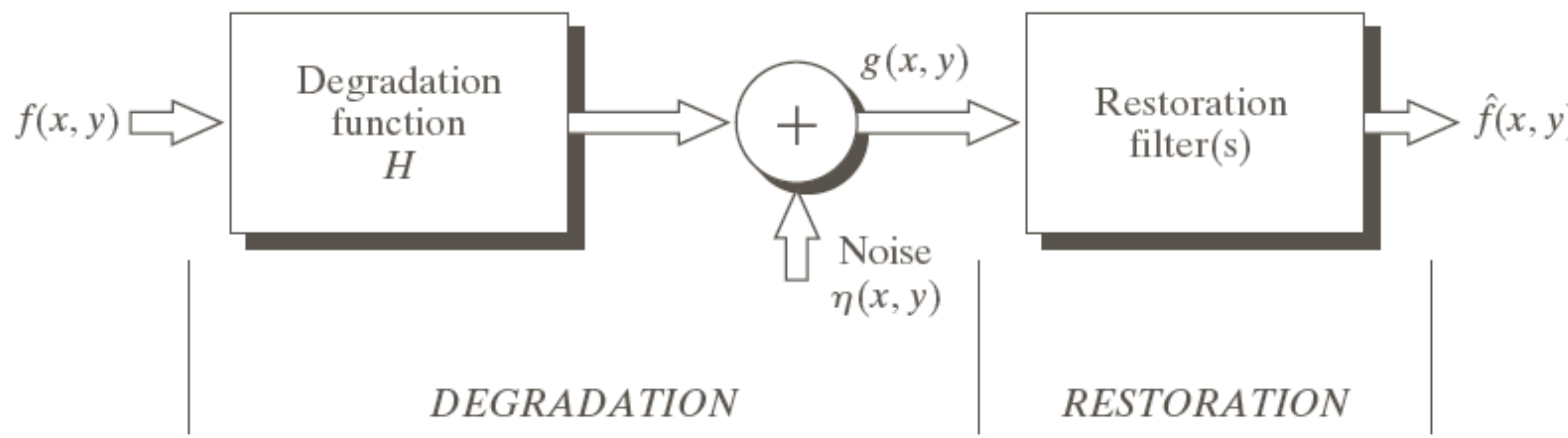
a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

Degradation Modeling

Degradation Modeling

FIGURE 5.1 A model of the image degradation/restoration process.

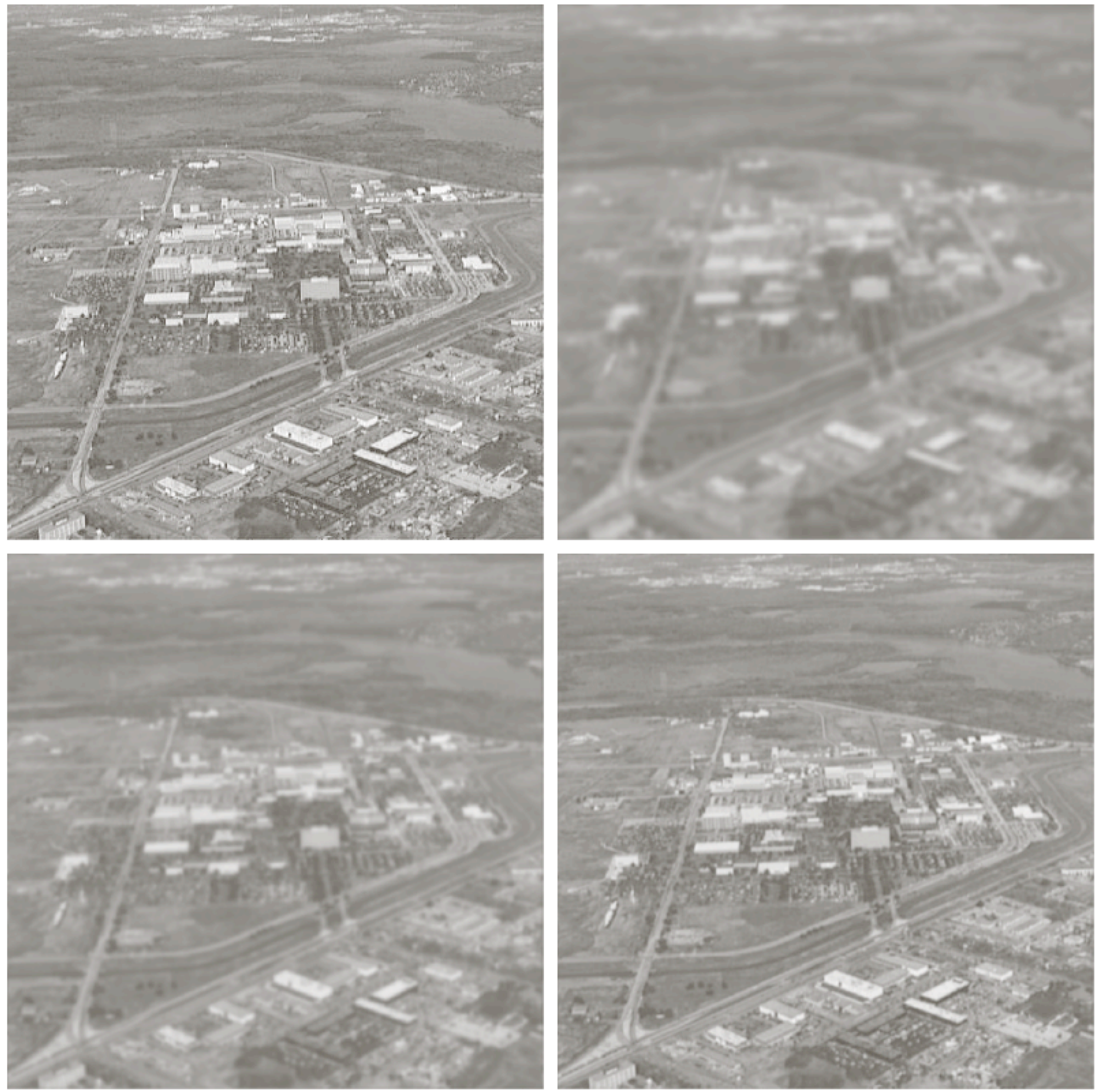


$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Modeling Degradation Due to Atmospheric Turbulence

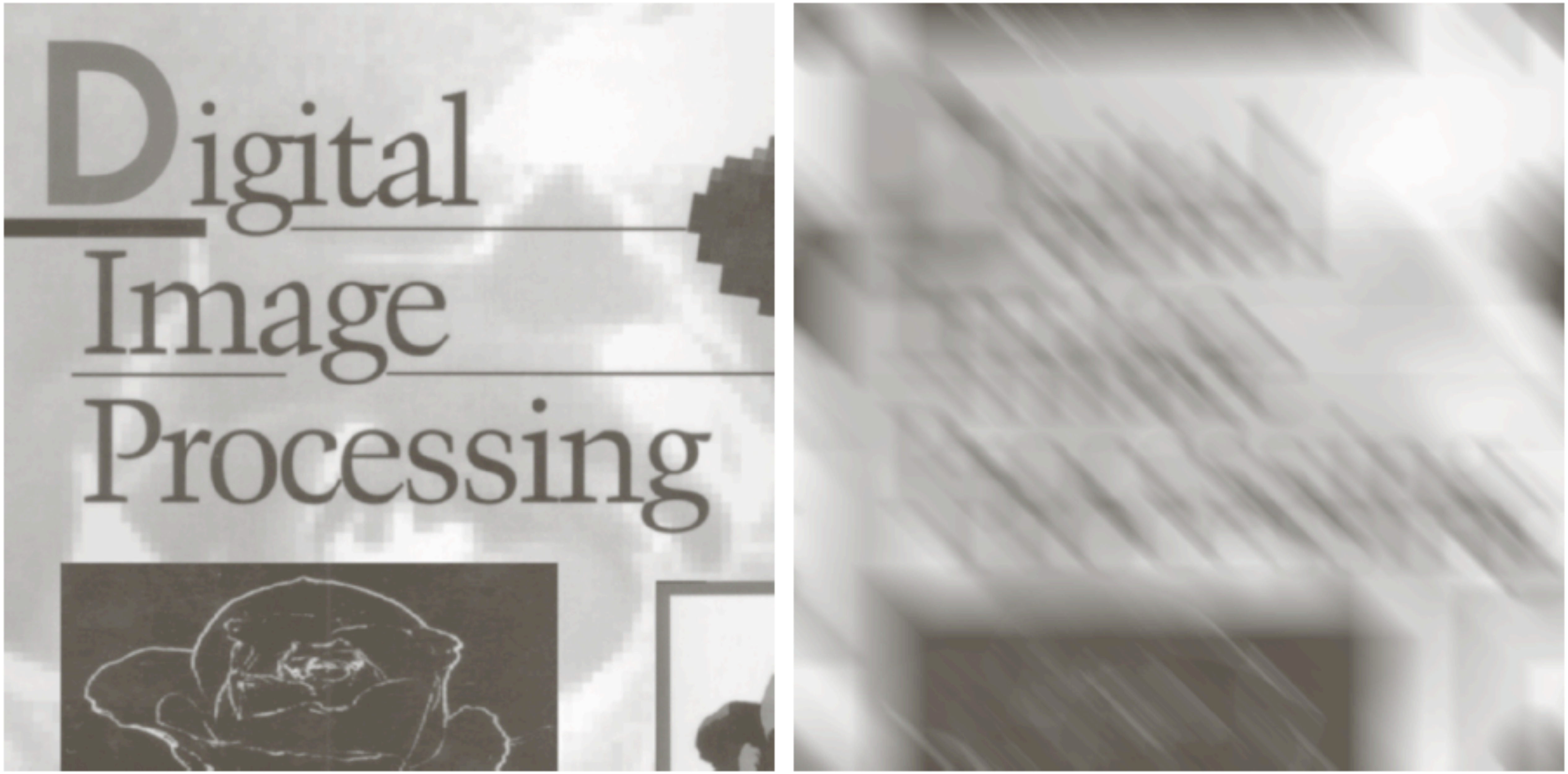
$$H(u, v) = e^{-k(u^2+v^2)^{5/6}}$$



a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Modeling Uniform Linear Motion Blur



a b

FIGURE 5.26
(a) Original image.
(b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Modeling Uniform Linear Motion Blur

T - duration of the exposure

blurred output:

$$g(x, y) = \int_0^T f(x - x_0(t), y - y_0(t))dt$$

Modeling Uniform Linear Motion Blur

T - duration of the exposure

blurred output:

$$g(x,y) = \int_0^T f(x - x_0(t), y - y_0(t))dt$$

$$G(u,v) = F(u,v) \int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt$$

Modeling Uniform Linear Motion Blur

T - duration of the exposure

blurred output:

$$g(x, y) = \int_0^T f(x - x_0(t), y - y_0(t))dt$$

$$G(u, v) = F(u, v) \int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt$$

$$\Rightarrow \; H(u, v) = \int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt$$

Modeling Uniform Linear Motion Blur

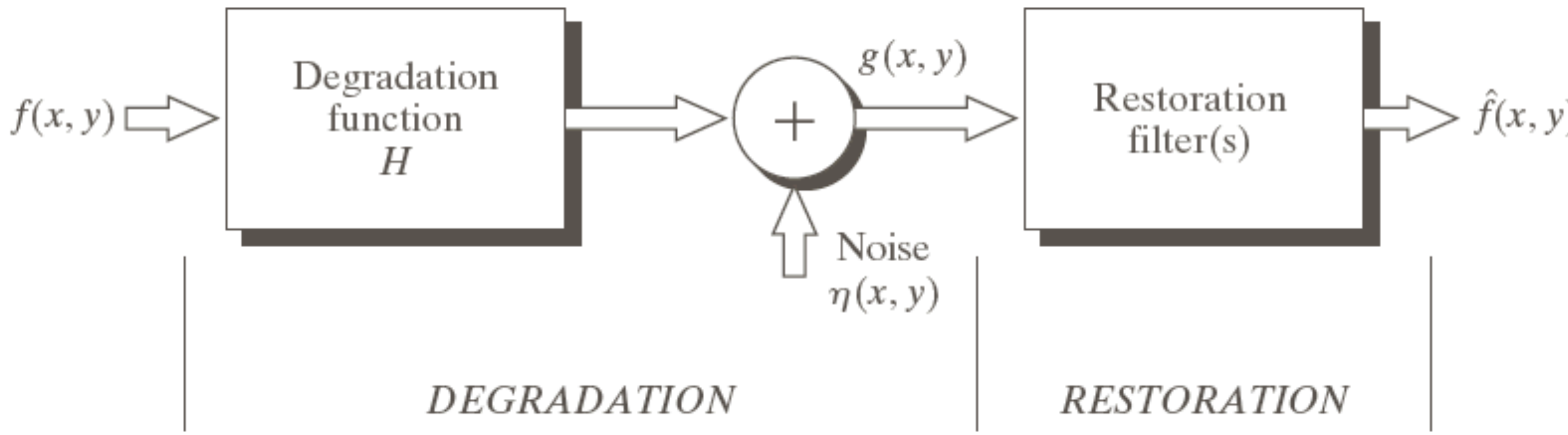
$$x_0(t) = \frac{at}{T} \qquad y_0(t) = \frac{bt}{T}$$

$$\begin{aligned} H(u,v) &= \int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt \\ &= \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)]\mathrm{e}^{-j\pi(ua+ub)} \end{aligned}$$

Image Restoration

Image Restoration by Inverse Filtering

FIGURE 5.1
A model of the image degradation/ restoration process.



$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)}$$

Image Restoration by Inverse Filtering

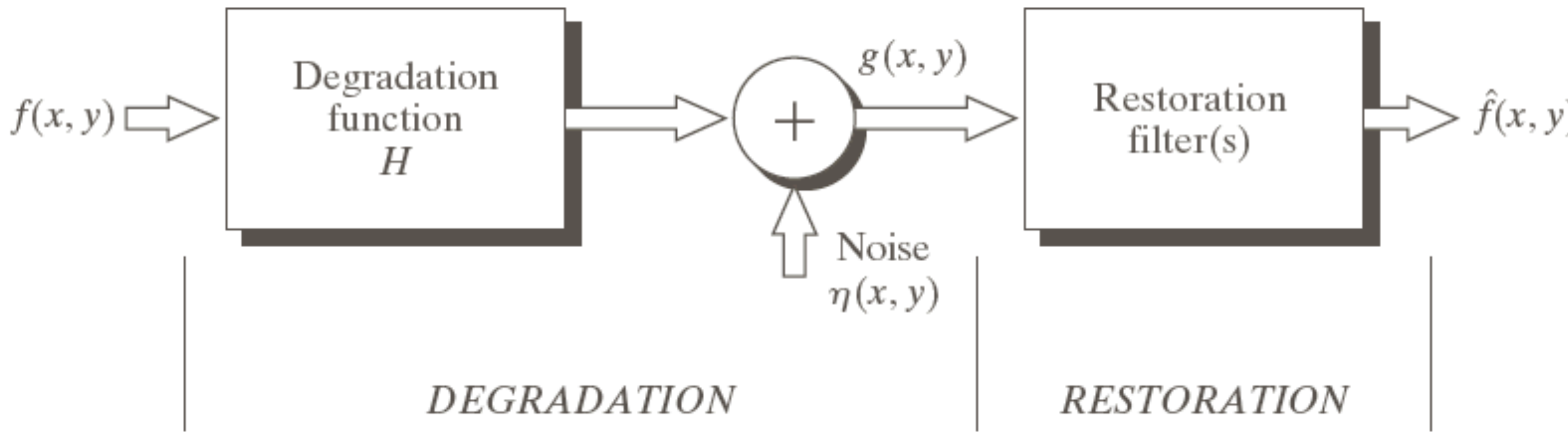
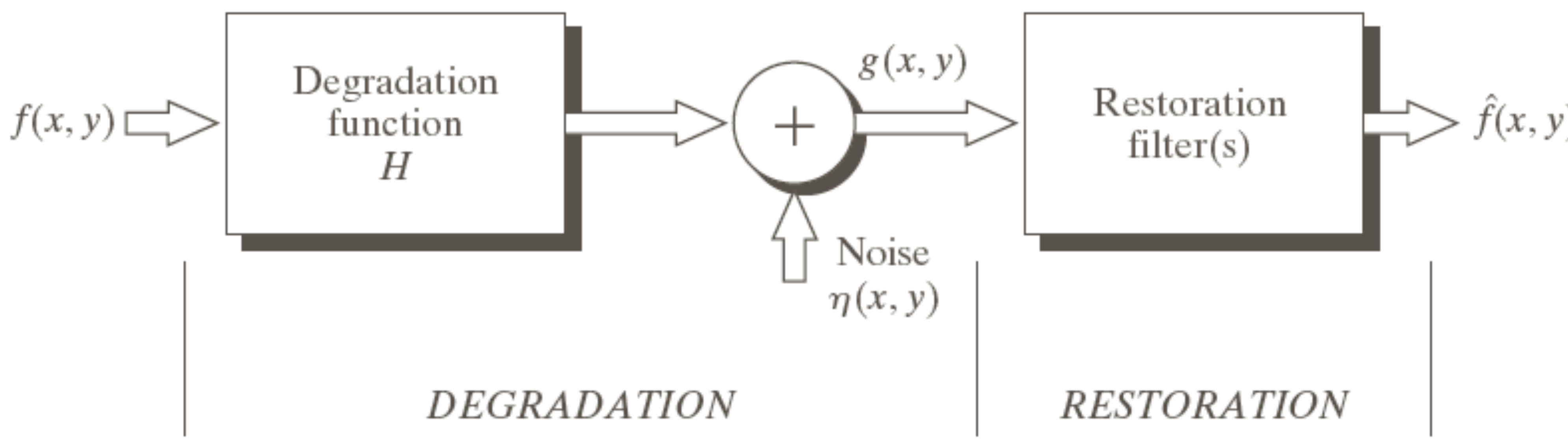


FIGURE 5.1 A model of the image degradation/restoration process.

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Image Restoration by Inverse Filtering

FIGURE 5.1 A model of the image degradation/restoration process.



$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

$$\hat{F}(u, v) = F(u, v) + \frac{N(u, v)}{H(u, v)}$$

Inverse Filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Bad news:

- Even when H(u,v) is known, there is always unknown noise
- Often H(u,v) has values close to zero

Example: Inverse Filtering

Atmospheric turbulence effect

$$H(u,v) = \exp\left\{-k\left[(u-M/2)^2 + (v-N/2)^2\right]^{5/6}\right\}$$

Example: Inverse Filtering

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

$$\frac{G(u,v)}{H(u,v)}$$

Wiener Filtering = Mean Squared Error Filtering

- Incorporates both:
  - Degradation function
  - Statistical characteristics of noise

- Assumption: noise and the image are uncorrelated

- Optimizes the filter so that MSE is minimized

$$e = \sum_x \sum_y (f(x,y) - \hat{f}(x,y))^2$$

Wiener Filter — Derivation

unknown original

after Wiener filtering

$$e = MN \sum_x \sum_y |f(x,y) - \hat{f}(x,y)|^2$$

Wiener Filter — Derivation

unknown original → $f(x,y)$; after Wiener filtering → $\hat{f}(x,y)$

$$e = MN \sum_x \sum_y |f(x,y) - \hat{f}(x,y)|^2$$

$$= \sum_u \sum_v |F(u,v) - \hat{F}(u,v)|^2$$

Parseval's Theorem

Wiener Filter — Derivation

$$e = MN \sum_x \sum_y |f(x,y) - \hat{f}(x,y)|^2$$

$$= \sum_u \sum_v |F(u,v) - \hat{F}(u,v)|^2$$

$$= \sum_u \sum_v |F(u,v) - [F(u,v)H(u,v) + N(u,v)]W(u,v))|^2$$

Unknown original

Corrupted original

Wiener filter

Wiener Filter — Derivation

independent signals

$$
\begin{aligned}
e &= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)] - N(u,v)W(u,v)|^2 \\
&= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)]|^2 + |N(u,v)W(u,v)|^2 \\
&= \sum_u \sum_v |F(u,v)|^2 |1 - H(u,v)W(u,v)|^2 + |N(u,v)|^2 |W(u,v)|^2
\end{aligned}
$$

Wiener Filter — Derivation

$$
\begin{aligned}
e &= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)] - N(u,v)W(u,v)|^2 \\
&= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)]|^2 + |N(u,v)W(u,v)|^2 \\
&= \sum_u \sum_v |F(u,v)|^2 |1 - H(u,v)W(u,v)|^2 + |N(u,v)|^2 |W(u,v)|^2
\end{aligned}
$$

$$
\frac{\partial e}{\partial W(u,v)} = 0 \quad \Rightarrow \quad W(u,v)
$$

Wiener Filter — Derivation

$$\frac{\partial}{\partial z}(zz^*) = 2z^*$$

$$\begin{aligned} e &= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)] - N(u,v)W(u,v)|^2 \\ &= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)]|^2 + |N(u,v)W(u,v)|^2 \\ &= \sum_u \sum_v |F(u,v)|^2 |1 - H(u,v)W(u,v)|^2 + |N(u,v)|^2 |W(u,v)|^2 \end{aligned}$$

$$\frac{\partial e}{\partial W(u,v)} = |F|^2[2(1 - W^*H^*)(-H)] + |N|^2[2W^*]$$

Wiener Filter — Derivation

$$\frac{\partial e}{\partial W(u,v)} = 0 \quad \Rightarrow \quad W^*(u,v) = \frac{|F(u,v)|^2 H(u,v)}{|H(u,v)|^2 |F(u,v)|^2 + |N(u,v)|^2}$$

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}} = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}$$

Wiener Filter — Derivation

$$\frac{\partial e}{\partial W(u,v)} = 0 \quad \Rightarrow \quad W^*(u,v) = \frac{|F(u,v)|^2 H(u,v)}{|H(u,v)|^2 |F(u,v)|^2 + |N(u,v)|^2}$$

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}} = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}$$

inverse filter

Wiener Filter -- Approximation

$$W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{1}{\mathrm{SNR}}}$$

Signal-to-noise ratio

Example: Wiener Filtering

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Example: Wiener Filtering

a b c
d e f
g h i

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a “curtain” of noise.

Next Class

- Image reconstruction from projections (Textbook 5.11)
- Radon Transform (Textbook 5.11.3)