ECE 468 / CS 519:
Digital Image Processing

Gradients, Harris Corners

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Image Gradient along X-axis

$$I_x(x,y) = I(x+1,y) - I(x,y)$$

$$= \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{D_x(x,y)} * I(x,y)$$

Image Gradient along Y-axis

$$I_y(x,y) = I(x,y+1) - I(x,y)$$

$$= \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{D_y(x,y)} * I(x,y)$$

Filtering Image Gradient

$$w(x, y; \sigma) * I_x(x, y) = w(x, y; \sigma) * D_x(x, y) * I(x, y)$$

$$= [w(x, y; \sigma) * D_x(x, y)] * I(x, y)$$

convolution is associative

Filtering Image Gradient

$$w(x, y; \sigma) * I_x(x, y) = w(x, y; \sigma) * D_x(x, y) * I(x, y)$$

$$= [w(x, y; \sigma) * D_x(x, y)] * I(x, y)$$

$$= [D_x(x, y) * w(x, y; \sigma)] * I(x, y)$$

convolution is commutative

Filtering Image Gradient

$$w(x, y; \sigma) * I_x(x, y) = w(x, y; \sigma) * D_x(x, y) * I(x, y)$$

$$= [w(x, y; \sigma) * D_x(x, y)] * I(x, y)$$

$$= [D_x(x, y) * w(x, y; \sigma)] * I(x, y)$$

$$= w_x(x, y; \sigma) * I(x, y)$$

derivative of the filter

Weighted Image Gradient

$$w(x, y; \sigma) * I_x(x, y) = w_x(x, y; \sigma) * I(x, y)$$

$$w(x, y; \sigma) * I_y(x, y) = w_y(x, y; \sigma) * I(x, y)$$

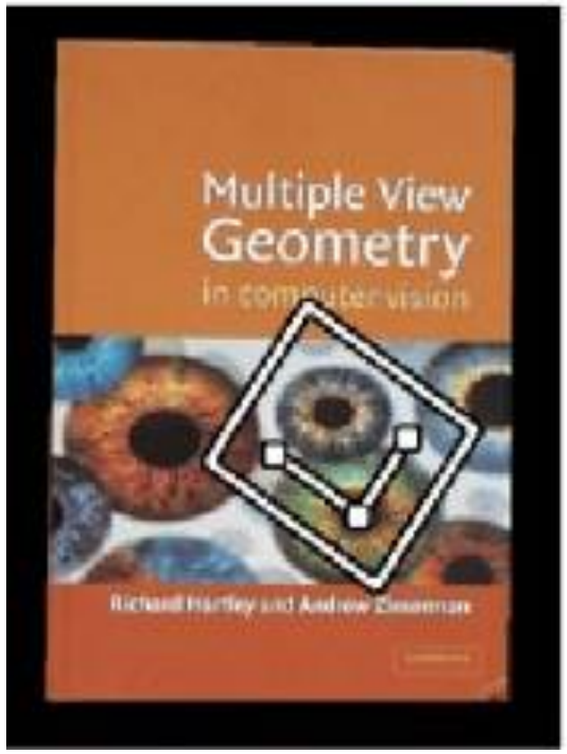
Image is discrete $\Rightarrow$ Gradient is approximate

We always find the gradient of the filter !

Interest Points

- Harris corners

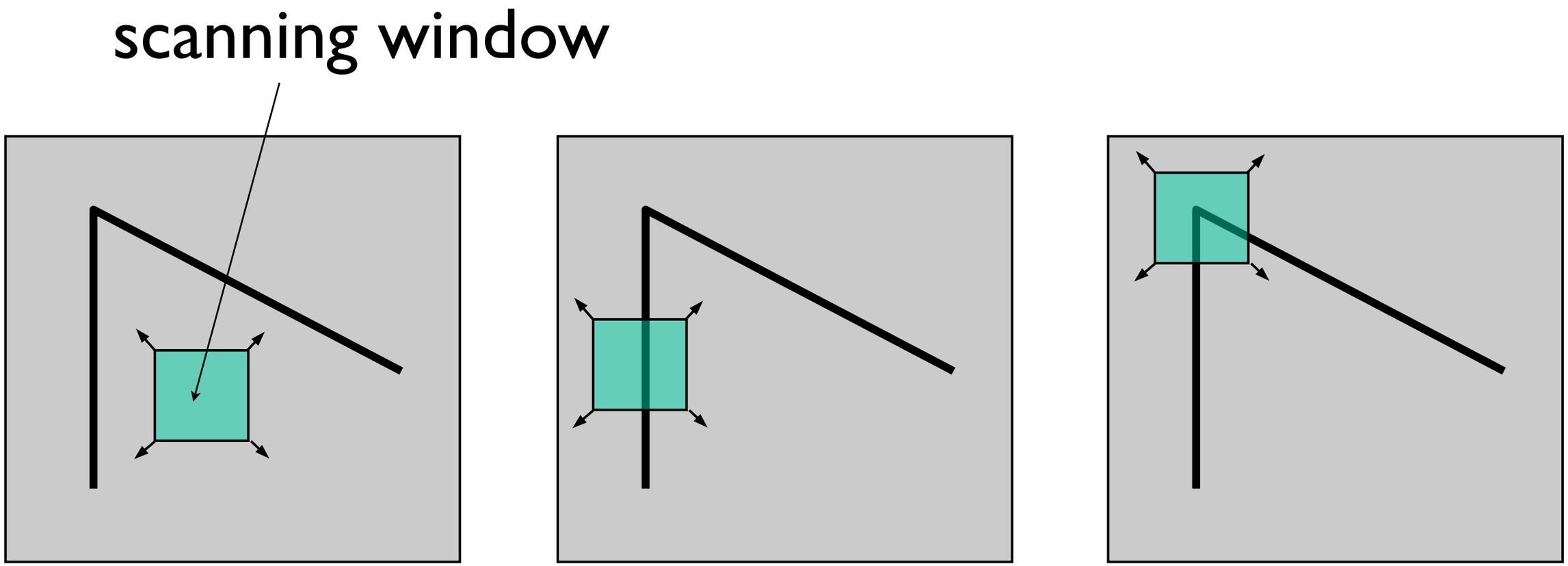
Properties of Interest Points



- Locality -- robust to occlusion, noise
- Saliency -- rich visual cue
- Stable under affine transforms
- Distinctiveness -- differ across distinct objects
- Efficiency -- easy to compute

Example of Detecting Harris Corners

Harris Corner Detector



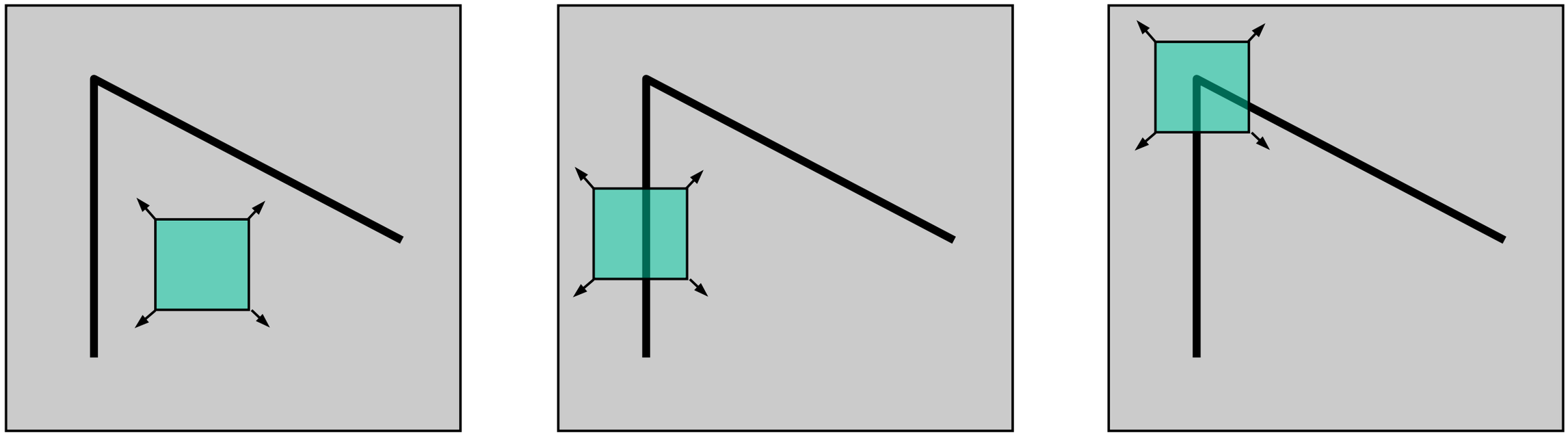
homogeneous region
⇓
no change in all directions

edge
⇓
no change along the edge

corner
⇓
change in all directions

Source: Frolova, Simakov, Weizmann Institute

Harris Corner Detector

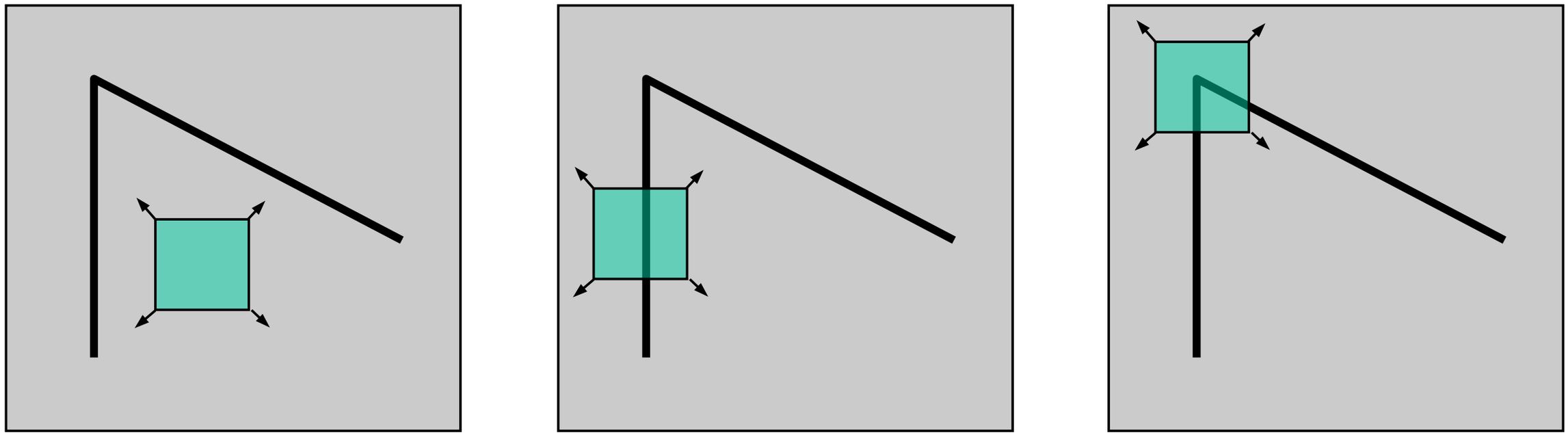


$$E(x,y) = w(x,y) * [I(x+u, y+v) - I(x,y)]^2$$

2D convolution

Source: Frolova, Simakov, Weizmann Institute

Harris Corner Detector



$$E(x,y) = \sum_{m,n} w(m,n)[I(x+u+m, y+v+n) - I(x+m, y+n)]^2$$

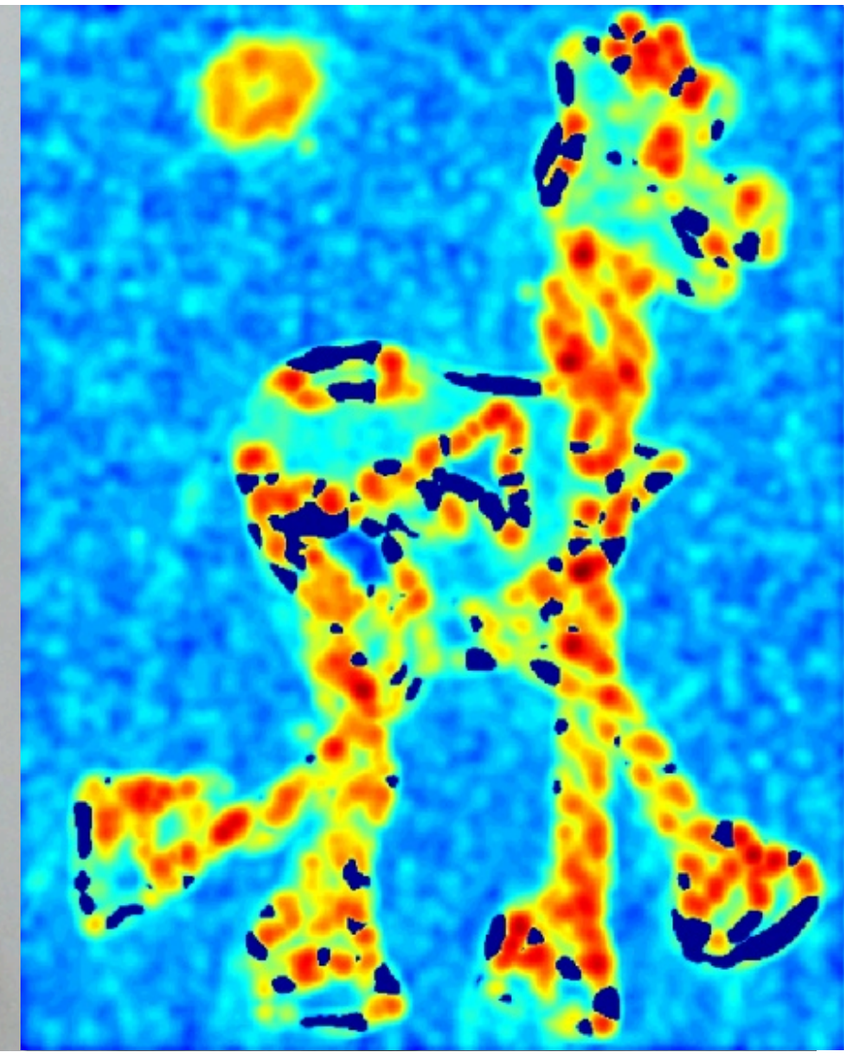
change weighting window shifted image original image

Source: Frolova, Simakov, Weizmann Institute

Harris Detector Example

$I(x, y)$

input image



$E(x, y)$

2D map of changes

Harris Corner Detector

Taylor series expansion

For small shifts

$$\begin{array}{rcl} u & \rightarrow & 0 \\ v & \rightarrow & 0 \end{array} \quad \Rightarrow \quad I(x+u, y+v) \approx I(x,y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v$$

$$I(x+u, y+v) \approx I(x,y) + [I_x \quad I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

image derivatives
along x and y axes

Harris Corner Detector

$$E(x,y) = w(x,y) * \left(I(x,y) + [I_x \quad I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x,y) \right)^2$$

$$= w(x,y) * \left([I_x \quad I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right)^2$$

$$= w(x,y) * \left([I_x \quad I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right)^{\mathrm{T}} \left([I_x \quad I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right)$$

Harris Corner Detector

$$E(x,y) = w(x,y) * [u \quad v] \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \quad I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

$$= [u \quad v] \underbrace{\left(w(x,y) * \begin{bmatrix} I_x^2(x,y) & I_x(x,y)I_y(x,y) \\ I_x(x,y)I_y(x,y) & I_y^2(x,y) \end{bmatrix} \right)}_{M(x,y)} \begin{bmatrix} u \\ v \end{bmatrix}$$

Filtering Image Gradient

$$w(x, y; \sigma) * I_x(x, y) = w(x, y; \sigma) * D_x(x, y) * I(x, y)$$

$$= [w(x, y; \sigma) * D_x(x, y)] * I(x, y)$$

$$= [D_x(x, y) * w(x, y; \sigma)] * I(x, y)$$

$$= w_x(x, y; \sigma) * I(x, y)$$

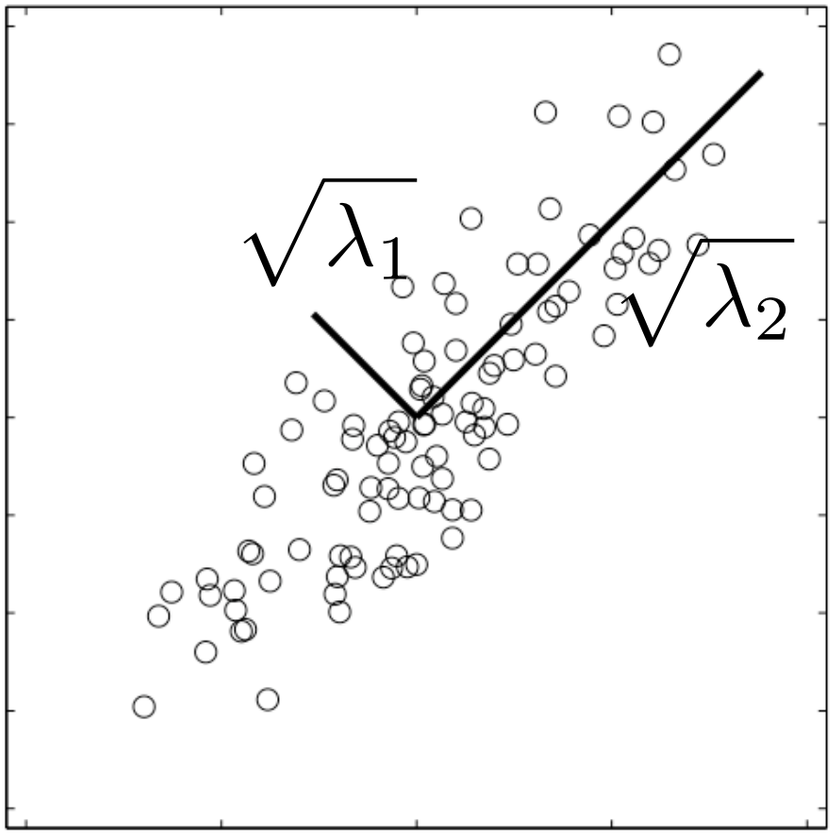
derivative of the filter

Harris Corner Detector

$$E(x,y) = [u \quad v] M(x,y) \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M(x,y) = \begin{bmatrix} (w_x * I)^2 & (w_x * I)(w_y * I) \\ (w_x * I)(w_y * I) & (w_y * I)^2 \end{bmatrix}_{(x,y)}$$

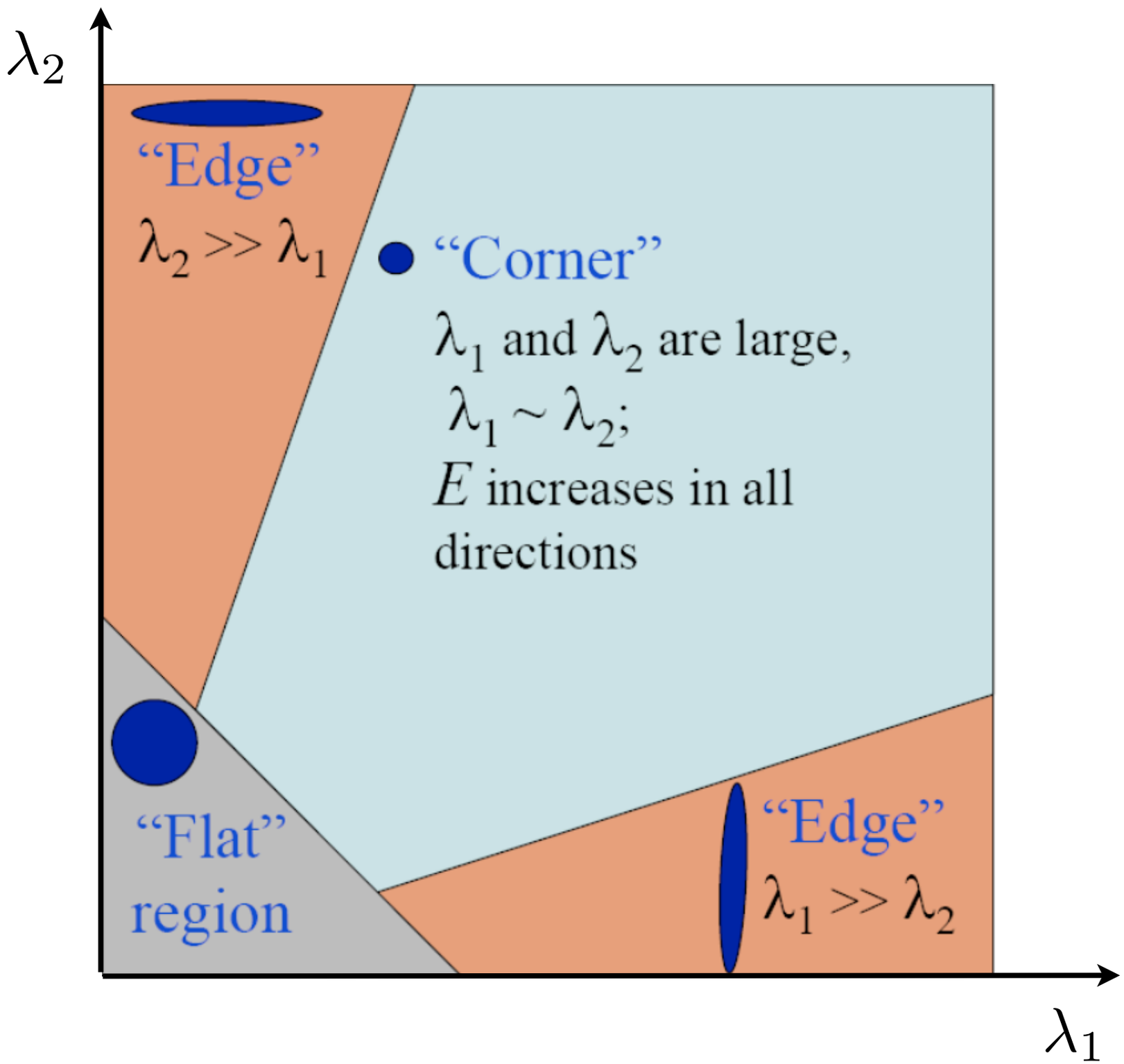
Eigenvalues and Eigenvectors of M



$$E(x,y) = [u \quad v] M(x,y) \begin{bmatrix} u \\ v \end{bmatrix}$$

- Eigenvectors of *M* -- directions of the largest change of *E(x,y)*
- Eigenvalues of *M* -- the amount of change along eigenvectors

Detection of Harris Corners using the Eigenvalues of M



Harris Detector

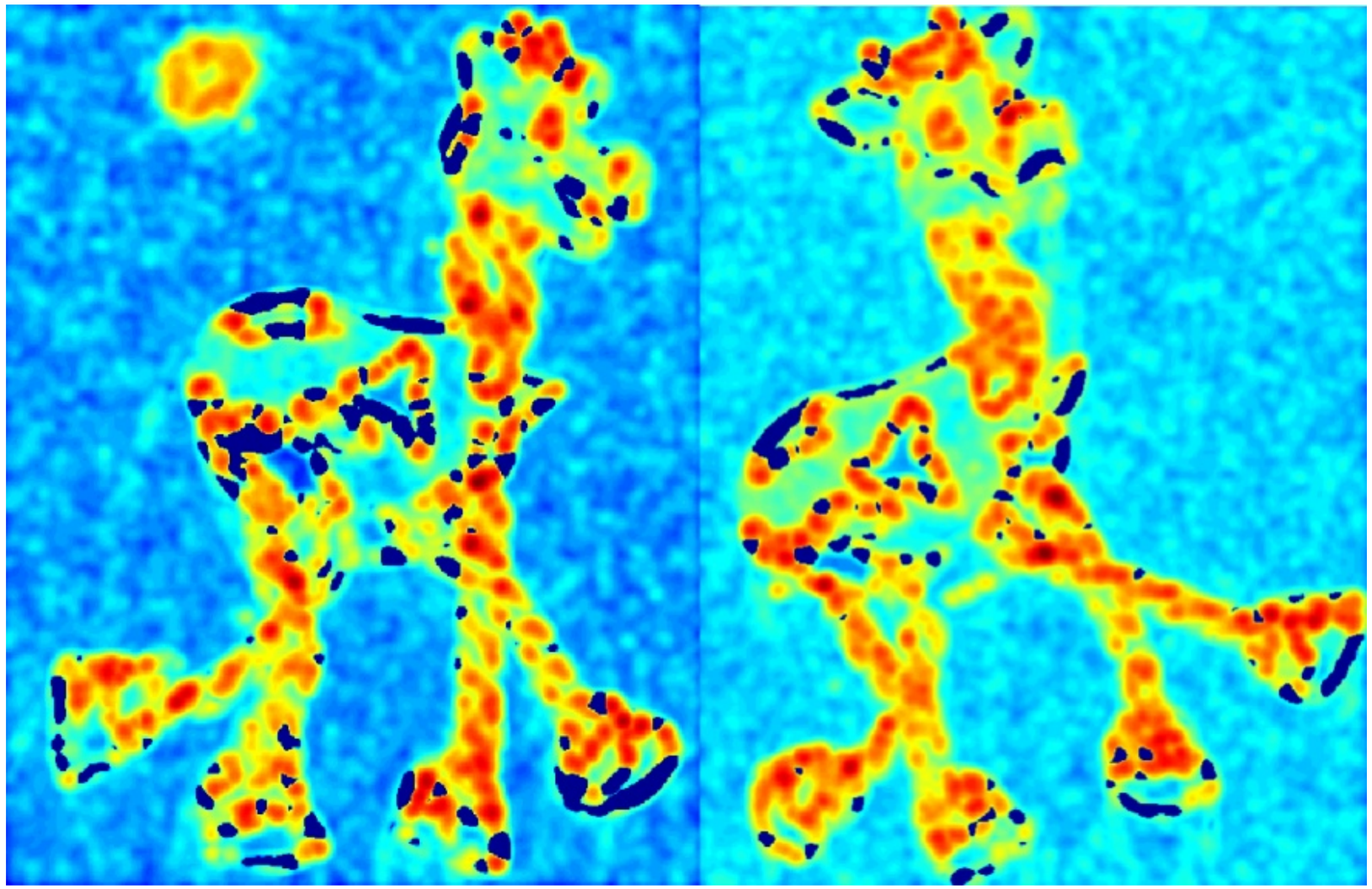
$$f(\sigma) = \frac{\lambda_1(\sigma)\lambda_2(\sigma)}{\lambda_1(\sigma) + \lambda_2(\sigma)} = \frac{\det(M(\sigma))}{\mathrm{trace}(M(\sigma))}$$

objective function

Harris Detector Example

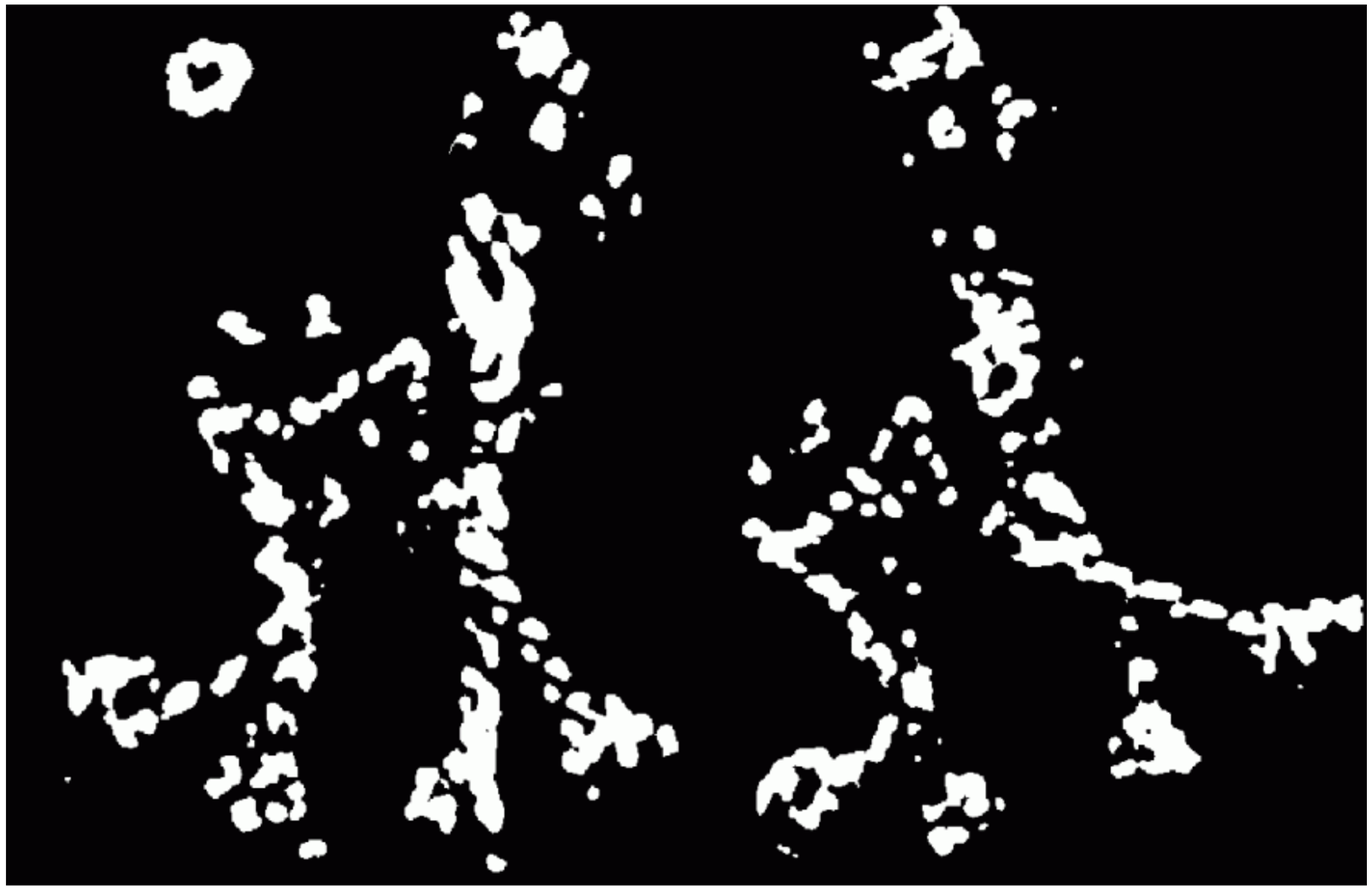
input image

Harris Detector Example



f values color coded
red = high values
blue = low values

Harris Detector Example



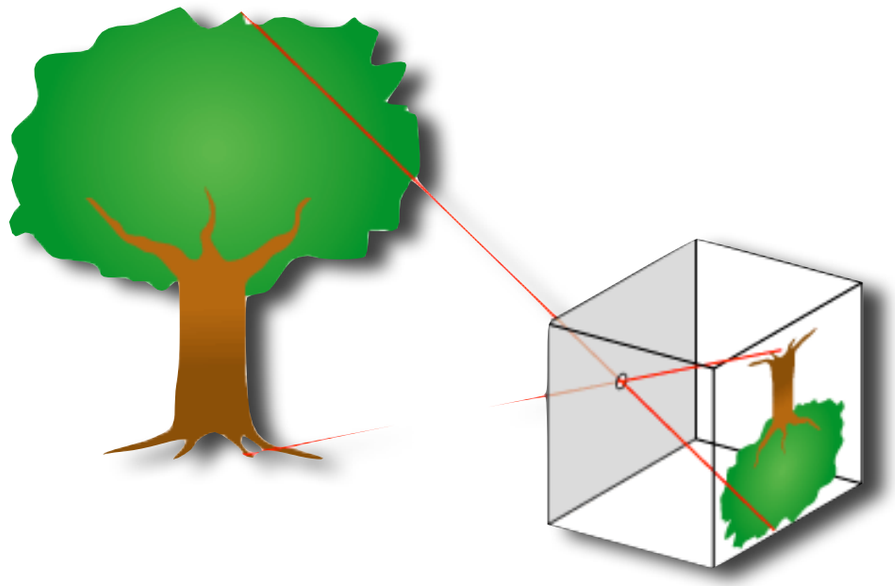
f values thresholded

Harris Detector Example

Harris features = Spatial maxima of f values

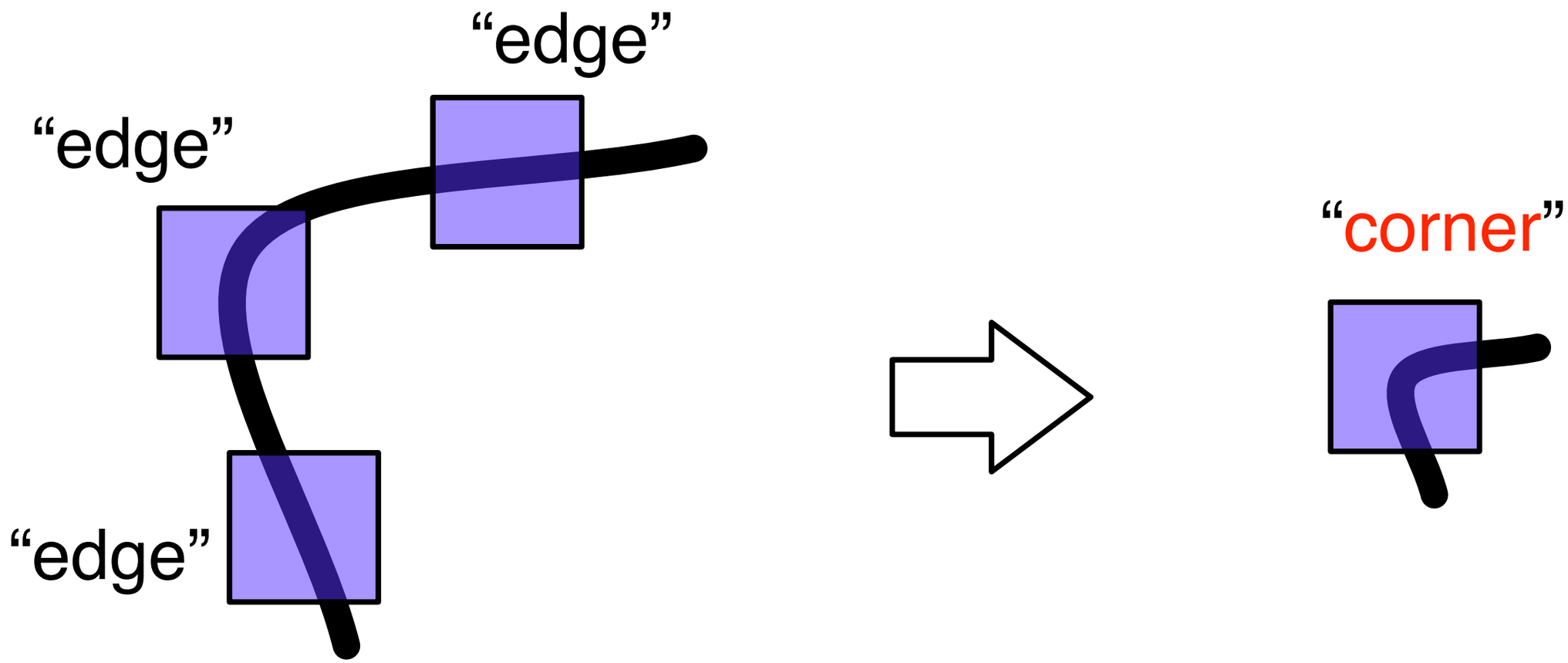
Example of Detecting Harris Corners

Properties of Harris Corners



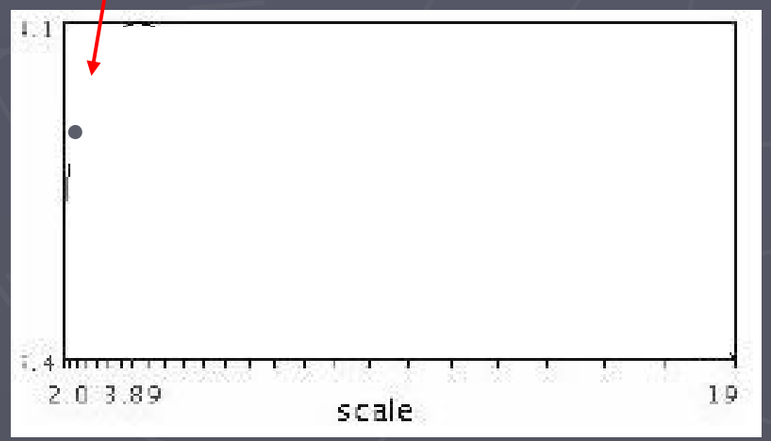
- Invariance to variations of imaging parameters:
  - Illumination?
  - Camera distance, i.e., scale ?
  - Camera viewpoint, i.e., affine transformation?

Harris/Hessian Detector is NOT Scale Invariant



Source: L. Fei-Fei

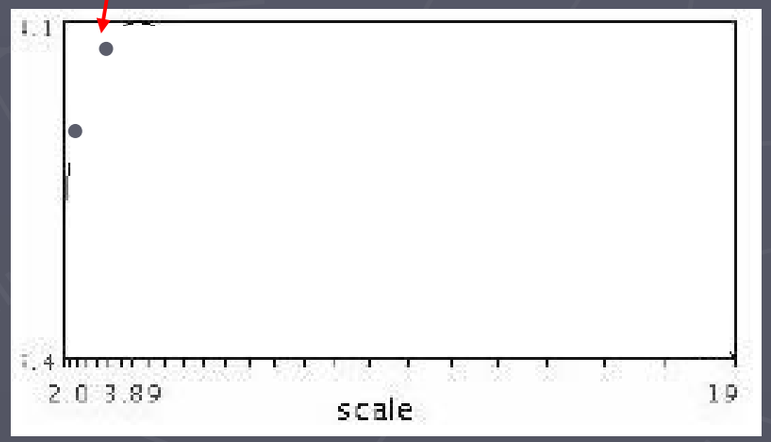
Automatic scale selection



$$f(I_{i_1..i_m}(x,\sigma))$$

Source: Tuytelaars

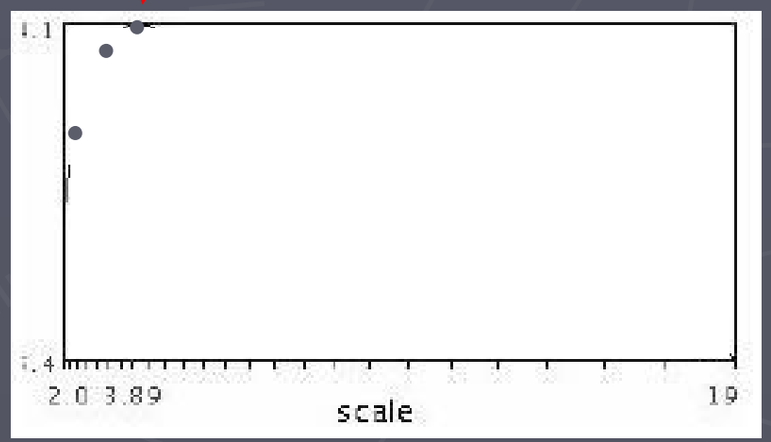
Automatic scale selection



$$f(I_{i_1 \ldots i_m}(x,\sigma))$$

Source: Tuytelaars

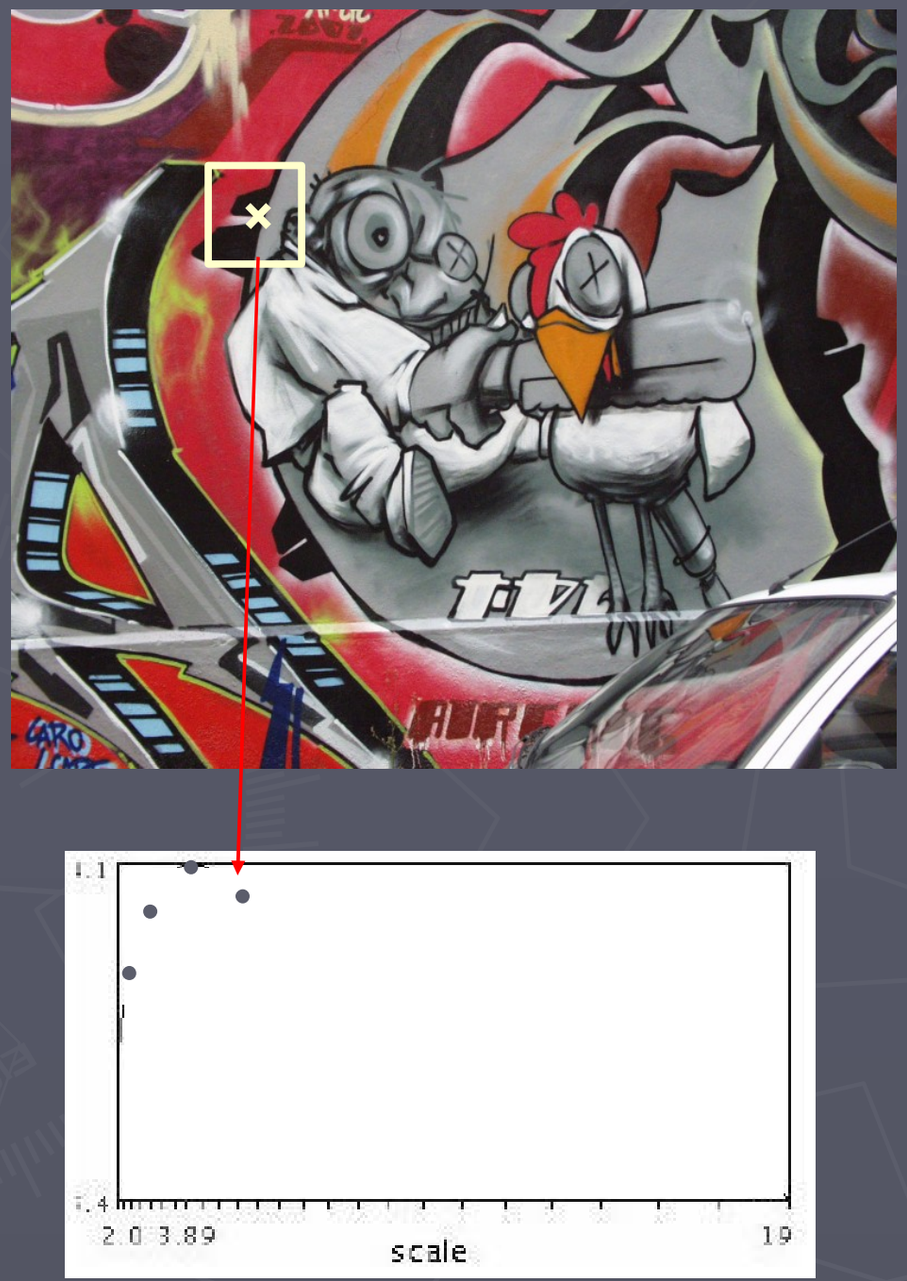
Automatic scale selection



$f(I_{i_1..i_m}(x,\sigma))$

Source: Tuytelaars

Automatic scale selection



$$f(I_{i_1 \ldots i_m}(x,\sigma))$$

Source: Tuytelaars

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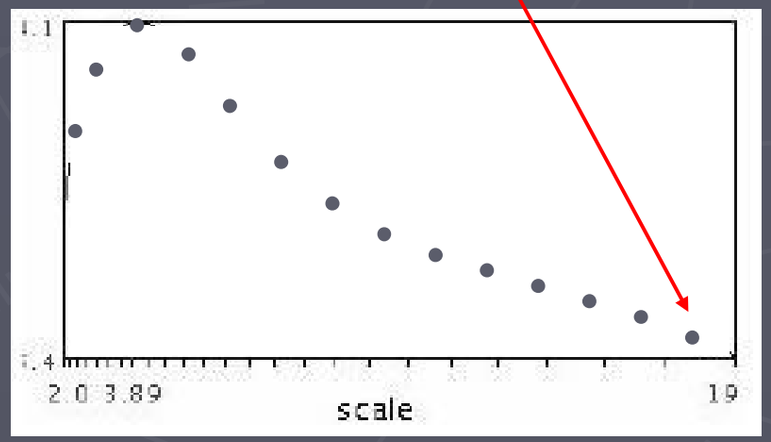

$$f(I_{i_1..i_m}(x,\sigma))$$

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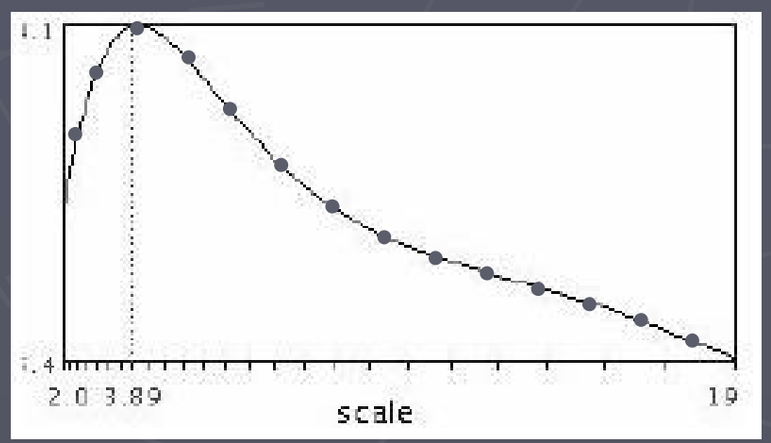
Automatic scale selection



$$f(I_{i_1..i_m}(x,\sigma))$$

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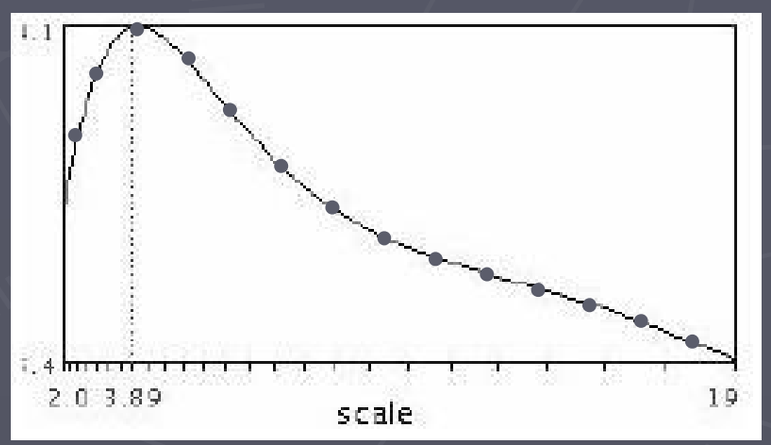
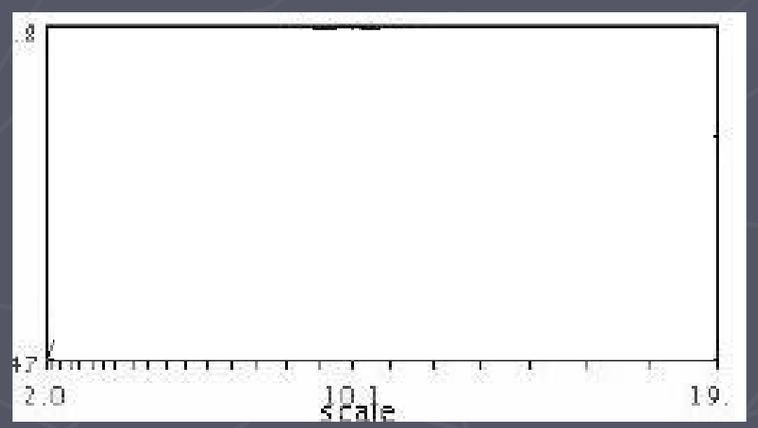


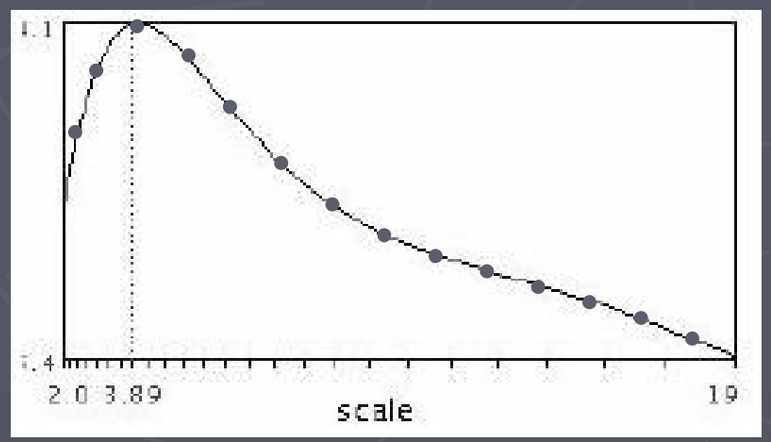
$f(I_{i_1 \ldots i_m}(x,\sigma))$



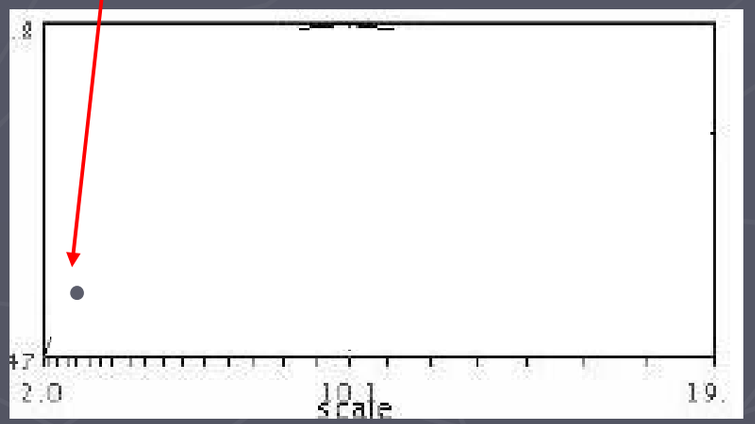
$f(I_{i_1 \ldots i_m}(x',\sigma))$

Source: Tuytelaars

Automatic scale selection



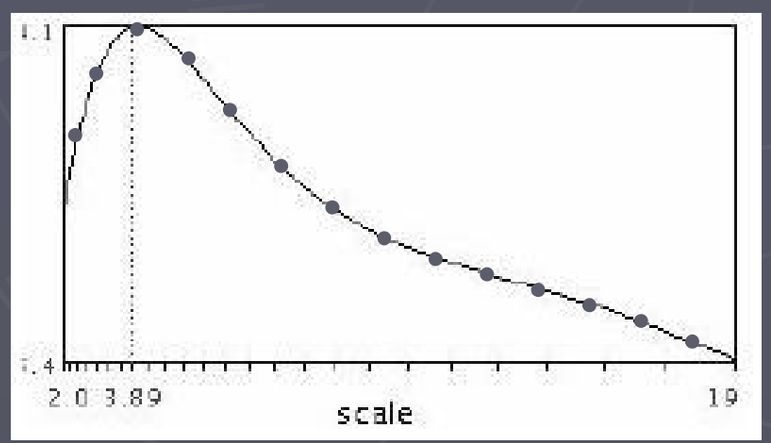
$f(I_{i_1 \ldots i_m}(x,\sigma))$



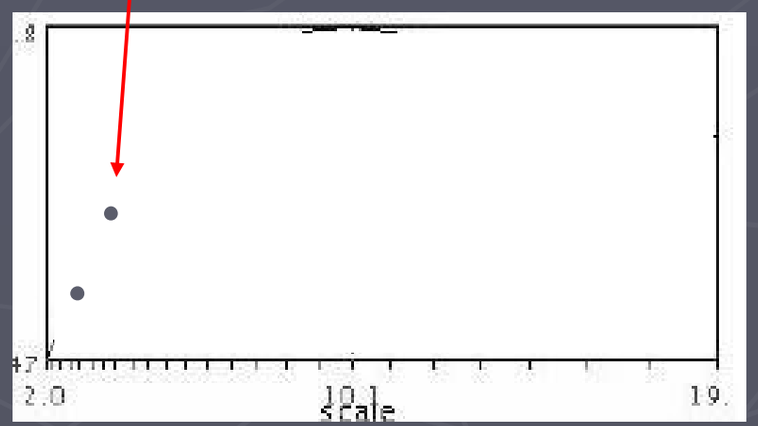
$f(I_{i_1 \ldots i_m}(x',\sigma))$

Source: Tuytelaars

Automatic scale selection



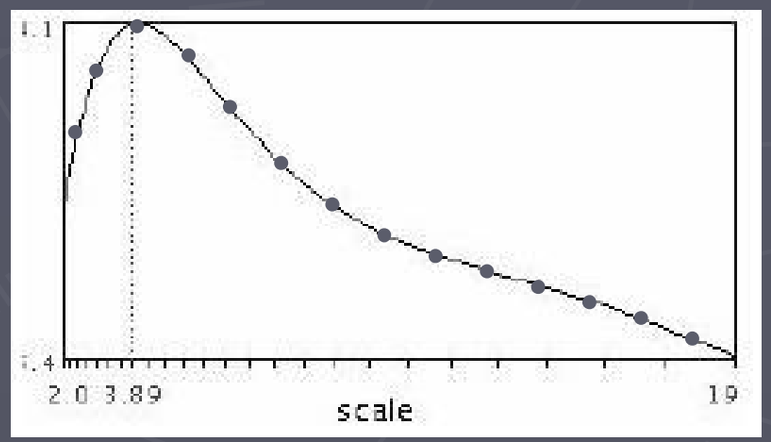
$f(I_{i_1 \ldots i_m}(x,\sigma))$



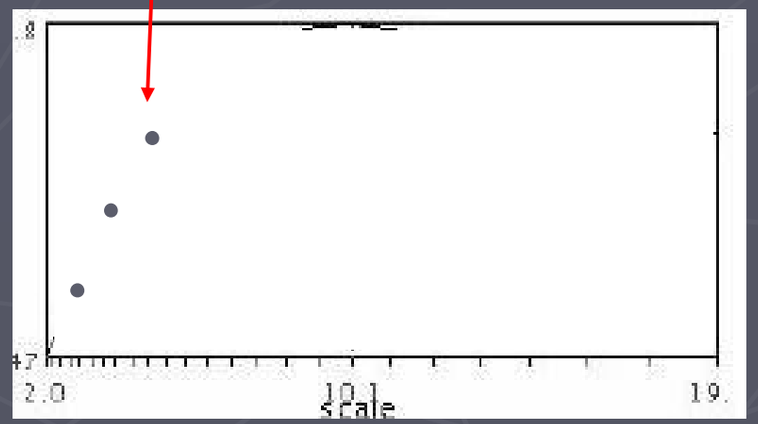
$f(I_{i_1 \ldots i_m}(x',\sigma))$

Source: Tuytelaars

Automatic scale selection



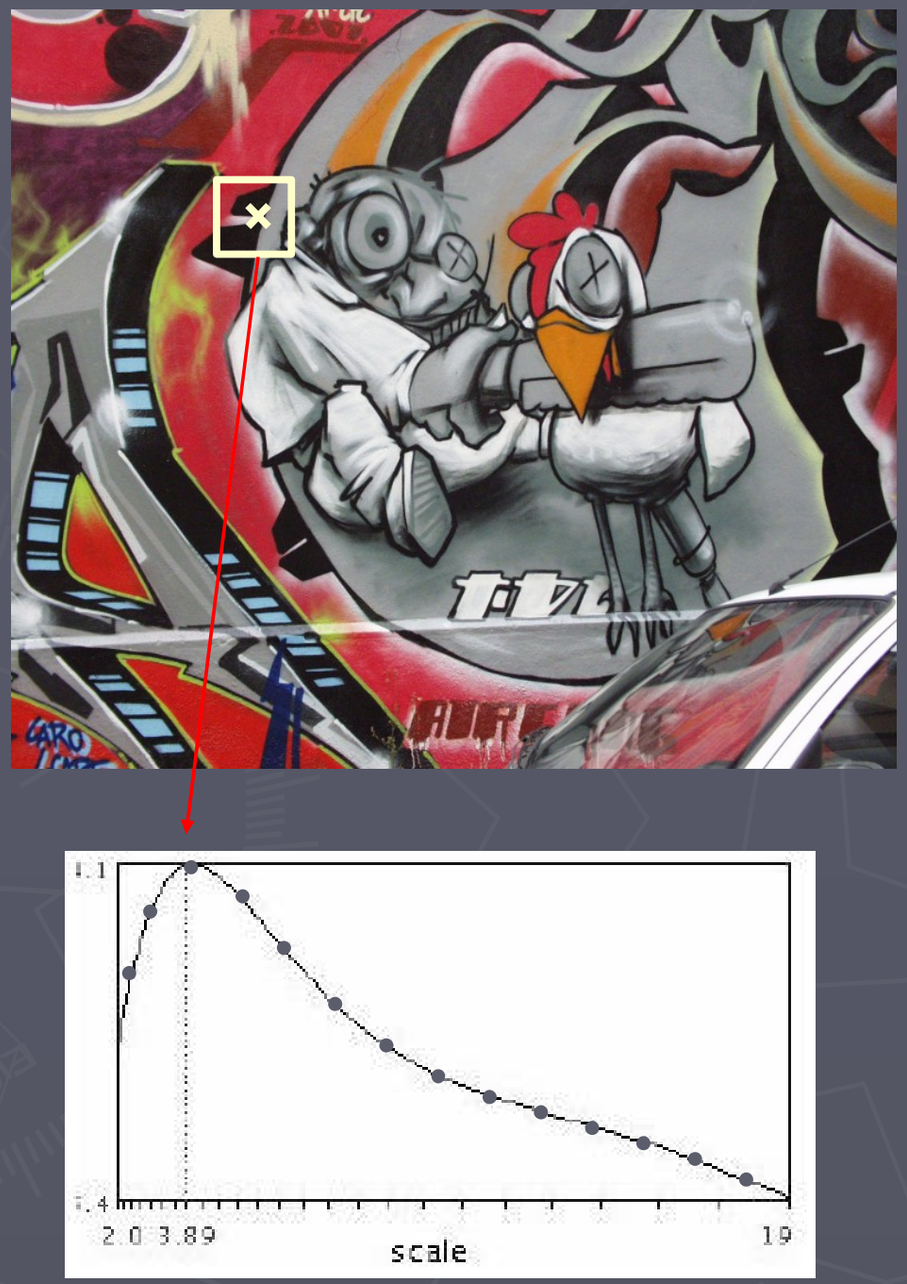
$f(I_{i_1 \ldots i_m}(x,\sigma))$



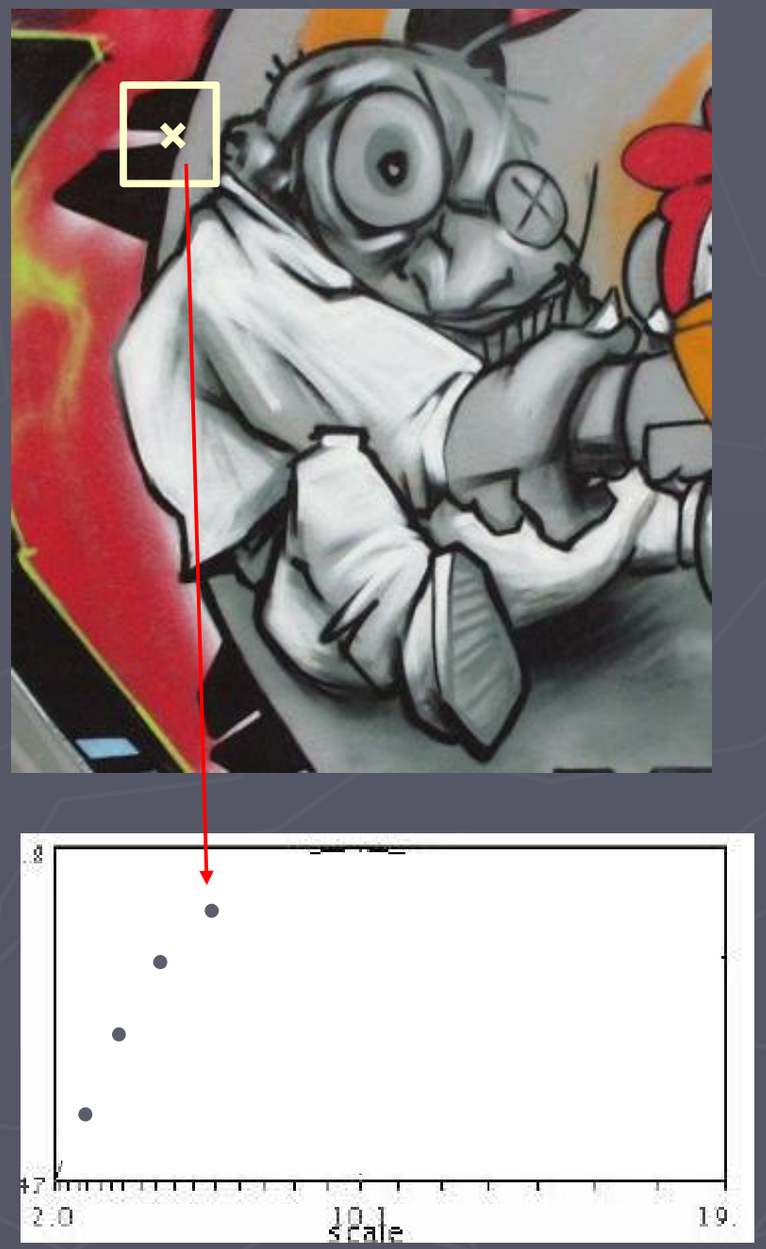
$f(I_{i_1 \ldots i_m}(x',\sigma))$

Source: Tuytelaars

Automatic scale selection



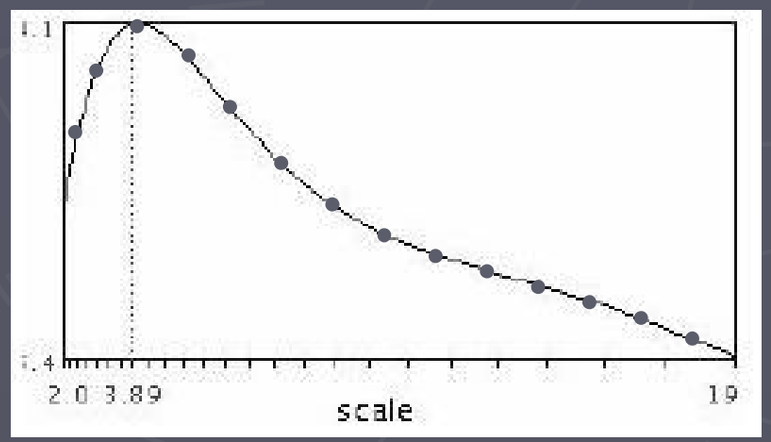
$f(I_{i_1 \ldots i_m}(x,\sigma))$



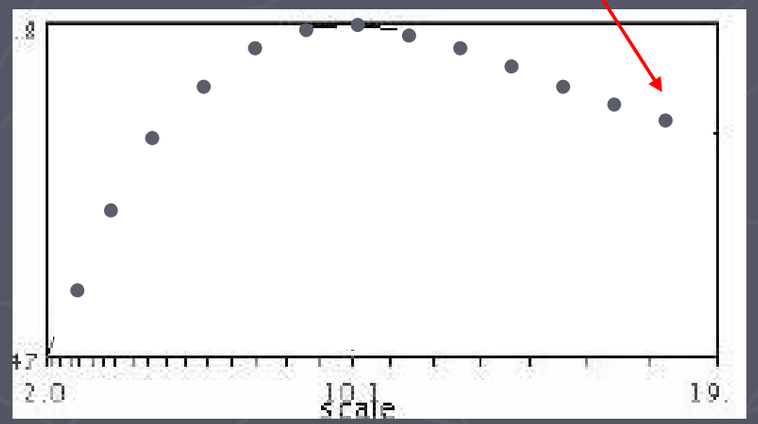
$f(I_{i_1 \ldots i_m}(x',\sigma))$

Source: Tuytelaars

Automatic scale selection



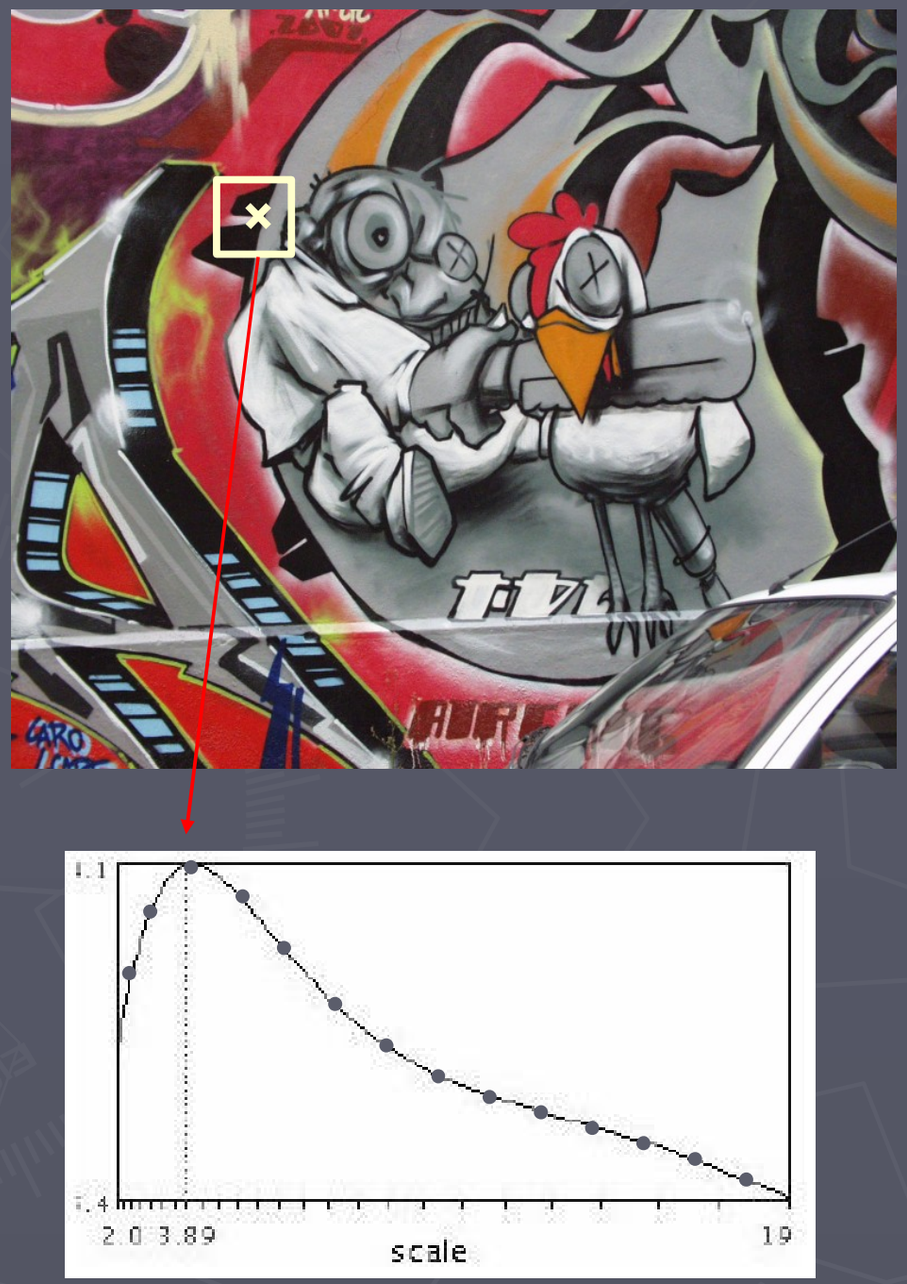
$f(I_{i_1 \ldots i_m}(x,\sigma))$



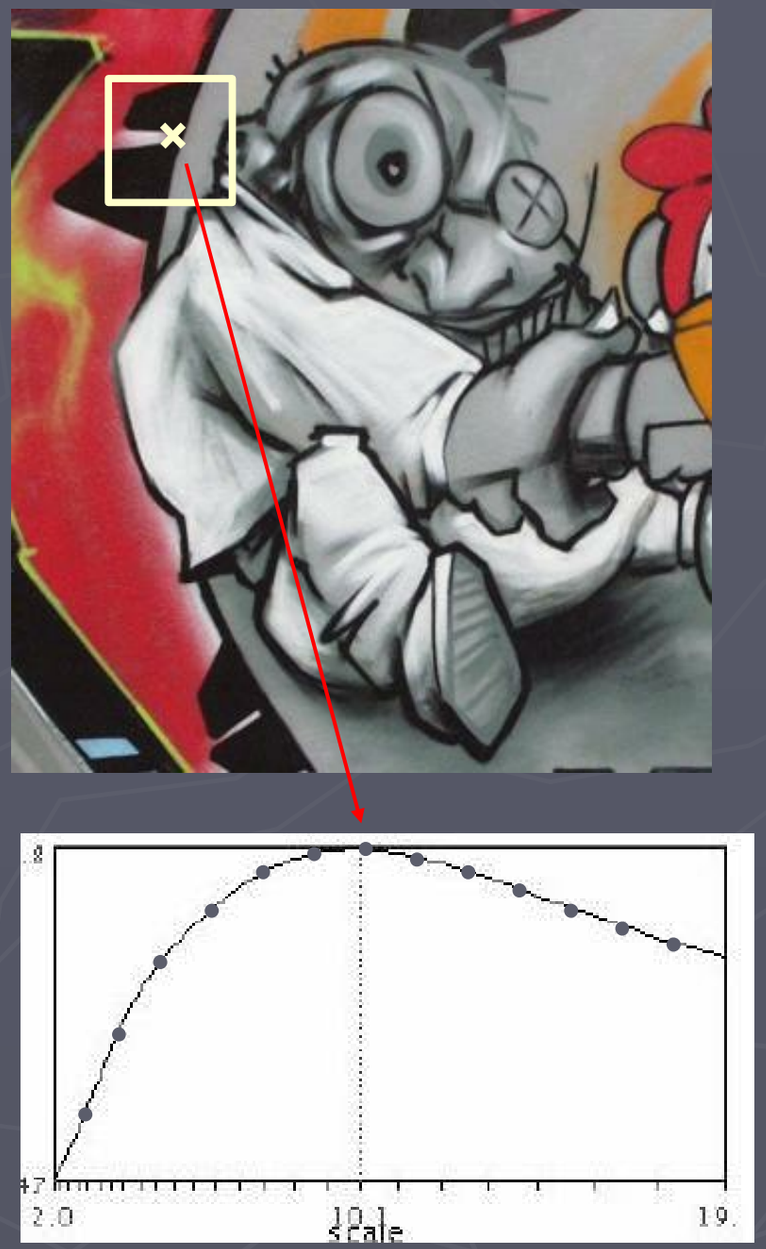
$f(I_{i_1 \ldots i_m}(x',\sigma))$

Source: Tuytelaars

Automatic scale selection



$f(I_{i_1 \ldots i_m}(x,\sigma))$



$f(I_{i_1 \ldots i_m}(x',\sigma'))$

Source: Tuytelaars