ECE 468: Digital Image Processing

Lecture 8

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Point Descriptors

Point Descriptors

- Describe image properties in the neighborhood of a keypoint
- Descriptors = Vectors that are ideally affine invariant
- Popular descriptors:
    - Scale invariant feature transform (SIFT)
    - Steerable filters
    - Shape context and geometric blur
    - Gradient location and orientation histogram (GLOH)
    - Histogram of Oriented Gradients (HOGs)

SIFT

The key idea:

a point

can be described by

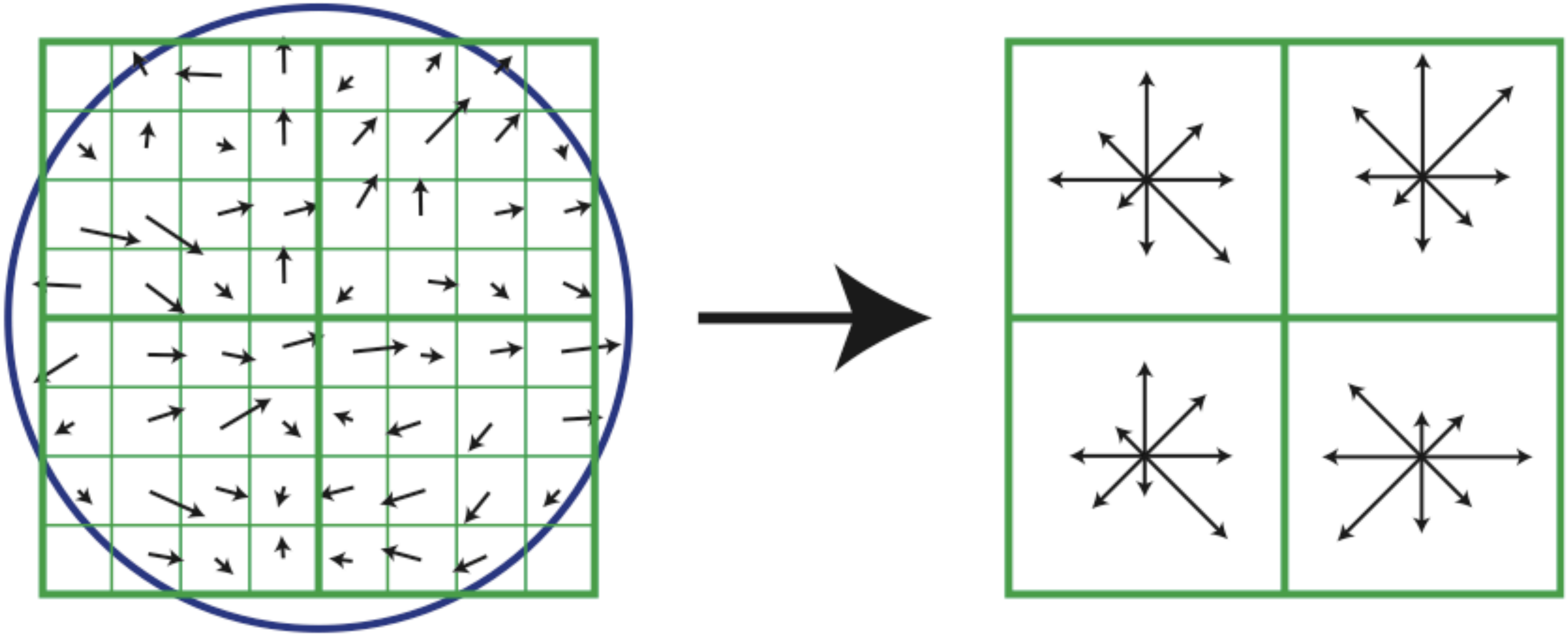
a distribution of

intensity gradients

in the neighborhood of that point

SIFT Descriptor

128-D vector = (4x4 blocks) x (8 bins of histogram)



gradients of a 16x16 patch centered at the point

histogram of gradients at certain angles of a 4x4 subpatch

The figure illustrates only 8x8 pixel neighborhood that is transformed into 2x2 blocks, for visibility

MATLAB Code for SIFT

Matching Cost of Two Descriptors

- Euclidean distance: $\psi(\mathbf{d}_1, \mathbf{d}_2) = \|\mathbf{d}_1 - \mathbf{d}_2\|^2$

- Chi-squared distance: $\psi(\mathbf{d}_1, \mathbf{d}_2) = \sum_i \frac{(\mathbf{d}_{1i} - \mathbf{d}_{2i})^2}{\mathbf{d}_{1i} + \mathbf{d}_{2i}}$

Matching Formulation

Given two sets of descriptors to be matched

$$V = \{\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_N\}, \text{ and } V' = \{\mathbf{d}'_1, \mathbf{d}'_2, \ldots, \mathbf{d}'_M\}$$

Find the legal mapping $f \in \mathcal{F}$

$$f := \{(\mathbf{d}, \mathbf{d}') : \ \mathbf{d} \in V, \ \ \mathbf{d}' \in V'\}$$

Which minimizes the total cost of matching

$$\hat{f} = \min_{f \in \mathcal{F}} \sum_{(\mathbf{d}, \mathbf{d}') \in f} \psi(\mathbf{d}, \mathbf{d}'), \ \ \psi(\mathbf{d}, \mathbf{d}') \geq \mathbf{0}$$

Total Cost of Matching

$$A = \begin{bmatrix} \psi_{11'} & \psi_{12'} & \psi_{13'} & \dots & \psi_{1M} \\ \psi_{21'} & \psi_{22'} & \psi_{23'} & \dots & \psi_{2M} \\ & & \dots & & \end{bmatrix}_{N \times M}$$

cost matrix

$$\sum_{(\mathbf{d},\mathbf{d}')} \psi(\mathbf{d}, \mathbf{d}') = \mathrm{tr}(A^T \mathbf{1})$$

matrix of all ones

Total Cost of Matching

$$A = \begin{bmatrix} \psi_{11'} & \psi_{12'} & \psi_{13'} & \dots & \psi_{1M} \\ \psi_{21'} & \psi_{22'} & \psi_{23'} & \dots & \psi_{2M} \\ & & \dots & & \end{bmatrix}_{N \times M}$$

cost matrix

$$\sum_{(\mathbf{d},\mathbf{d}')} \psi(\mathbf{d},\mathbf{d}') = \mathrm{tr}(A^T \mathbf{1})$$

matrix of all ones

$$\sum_{(\mathbf{d},\mathbf{d}') \in f} \psi(\mathbf{d},\mathbf{d}') = ?$$

Linearization

$$\sum_{(\mathbf{d},\mathbf{d}')\in f} \psi(\mathbf{d},\mathbf{d}') = \sum_{(\mathbf{d},\mathbf{d}')\in f} \psi(\mathbf{d},\mathbf{d}') \cdot 1$$

$$= \sum_{(\mathbf{d},\mathbf{d}')} \psi(\mathbf{d},\mathbf{d}') \cdot x(\mathbf{d},\mathbf{d}')$$

$$x(\mathbf{d},\mathbf{d}') = 1, \text{ if } (\mathbf{d},\mathbf{d}') \in f \text{ matched pair}$$

$$x(\mathbf{d},\mathbf{d}') = 0, \text{ if } (\mathbf{d},\mathbf{d}') \notin f \text{ unmatched pair}$$

Linearization

Linearization by introducing an indicator matrix

$$X = \begin{bmatrix} 0 & 0 & 1 & ... & 0 & 0 \\ 0 & 1 & 0 & ... & 0 & 0 \\ & & ... & & & \end{bmatrix}_{N \times M}$$

$$x(\mathbf{d}, \mathbf{d}') = 1, \text{ if } (\mathbf{d}, \mathbf{d}') \in f \text{ matched pair}$$

$$x(\mathbf{d}, \mathbf{d}') = 0, \text{ if } (\mathbf{d}, \mathbf{d}') \notin f \text{ unmatched pair}$$

Linearization

$$A = \begin{bmatrix} \psi_{11'} & \psi_{12'} & \psi_{13'} & \dots & \psi_{1M} \\ \psi_{21'} & \psi_{22'} & \psi_{23'} & \dots & \psi_{2M} \\ & & \dots & & \end{bmatrix}_{N \times M}$$

$$X = \begin{bmatrix} 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ & & \dots & & & \end{bmatrix}_{N \times M}$$

$$\sum_{(\mathbf{d},\mathbf{d}') \in f} \psi(\mathbf{d}, \mathbf{d}') = \sum_{(\mathbf{d},\mathbf{d}')} \psi_{\mathbf{d},\mathbf{d}'} x_{\mathbf{d},\mathbf{d}'} = \mathrm{tr}(A^T X)$$

Matching Formulation

$$\hat{X} = \min_{X} \ \mathrm{tr}(A^T X)$$

$$\Downarrow$$

$$\hat{X} = \mathbf{0}$$ trivial solution

$$\Downarrow$$

we need to constrain the formulation

Matching Formulation

$$\min_{X} \ \mathrm{tr}(A^T X)$$

subject to:

$$\forall \mathbf{d} \in V, \ \forall \mathbf{d}' \in V', \ x_{\mathbf{dd}'} \in \{0, 1\}$$

$$\forall \mathbf{d}, \quad \sum_{\mathbf{d}'} x_{\mathbf{dd}'} = 1$$

$$\forall \mathbf{d}', \quad \sum_{\mathbf{d}} x_{\mathbf{dd}'} = 1$$

what is the meaning of this constraint?

Matching Formulation

$$\min_{X} \; \mathrm{tr}(A^T X)$$

subject to:

$$\forall \mathbf{d} \in V,\ \forall \mathbf{d}' \in V',\ x_{\mathbf{d}\mathbf{d}'} \in \{0,1\}$$

$$\forall \mathbf{d}, \quad \sum_{\mathbf{d}'} x_{\mathbf{d}\mathbf{d}'} = 1$$

one-to-one matching

$$\forall \mathbf{d}', \quad \sum_{\mathbf{d}} x_{\mathbf{d}\mathbf{d}'} = 1$$

Relaxation

$$\min_X \; \mathrm{tr}(A^T X)$$

subject to:

$$\forall \mathbf{d} \in V,\ \forall \mathbf{d}' \in V',\ x_{\mathbf{d}\mathbf{d}'} \in [0,1]$$

$$\forall \mathbf{d},\quad \sum_{\mathbf{d}'} x_{\mathbf{d}\mathbf{d}'} = 1$$

one-to-one matching

$$\forall \mathbf{d}',\quad \sum_{\mathbf{d}} x_{\mathbf{d}\mathbf{d}'} = 1$$

Linear Assignment Problem

$$\min_{X} \; \mathrm{tr}(A^T X)$$

subject to:

$$\forall \mathbf{d} \in V, \; \forall \mathbf{d}' \in V', \; x_{\mathbf{d}\mathbf{d}'} \in [0,1]$$

$$\forall \mathbf{d}, \quad \sum_{\mathbf{d}'} x_{\mathbf{d}\mathbf{d}'} = 1$$

one-to-one matching

$$\forall \mathbf{d}', \quad \sum_{\mathbf{d}} x_{\mathbf{d}\mathbf{d}'} = 1$$

Hungarian algorithm for the balanced problem |V| = |V'|

Hungarian Algorithm

The Hungarian Algorithm

1. From each row of A, find the row minimum, and subtract it from all elements in that row.
2. From each column of A, find the column minimum, and subtract it from all elements in that column.
3. Cross out the minimum number of rows and columns in A to cover all zero elements of A
4. If all rows of A are crossed out, we are done, and go to step 6.
5. Otherwise, find the minimum entry of A that is not crossed out. Subtract it from all entries of A that are not crossed out. Also, add it to all elements that are crossed out. Return to step 2 with the new matrix.
6. Solutions are zero elements of A. Go first for the zero element which is unique in its row and column. Then, delete that row and column from A. Repeat until you delete all rows or columns from A.

Example -- The Hungarian Algorithm

given a
cost matrix

$$A = \begin{bmatrix} 14 & 5 & 8 & 7 \\ 2 & 12 & 6 & 5 \\ 7 & 8 & 3 & 9 \\ 2 & 4 & 6 & 10 \end{bmatrix}$$

$$\Downarrow$$

step 1: find minimums
in each row and subtract

$$A = \begin{bmatrix} 9 & 0 & 3 & 2 \\ 0 & 10 & 4 & 3 \\ 4 & 5 & 0 & 6 \\ 0 & 2 & 4 & 8 \end{bmatrix}$$

Example -- The Hungarian Algorithm

step 2: find minimums
in each column and subtract

$$A = \begin{bmatrix} 9 & 0 & 3 & 0 \\ 0 & 10 & 4 & 1 \\ 4 & 5 & 0 & 4 \\ 0 & 2 & 4 & 6 \end{bmatrix}$$

Example -- The Hungarian Algorithm

step 3: cross out the zeros with a minimum number of lines

$$A = \begin{bmatrix} \mathbf{9} & \mathbf{0} & \mathbf{3} & \mathbf{0} \\ \mathbf{0} & 10 & 4 & 1 \\ \mathbf{4} & \mathbf{5} & \mathbf{0} & \mathbf{4} \\ \mathbf{0} & 2 & 4 & 6 \end{bmatrix}$$

bold means crossed out

we found 3 lines < 4 rows

Example -- The Hungarian Algorithm

step 5: find minimum
that is not crossed out

$$A = \begin{bmatrix} \mathbf{9} & \mathbf{0} & \mathbf{3} & \mathbf{0} \\ \mathbf{0} & 10 & 4 & 1 \\ \mathbf{4} & \mathbf{5} & \mathbf{0} & \mathbf{4} \\ \mathbf{0} & 2 & 4 & 6 \end{bmatrix}$$

Example -- The Hungarian Algorithm

step 5: subtract from non-crossed
and add to crossed out elements

$$A = \begin{bmatrix} \mathbf{10} & \mathbf{1} & \mathbf{4} & \mathbf{1} \\ \mathbf{1} & 9 & 3 & 0 \\ \mathbf{5} & \mathbf{6} & \mathbf{1} & \mathbf{5} \\ \mathbf{1} & 1 & 3 & 5 \end{bmatrix}$$

Example -- The Hungarian Algorithm

Return to step 1: find minimums
in each row and subtract

$$A = \begin{bmatrix} 9 & 0 & 3 & 0 \\ 1 & 9 & 3 & 0 \\ 4 & 5 & 0 & 4 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

Example -- The Hungarian Algorithm

Repeated step 2: find minimums in each column and subtract

$$A = \begin{bmatrix} 9 & 0 & 3 & 0 \\ 1 & 9 & 3 & 0 \\ 4 & 5 & 0 & 4 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

note: no change from the previous step

Example -- The Hungarian Algorithm

Repeated step 3: cross out the zeros
with a minimum number of lines

$$A = \begin{bmatrix} \mathbf{9} & \mathbf{0} & \mathbf{3} & \mathbf{0} \\ 1 & 9 & 3 & \mathbf{0} \\ \mathbf{4} & \mathbf{5} & \mathbf{0} & 4 \\ \mathbf{0} & \mathbf{0} & \mathbf{2} & 4 \end{bmatrix}$$

bold means crossed out

we found 4 lines = 4 rows

Example -- The Hungarian Algorithm -- Solution

go for the unique solution first

step 6:

$$A = \begin{bmatrix} 9 & 0 & 3 & 0 \\ 1 & 9 & 3 & 0 \\ 4 & 5 & 0 & 4 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$f = \{(\mathbf{d}_3, \mathbf{d}'_3)\}$$

Example -- The Hungarian Algorithm -- Solution

go for the unique solution first

step 5:

$$A = \begin{bmatrix} 9 & 0 & 3 & 0 \\ 1 & 9 & 3 & 0 \\ 4 & 5 & 0 & 4 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$f = \{(\mathbf{d}_3, \mathbf{d}'_3),\ (\mathbf{d}_4, \mathbf{d}'_1)\}$$

Example -- The Hungarian Algorithm -- Solution

go for the unique solution first

step 5:

$$A = \begin{bmatrix} 9 & 0 & 3 & 0 \\ 1 & 9 & 3 & 0 \\ 4 & 5 & 0 & 4 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$f = \{(\mathbf{d}_3, \mathbf{d}'_3),\ (\mathbf{d}_4, \mathbf{d}'_1),\ (\mathbf{d}_1, \mathbf{d}'_2)\}$$

Example -- The Hungarian Algorithm -- Solution

go for the unique solution first

step 5:

$$A = \begin{bmatrix} 9 & 0 & 3 & 0 \\ 1 & 9 & 3 & 0 \\ 4 & 5 & 0 & 4 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$f = \{(\mathbf{d}_3, \mathbf{d}'_3),\ (\mathbf{d}_4, \mathbf{d}'_1),\ (\mathbf{d}_1, \mathbf{d}'_2),\ (\mathbf{d}_2, \mathbf{d}'_4)\}$$

There is a number of alternative solutions!