CS517 Homework 1 Spring 2013 Due: 4/15/2013

- (a) Let f(w) = 1 if w has twice as many 0's as 1's and 0 otherwise. Give a Turing machine to compute f. Assume a simpler model of the Turing machine where there is only one input-output tape, and the input initially is written on the tape starting with the first position. The tape should contain just the output bit at the end of computation. Since there is a single input/output tape, the Turing machine can be fully described using the transition function δ : Γ × Q → Γ × Q × {L, S, R}.
 - (b) How long does it take to compute f as a function of n in O notation with your machine?
- 2. For this question consider one tape Turing machine model M described above. In some texts, the Turing Machine is not allowed to keep its head in its current cell after the transition. Call this model M'. Show that any function that can be computed by M in time T(n) can be computed in at most time 2T(n) by M'. In particular you need to give an algorithm for translating the transition function of M into a transition function for M' and compute an upper bound on the time.
- 3. Do problem 1.9 in A&B.
- 4. Show that the following problem HALT0 is uncomputable by reducing HALT to HALT0. HALT0(i) = 1 if M_i halts on blank input and 0 otherwise.
- 5. Consider the following function LeftMove.

LeftMove(M, w) = 1 if M ever moves its head left on input w and 0 otherwise.

Show that LeftMove is computable, i.e., show that there is a Turing machine that takes as input the code of another arbitrary Turing machine M and input w, halts in a finite time and ouputs 1 if M ever moves its head left on input w and 0 otherwise.

- 6. Show that the function EvenTM is not decidable (computable). EvenTM(M) = 1 if M accepts an even length string and 0 otherwise. Do not use Rice's theorem.
- 7. Show that the following language is not semi-decidable (also called "recursively enumerable" or "Turing-recognizable" (Sipser)). $\{i \mid \text{Turing Machine } M_i \text{ does not accept the string "0000"} \}$
- 8. Show that the following language is semi-decidable: $\{i | \text{ Turing Machine } M_i \text{ accepts at least } 100 \text{ strings } \}$
- 9. Show that the following language is not semi-decidable: $\{i | \text{ Turing Machine } M_i \text{ accepts at most } 2 \text{ strings } \}$