THE PRINCIPLES OF ΔΣ DATA CONVERTERS

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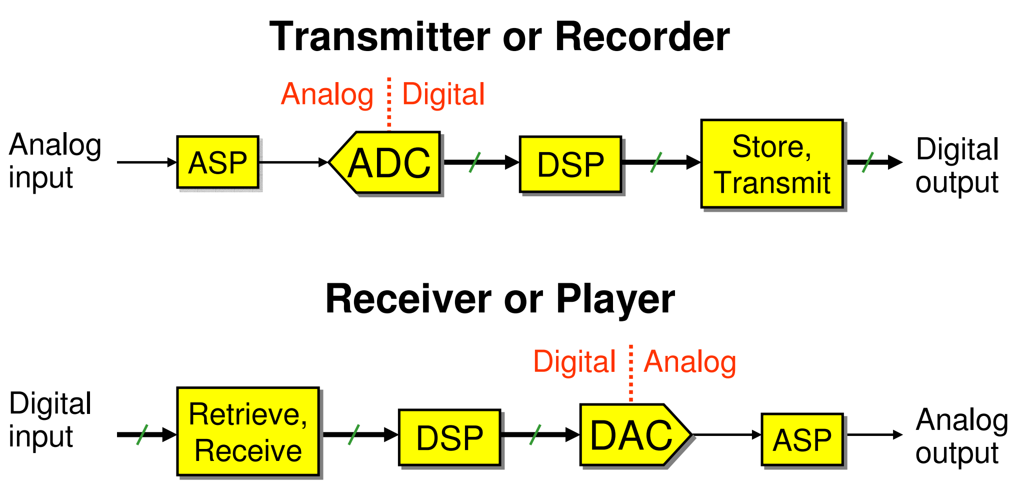
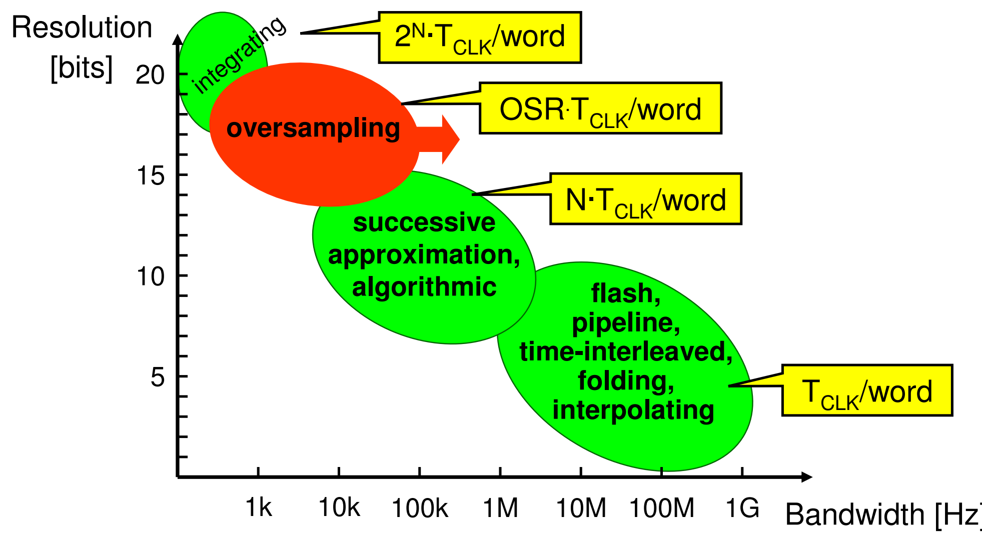
Mixed-Mode Electronic System



- Usually, the ADC, DAC and ASP blocks limit the accuracy and bandwidth.

Popular Analog-to-Digital Converters

- For ADCs, trade-off exists between speed and accuracy.
- Oversampling ΔΣ converters have been typically used for high-resolution, low-bandwidth applications.
- Recently, there is a trend towards higher-bandwidth applications.



A Few Applications

Consumer Electronics

Multimedia

Sensors

Communications

Instrumentation

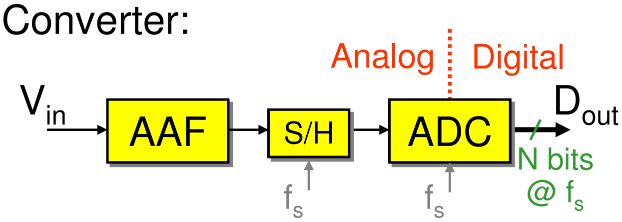
Medical

Music
Production

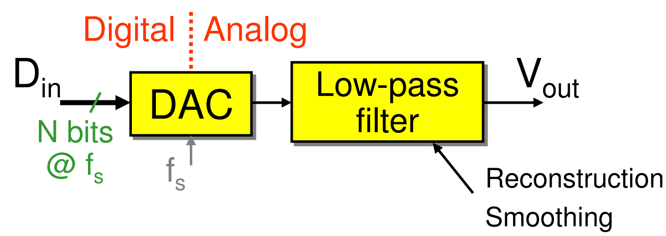
Nyquist-Rate Converters

Nyquist-Rate ⇔ Static, Memoryless, Sample-by-Sample

- A/D Converter:



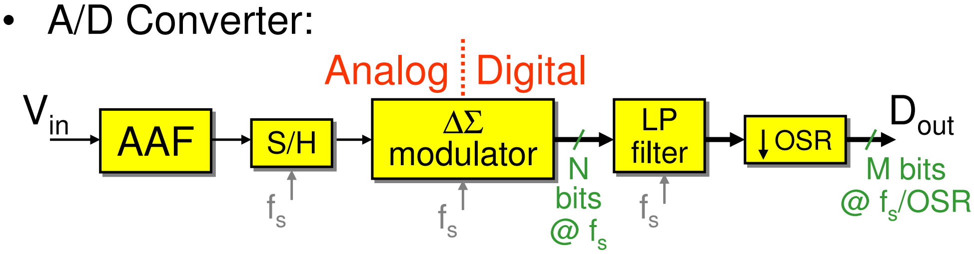
- D/A Converter:



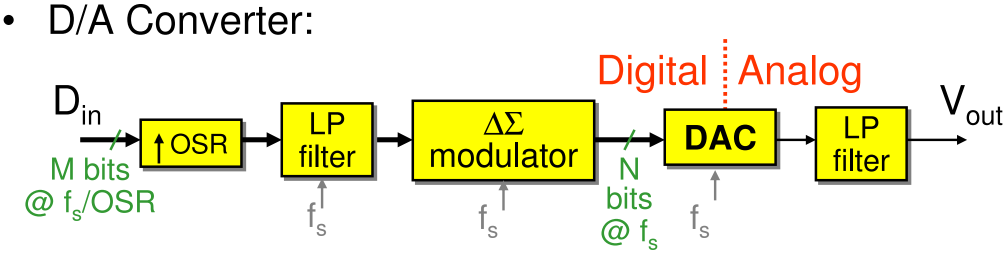
- The accuracy is limited by the matching of analog components.

Oversampling Converters

- A/D Converter:

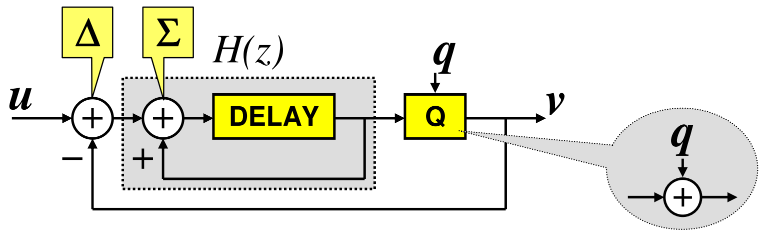


- D/A Converter:



- Advantages:
  - Simpler anti-aliasing filter (AAF).
  - Relaxed requirements on analog circuitry.
  - Resolution ↔ bandwidth trade-off possible.

First-Order Noise Shaping ADC (1)



- Assume that q is random white noise, uncorrelated with u. (Valid only for large and fast input to Q. May cause problems otherwise) Then:
- Signal Transfer Function: $STF = \dfrac{V}{U} = \dfrac{H}{1+H} = z^{-1}$
- Noise Transfer Function: $NTF = \dfrac{V}{Q} = \dfrac{1}{1+H} = 1 - z^{-1}$
- Negative feedback loop; *H(z)* is the gain block: $H(z) = \dfrac{z^{-1}}{1 - z^{-1}}$

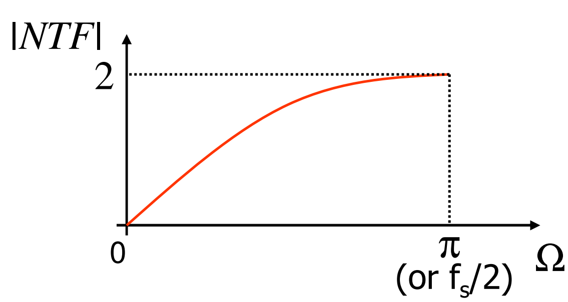
First-Order Noise Shaping ADC (2)

- Squared magnitude of signal transfer function: $\left|STF\right|^2 = \left|z^{-1}\right|^2 = 1$
- Squared magnitude of noise transfer function:

$$\left|NTF\right|^2 = \left|1 - z^{-1}\right|^2 = \left|1 - e^{-j\Omega}\right|^2 = \left|1 - \cos\Omega + j\sin\Omega\right|^2$$

…where: $\Omega = 2\pi f / f_s$

$$\left|NTF\right|^2 = (1 - \cos\Omega)^2 + \sin^2\Omega = 2 - 2\cos\Omega = \left(2\sin\tfrac{\Omega}{2}\right)^2$$

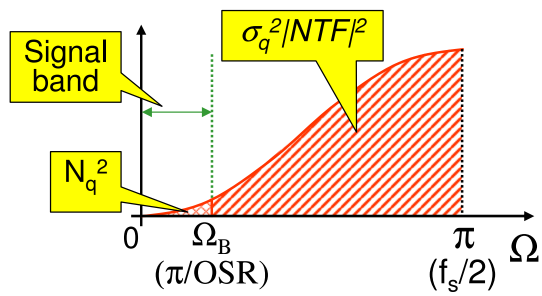


First-Order Noise Shaping ADC (3)

- For quantization noise PSD σ_q^2, the in-band noise power is:

$$N_q^2 = \frac{1}{\pi}\int_0^{\Omega_B}\sigma_q^2|NTF|^2 d\Omega = \frac{\sigma_q^2}{\pi}\int_0^{\pi/OSR}\left(2\sin\tfrac{\Omega}{2}\right)^2 d\Omega$$

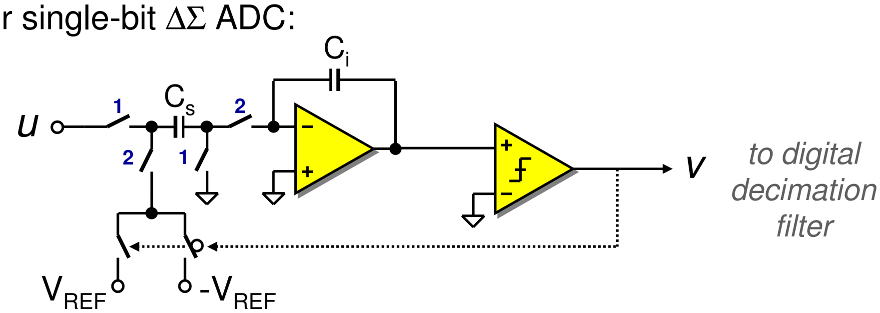
$$\approx \frac{\sigma_q^2}{\pi}\int_0^{\pi/OSR}\Omega^2 d\Omega = \frac{\sigma_q^2\pi^2}{3\mathrm{OSR}^3}$$

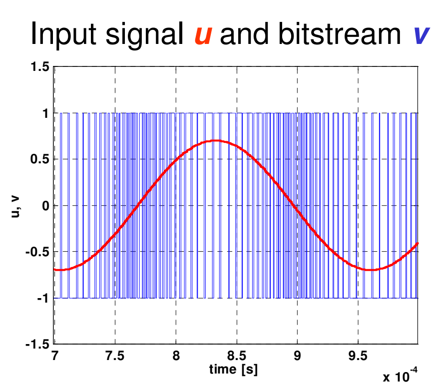


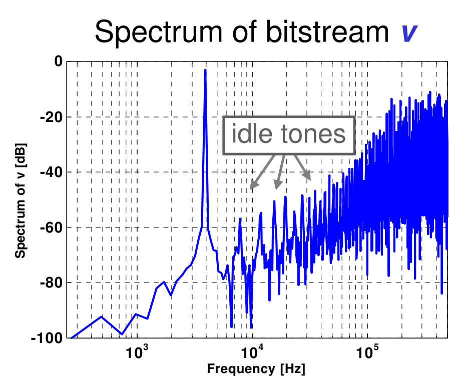
Doubling OSR increases SNR by 9 dB (1.5 bit/oct)

Circuit Implementation and Simulations

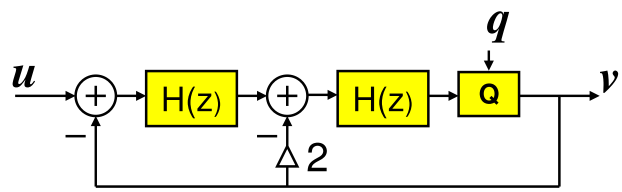
- First-order single-bit ΔΣ ADC:





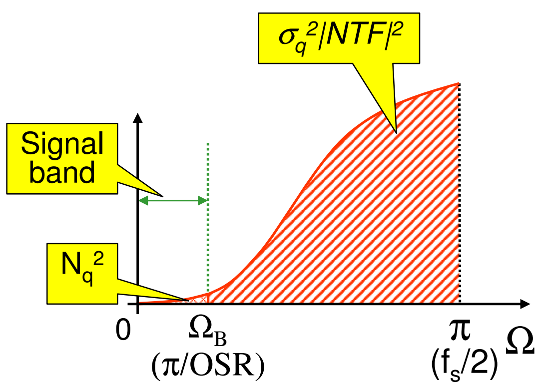


Second-Order Noise Shaping ADC (1)



- Signal Transfer Function: $STF = \dfrac{H^2}{1+2H+H^2} = z^{-2}$
- Noise Transfer Function: $NTF = \dfrac{1}{1+2H+H^2} = (1-z^{-1})^2$
- Magnitude of noise transfer function: $|NTF|^2 = \left(2\sin\dfrac{\Omega}{2}\right)^4$

Second-Order Noise Shaping ADC (2)



- In-band noise power: $N_q^2 \approx \dfrac{\sigma_q^2 \pi^4}{5\mathrm{OSR}^5}$

Doubling OSR increases SNR by 15 dB (2.5 bit/oct)

Generalization (1)

- For an L-order NTF, the in-band noise power is:

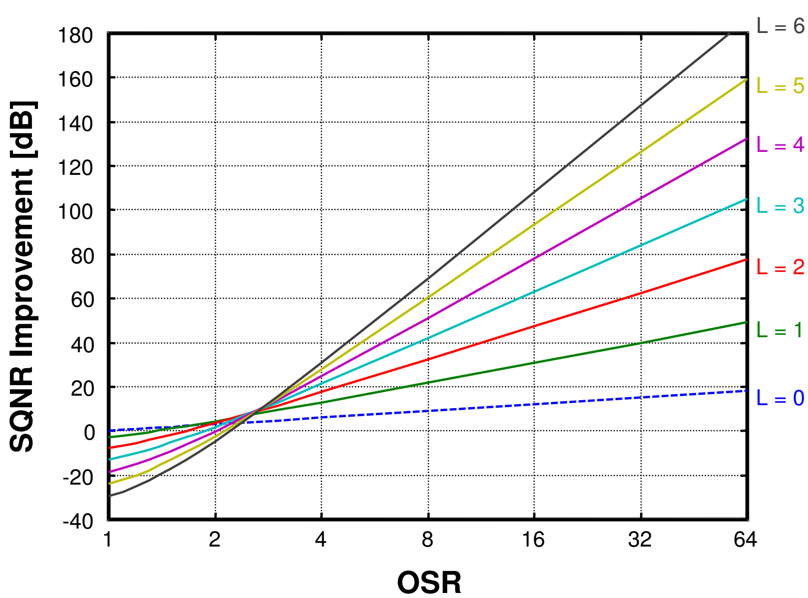
$$N_q^2 \approx \frac{\sigma_q^2 \pi^{2L}}{(2L+1)\cdot \mathrm{OSR}^{2L+1}}$$

Doubling OSR increases SNR by 6L+3 dB (L+0.5 bit/oct)

- Maximum SQNR (valid for large OSR):

$$SQNR_{\max} \approx \underbrace{6.02N + 1.76}_{\text{Quantizer resolution}} + \underbrace{(20L+10)\log_{10} OSR - 10\log_{10}\frac{\pi^{2L}}{2L+1}}_{\text{SQNR improvement}} \quad \text{[dB]}$$

Generalization (2)



Non-Ideal Effects (1)

- So, to get a high SNR:
  – Increase number of bits in the quantizer (*N*)
  – Increase order of noise-shaping function (*L*)
  – Increase oversampling ratio (*OSR*)
- But there are non-ideal effects to take into account:
  – Quantization noise is not the only noise source (1/f, thermal, digital crosstalk, etc).
  – Quantization noise is not truly white (tones, limit cycles).
  – Noise transfer function is not ideal (mismatches, finite opamp gain).
- And these deserve special attention:
  – DAC with $N > 1$ causes linearity problems.
  – $L > 2$ causes stability problems.

Non-Ideal Effects (2)

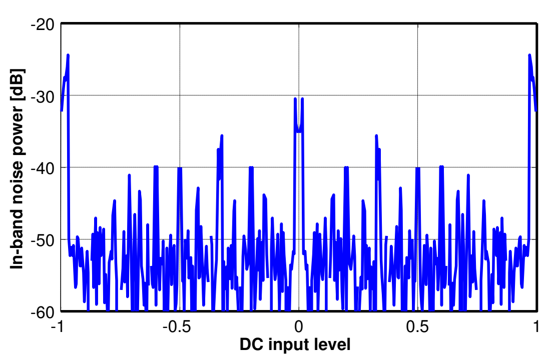
LIMIT CYCLES

- Limit cycles appear for DC or slow varying signals, if the input voltage is near a rational multiple of V_{REF}, i.e.:

$$u \approx \frac{n}{m} V_{REF} \qquad \text{where } n \text{ and } m \text{ are integers}$$

… which causes the output ***v*** to repeat itself with a certain period.

- If frequency of repetition falls in band, SNR can be severely degraded.

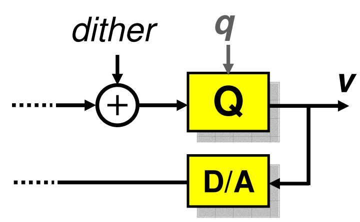


Non-Ideal Effects (3)

TONES

- Tones are caused by correlation with input signal *u*.
- Amplitude of tones increases with the frequency and amplitude of input signal *u*, and decreases with higher order *L* of modulator.

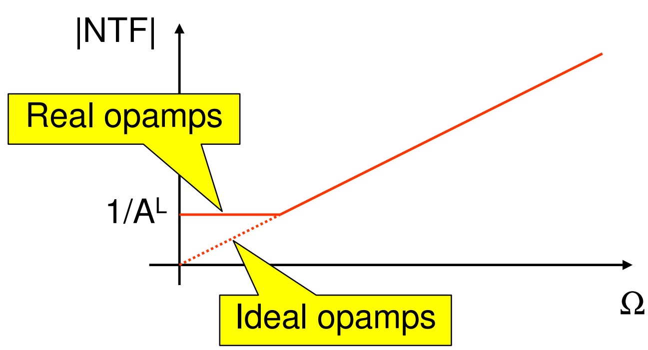
- Easiest way to prevent limit cycles and tones is to add random noise (dither) at input of quantizer:



Non-Ideal Effects (4)

FINITE OPAMP GAIN

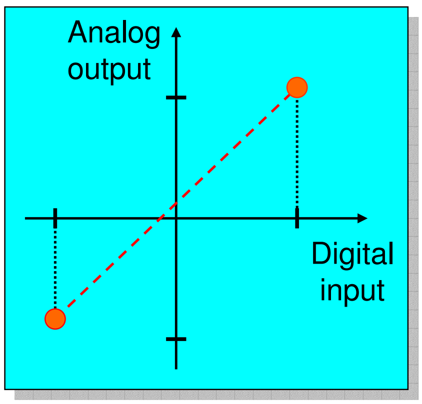
- For ΔΣ ADCs, the gain of the opamps determines how much the noise is suppressed in the baseband.



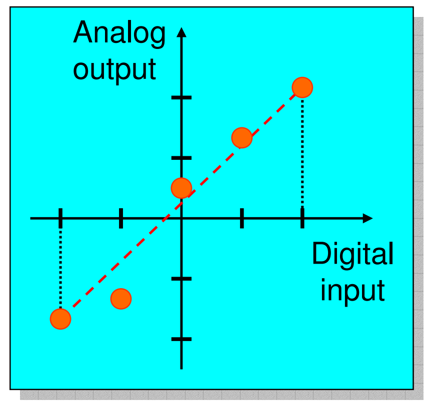
DAC Linearity (1)

- Single-bit DAC ($N = 1$) is always linear. (Only gain and offset error.)
- Multi-bit DAC ($N > 1$) is only as linear as its analog circuit elements match (typically 9 to 12 bits)

Single-bit DAC



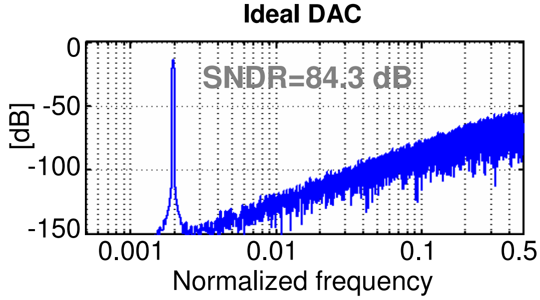
Multi-bit DAC



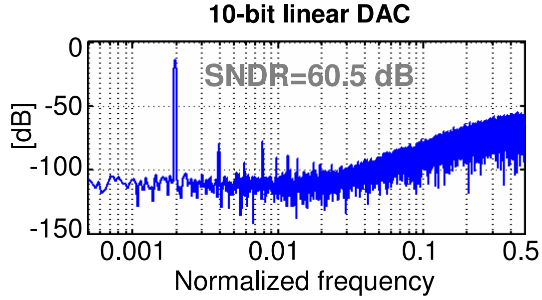
DAC Linearity (2)

- Second-order ΔΣ modulator with 4-bit quantizer and DAC
- OSR = 32 ⇒ Ideal SNDR ≈ 14 bit

Ideal DAC



10-bit linear DAC



- Nonlinear DAC causes higher noise floor and harmonics.
- 10-bit linear DAC causes 10-bit level SNDR.

In general, linearity of ΔΣ modulator is no better than linearity of DAC

Improving DAC Linearity

1. Element calibration:

- During fabrication (e.g., laser trimming) – expensive, not effective for long-term process variations (temperature, aging, etc).
- During circuit operation – can be performed periodically, but increases analog design difficulty .

2. Dynamic element matching (DEM):

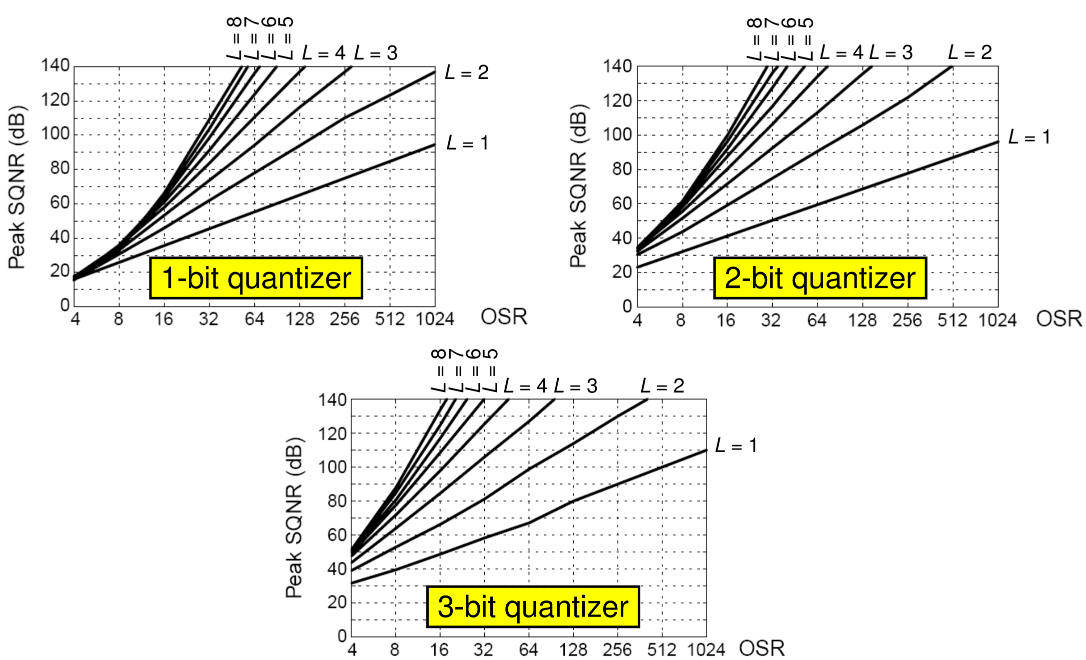
- Randomize usage of analog elements, so that DAC errors are averaged.
- Many different flavors are available (barrel-shifting, individual-level averaging, data-weighted averaging, tree-structure, etc).
- Works well, but only for high OSR (OSR > 16).

3. Digital Estimation and Correction of DAC errors:

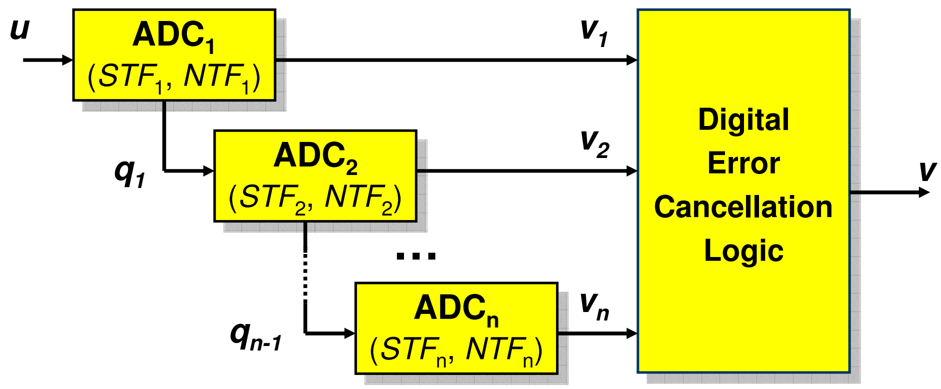
- Correlation based method. Works for any OSR.

Empirical Results

- SQNR limits for modulators of order *L*:



Multi-Stage Noise Shaping (1)



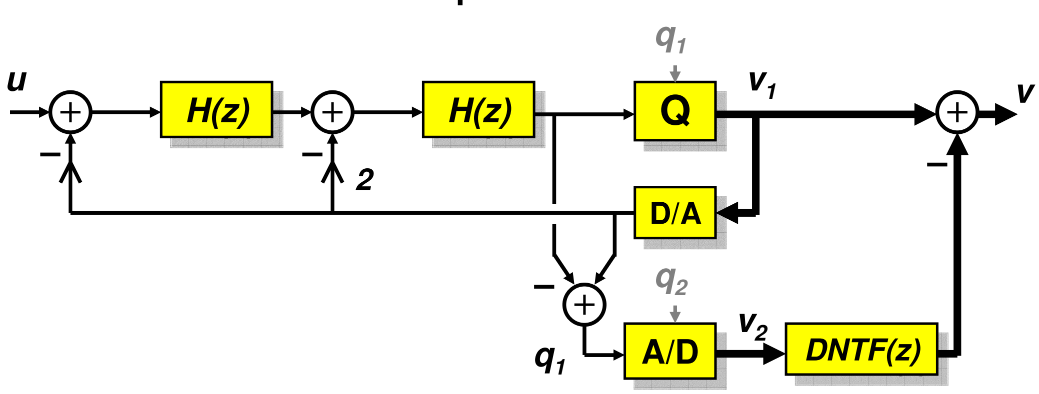
- Purpose of error cancellation logic is to eliminate quantization noise from all stages, except the last, so that:

$$V = U \cdot STF_1 STF_2 \cdots STF_n + Q_n \cdot NTF_1 NTF_2 \cdots NTF_n$$

- Order of NTF is the product of the individual orders L_1 to L_n.
- Stability is easily guaranteed if $L \leq 2$ for every stage.

Multi-Stage Noise Shaping (2)

Example: 2-0 MASH



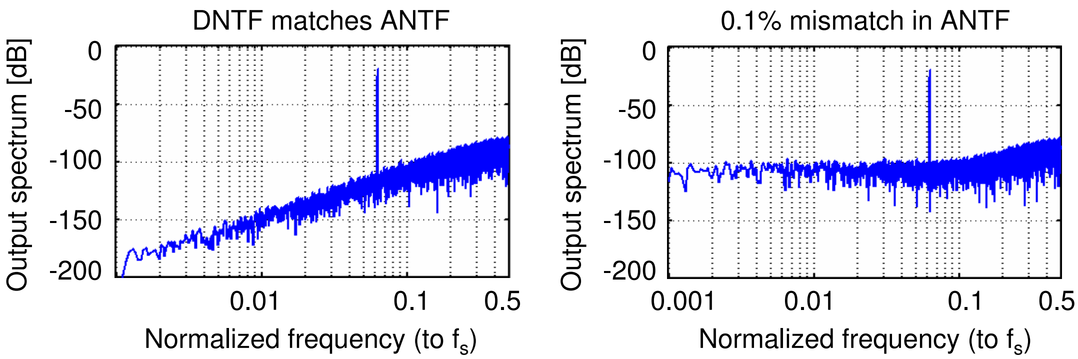
$$V = U \cdot STF_1 + Q_1(ANTF - DNTF) - Q_2 \cdot DNTF$$

- Assuming that everything is ideal, and *DNTF* = *ANTF*, the quantization noise q_1 is cancelled at the output:

$$V = U \cdot STF_1 - Q_2 \cdot DNTF$$

Multi-Stage Noise Shaping (3)

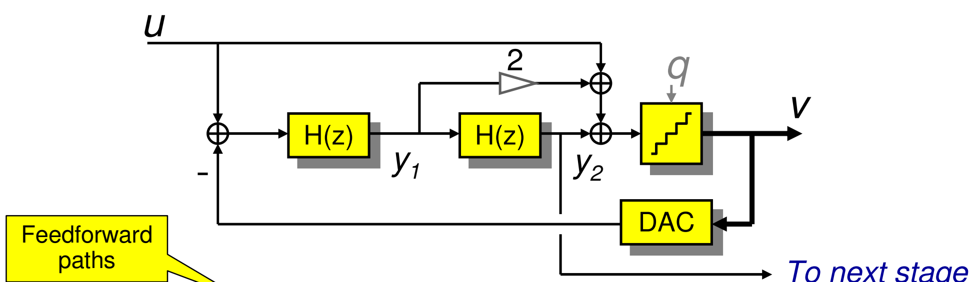
- *ANTF* is an analog transfer function. *DNTF* is a digital transfer function. What if *ANTF* and *DNTF* do not match exactly?



- A certain amount of q_1 will show at the output v, degrading SNR.
- This is commonly referred to as “quantization noise leakage”.
- Can be suppressed using digital correlator.

Low-Distortion ΔΣ Topology (1)

- In traditional topologies, integrators have to process some of input signal.
- This can be avoided by making STF = 1.

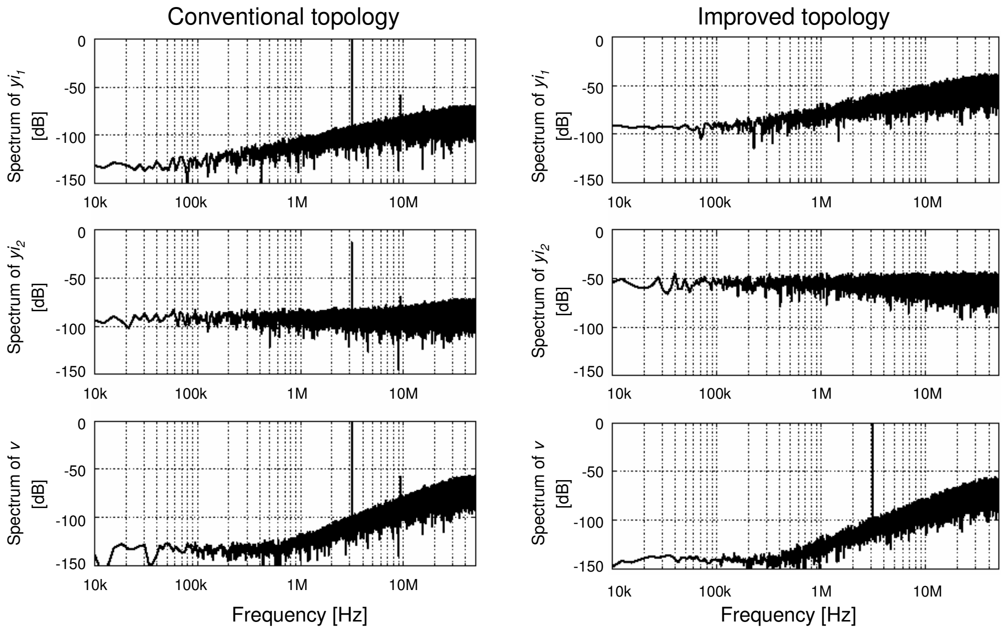


$$STF = \frac{1+2H+H^2}{1+2H+H^2} = 1 \qquad NTF = \frac{1}{1+2H+H^2} = \left(1-z^{-1}\right)^2$$

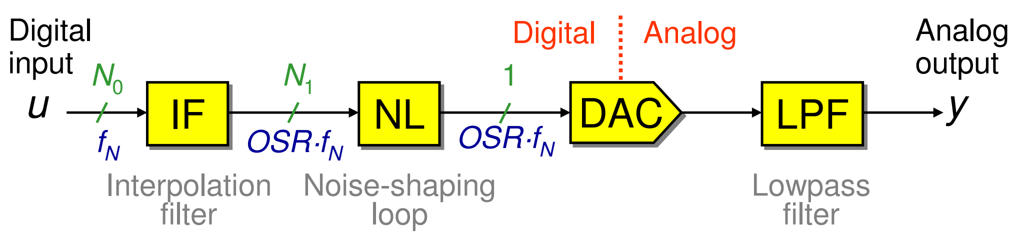
- Integrators do not process input signal, only quantization noise.
  No signal ⇒ No distortion.
- Bonuses:
  - Low area and power consumption due to integrator gain scaling.
  - Quantization noise readily available at y_2. No analog subtraction needed for MASH.

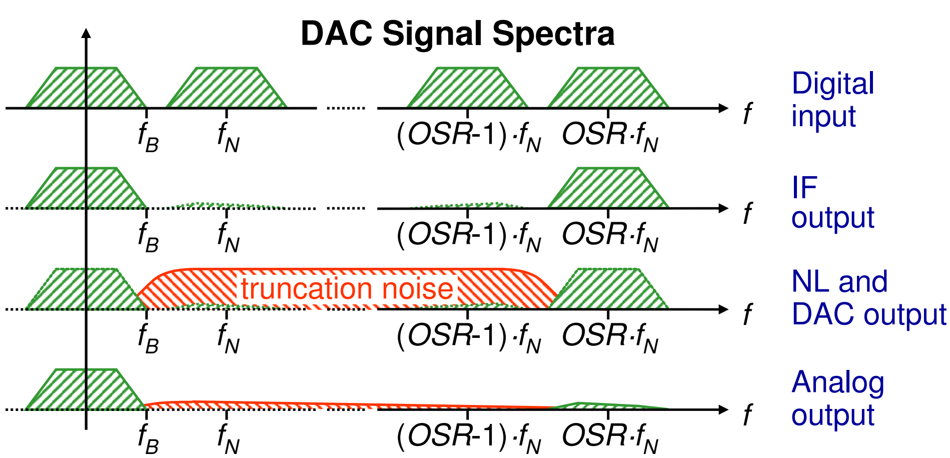
Low-Distortion ΔΣ Topology (2)

- MATLAB simulations with nonlinear function included in first integrator:



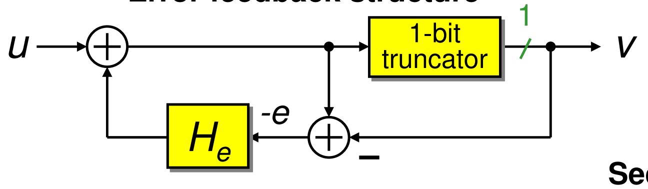
ΔΣ DAC System



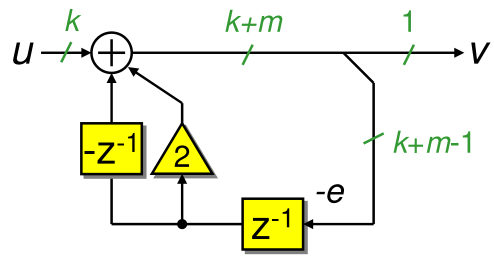


Error Feedback Architecture

Error feedback structure



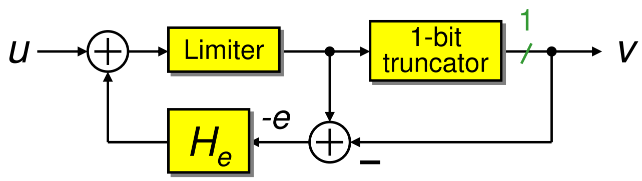
Second-order error-feedback noise-shaping loop



$$V(z) = U(z) + [1 - H_e(z)E(z)]$$

$$H_e(z) = 1 - (1 - z^{-1})^2 = z^{-1}(2 - z^{-1})$$

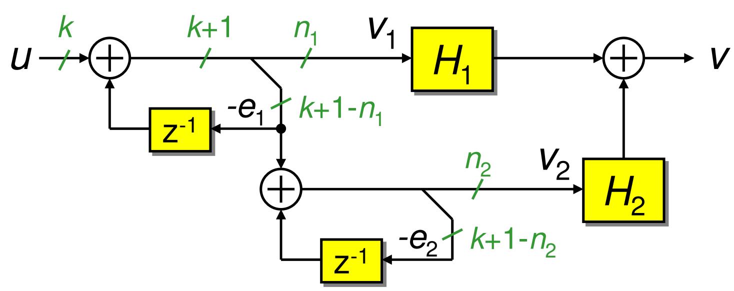
Error feedback with limiter



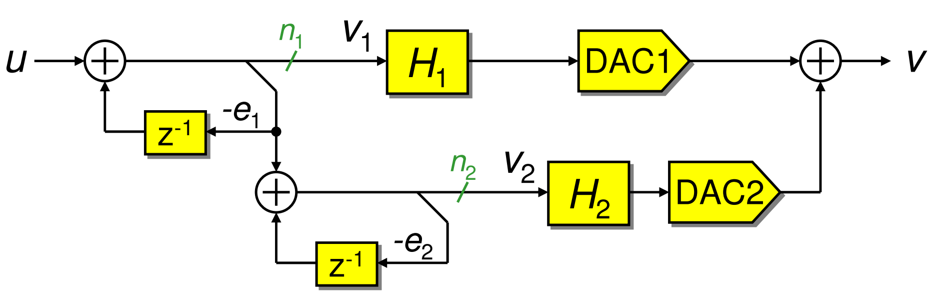
Also, all ΔΣ configurations can be used in the noise-shaping loop.

Cascade DACs

- Cascade structure for a second-order noise shaping loop:

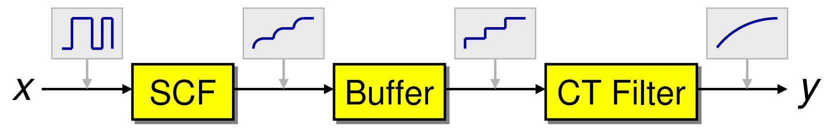


- A cascade DAC using analog recombination:

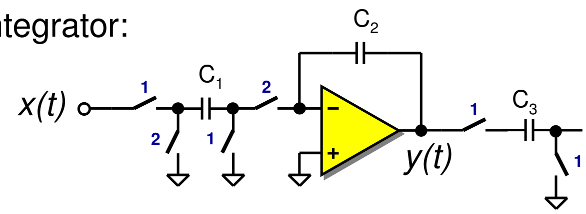


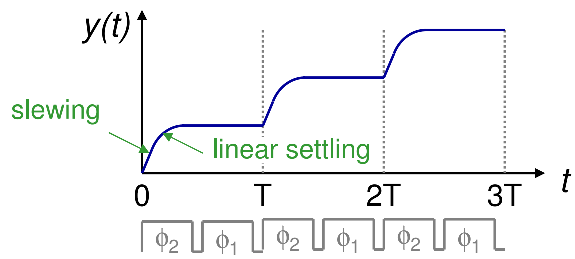
Post-Filter Design

- Post filter for a 1-bit ΔΣ DAC and associated signals:



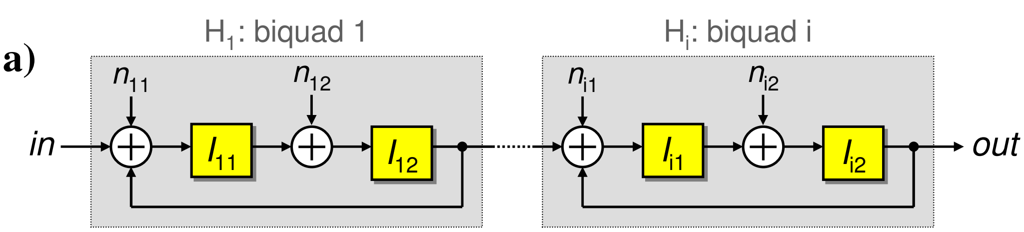
- An SC integrator:



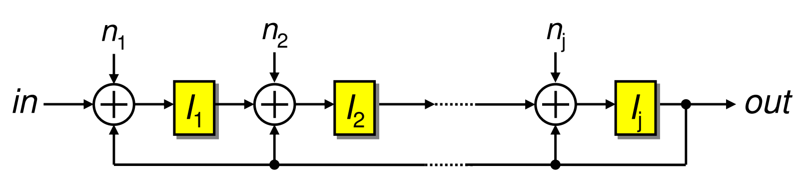


Reconstruction Filter Architectures

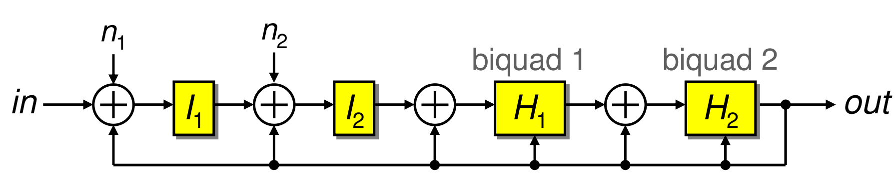
a)



b)

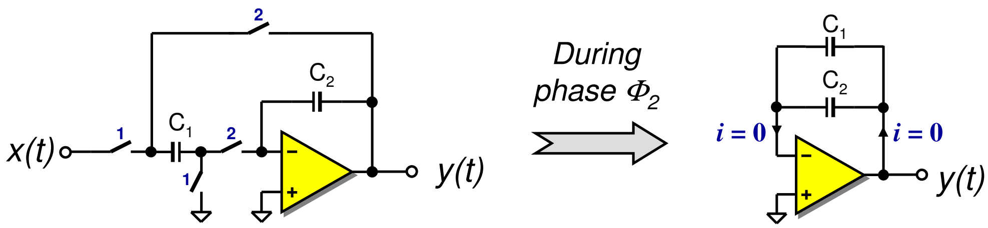


c)

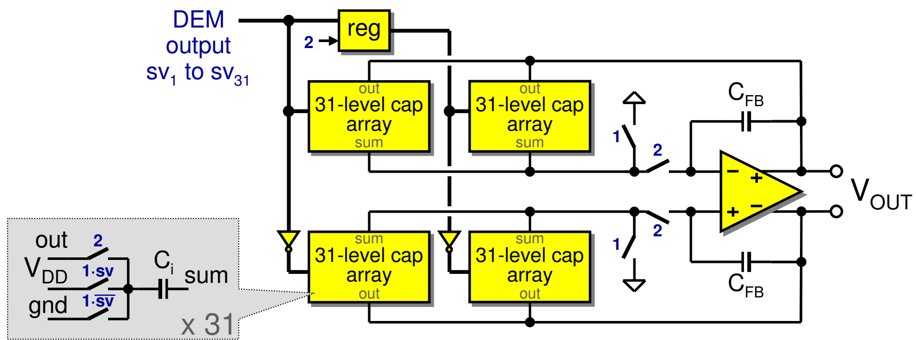


Design Example

A direct-charge-transfer (DCT) stage:

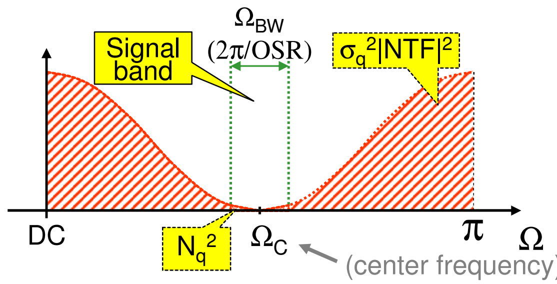


Combined DAC, DCT and filter for a multi-bit ΔΣ DAC:



Other Forms of Noise Shaping

- Continuous-time ΔΣ modulation:
  - Only quantizer and DAC are clocked.
  - Inherent anti-alias filtering, higher speeds, lower power consumption.
- Bandpass ΔΣ modulation:
  - Useful for signal bands that are not at DC.



- Complex ΔΣ modulation
  - For quadrature signal pairs. Useful in wireless applications.
- Fractional-N dividers:
  - Used in PLLs to generate accurate frequency ratios.
  - Useful for clock recovery, channel tuning in radios, etc.

Conclusions

- Oversampling data converters offer a trade-off: fast clocking and increased digital complexity are traded for relaxed analog tolerance and/or conversion accuracy.
- This trade-off is attractive for high-resolution converters with narrow (< 5 MHz) bandwidths, allowing many applications in instrumentation, consumer electronics and communications. The bandwidth (and hence the field of applications) is continuously widening with faster IC technologies.
- The design of $\Delta\Sigma$ converters is based on a qualitative understanding of the noise shaping process based on idealized assumptions, followed by high-level simulations, and transistor-level simulation of the individual stages.
- High-resolution oversampling converters require an understanding of nonideal effects (noise coupling, signal dependent quantization errors and reference loading, noise leakage, etc.) and available methods for their prevention. This was not covered in this lecture, but can be found in the references given.