Embedded Zerotree Wavelet
EZW

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Outline

- Introduction
- Concept of EZW
- Algorithm
- Examples

Introduction

- **E**mbedded Image coding using **Z**erotrees of **W**avelet coefficients by J. M. Shapiro.
- Uses “parent-child” dependencies between subband coefficients at the same spatial location.
- Bit-plane coding: enables an embedded bitstream wrt distortion

Concepts of EZW

- (1) a discrete wavelet transform or hierarchical subband decomposition,
- (2) prediction of the absence of significant information across scales by exploiting the self-similarity inherent in images,
- (3) entropy-coded successive-approximation quantization, and
- (4) universal lossless data compression which is achieved via adaptive arithmetic coding

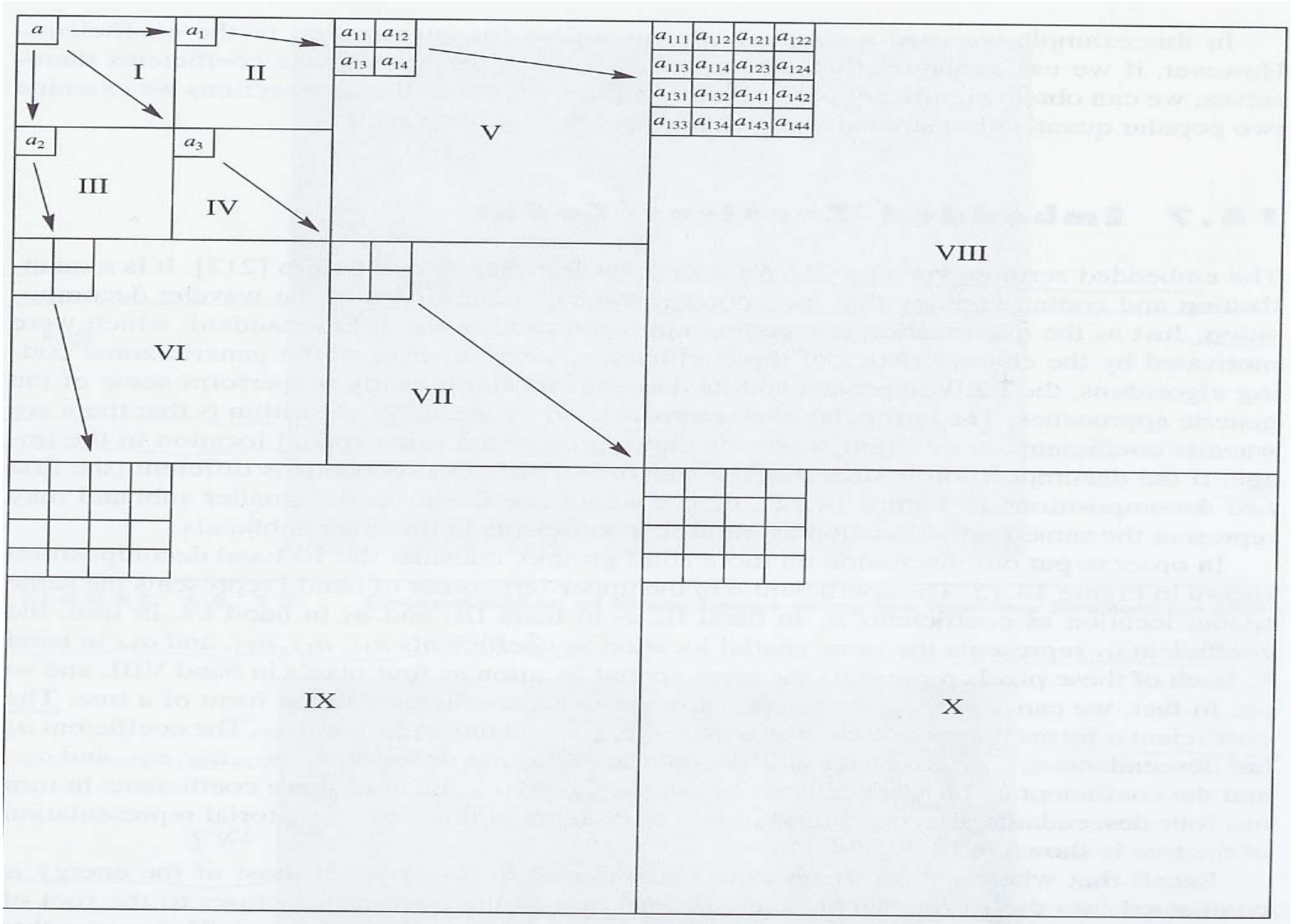


FIGURE 14.15 **A 10-band wavelet decomposition.**

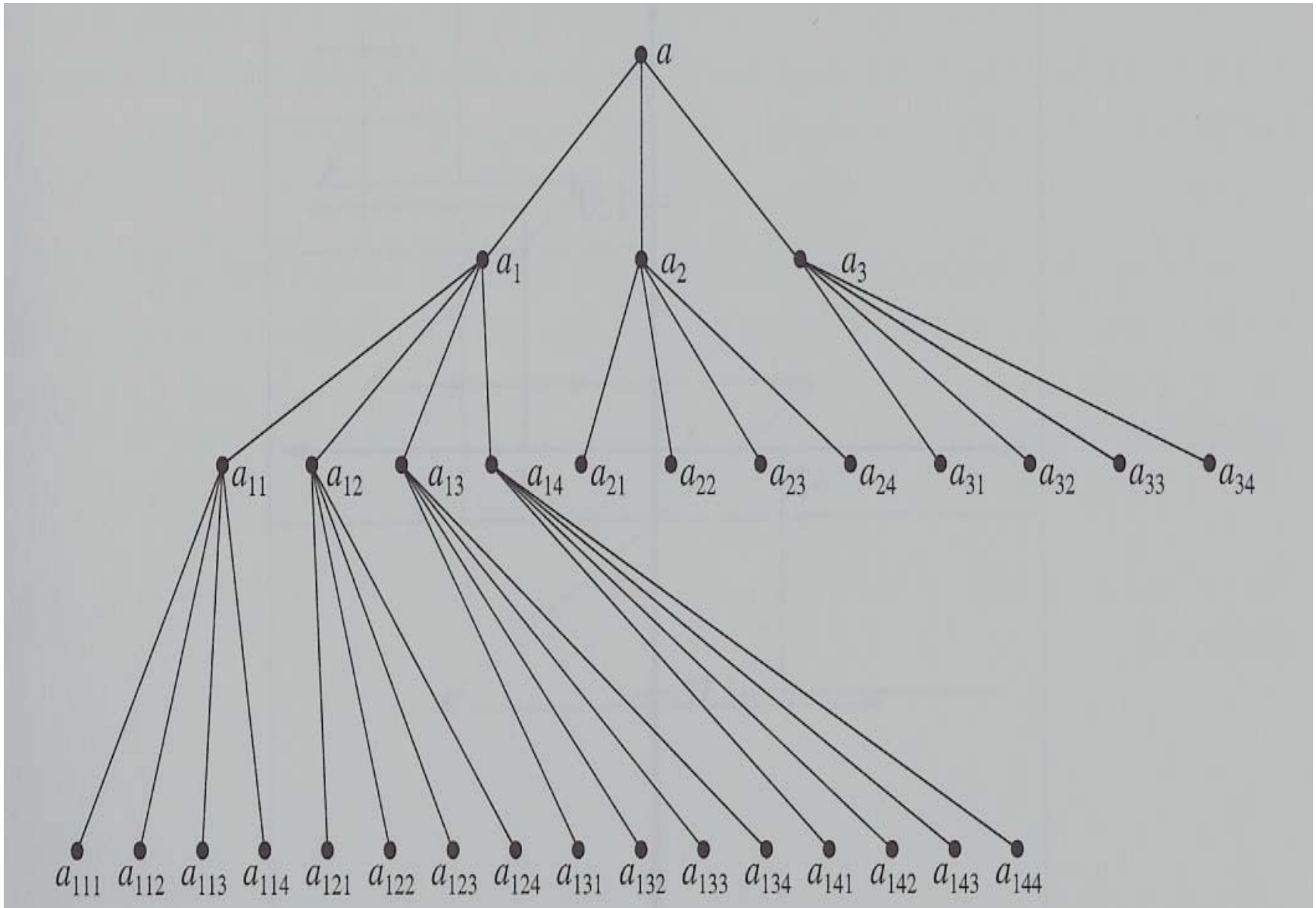
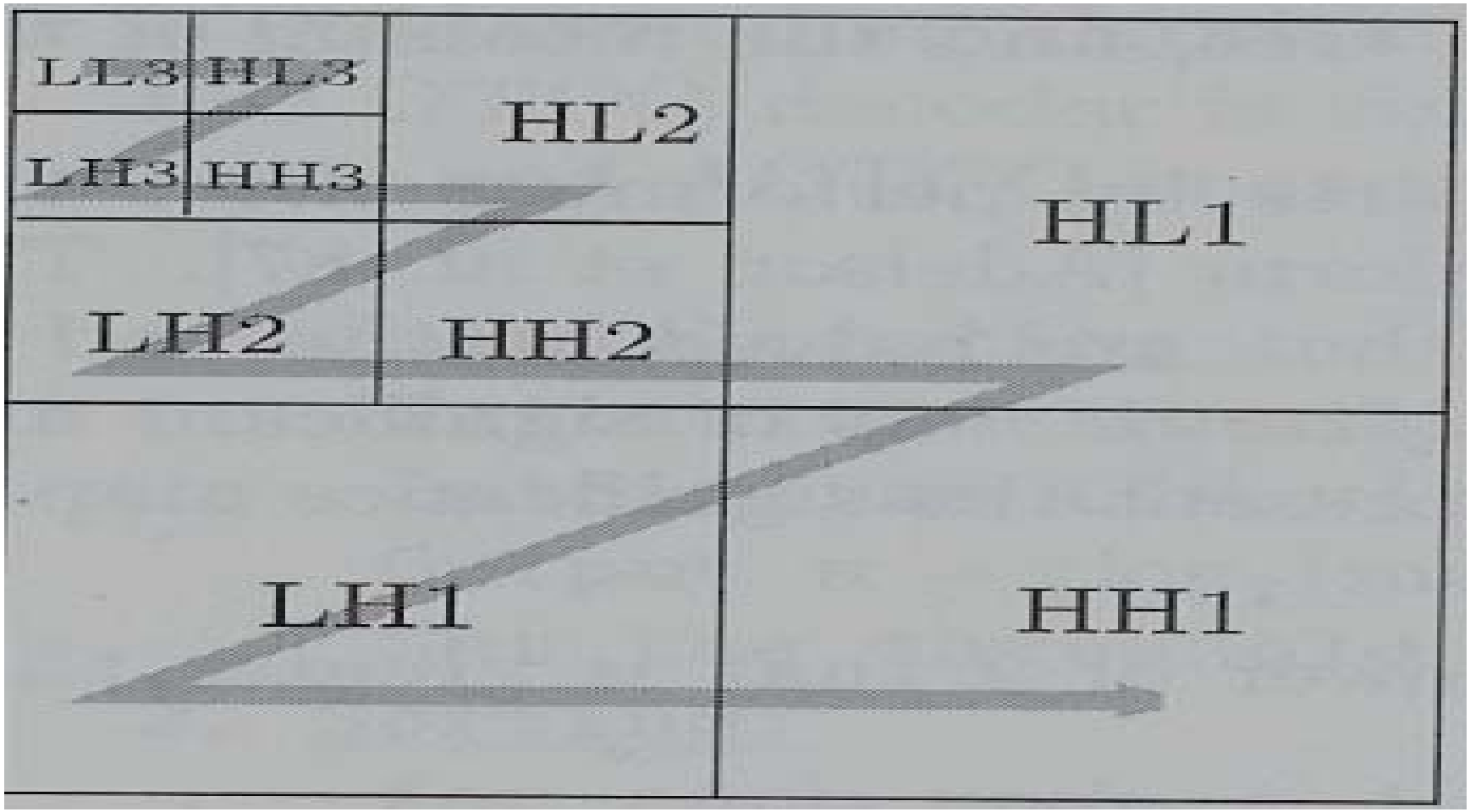


FIGURE 14.16 **Data structure used in the EZW coder.**

How does it work?

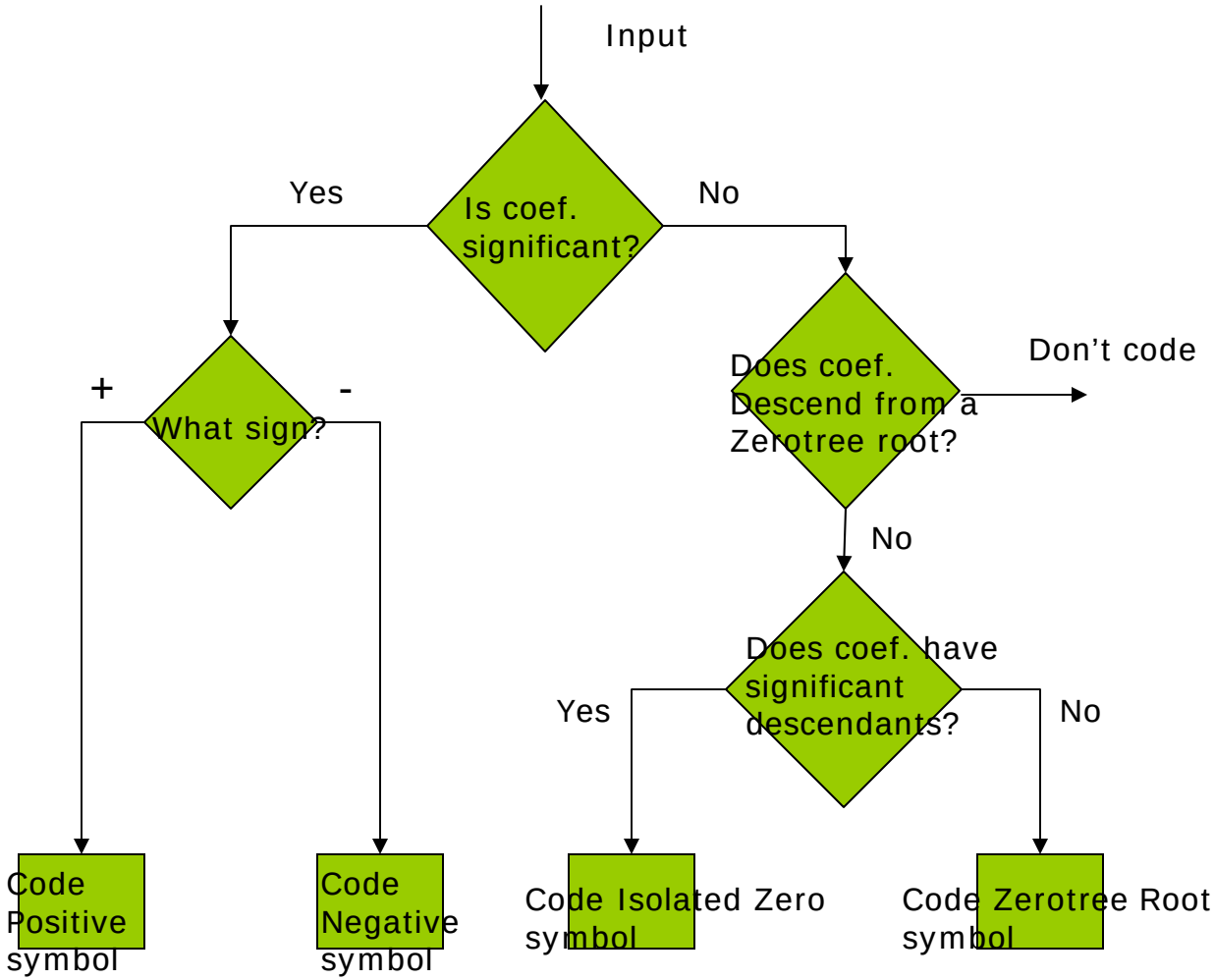
Scanning a zerotree



Terminology

- ***sp***: Given a threshold *T,* if a given coefficient has a magnitude greater than *T* , it is called a ***significant coefficient*** at level *T*
- ***sn***: ***negative signigicant***
- ***zr****: If the magnitude of the coefficient is less than T (it is insignificant), and all its descendants have magnitudes less than T, then the coefficient is called a* ***zerotree root****.*
- ***iz***: it might happen that the coefficient itself is less than *T* but some of its descendants have a value greater than *T.* Such a coefficient is called an ***isolated zero****.*

Algorithm Chart:



EZW Example (1): seven-level decomposition shown below to demonstrate the various steps of EZW

26	6	13	10
-7	7	6	4
4	-4	4	-3
2	-2	-2	0

- Initial threshold

$$T_0 = 2^{\lfloor \log_2 26 \rfloor} = 16$$

- 8 bits from bit budget

EZW Example (1): seven-level decomposition shown below to demonstrate the various steps of EZW

- 26 > 16 → sp
- 6 < 16 →

  descendants < 16 → zr
- -7 < 16 →

  descendants < 16 → zr
- 7 < 16 →

  descendants < 16 → zr
- labels to be transmitted *sp zr zr zr*

26	6	13	10
-7	7	6	4
4	-4	4	-3
2	-2	-2	0

- Initial threshold

$$T_0 = 2^{\lfloor \log_2 26 \rfloor} = 16$$

- 8 bits from bit budget

EZW Example (1): Subordinate Pass

- *Ls* = {26}
- *The significant coefficient reconstructed value*

  1.5To = 24
- reconstructed bands

24	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

EZW Example (1): Subordinate Pass

- *Ls* = {26}
- *The significant coefficient*

  1.5To = 24
- reconstructed bands

24	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

- Difference 26 – 24
- Using a 2-level quantizer with reconstruction levels ±*To/4* , correction term of 4
- Reconstruction

  24 + 4 = 28
- Transmitting the correction term costs a single bit.

28	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

EZW Example (1):

- *T1 = ½ * T0 = ½ * 16 = 8*
- 6 < 8 →

  descendants > 8 → iz
- -7 < 8 →

  descendants < 8 → zr
- 7 < 8 →

  descendants < 8 → zr
- 13 no descendants > 8 → sp
- 10 no descendants > 8 → sp
- 6 no descendants < 8 → iz
- 4 no descendants < 8 → iz

*	6	***13***	***10***
-7	7	6	4
4	-4	4	-3
2	-2	-2	0

EZW Example (1):

- *T1 = ½ * T0 = ½ * 16 = 8*
- 6 < 8 →

  descendants > 8 → iz
- -7 < 8 →

  descendants < 8 → zr
- 7 < 8 →

  descendants < 8 → zr
- 13 no descendants > 8 → sp
- 10 no descendants > 8 → sp
- 6 no descendants < 8 → iz
- 4 no descendants < 8 → iz

*	6	***13***	***10***
-7	7	6	4
4	-4	4	-3
2	-2	-2	0

- labels to be transmitted

  iz zr zr sp sp iz iz
- Requires 14 bits
- Total bits = 9 + 14 = 23

EZW Example (1): Subordinate Pass

- *The significant coefficient*

  1.5T1 = 1.5 * 8 = 12
- *Ls* = {26, 13, 10}
- reconstructed bands

28	0	12	12
0	0	0	0
0	0	0	0
0	0	0	0

EZW Example (1): Subordinate Pass

- *The significant coefficient*

  1.5T1 = 1.5 * 8 = 12
- *Ls* = {26, 13, 10}
- reconstructed bands

28	0	12	12
0	0	0	0
0	0	0	0
0	0	0	0

with a 2-level'quantizer with reconstruction levels ±*T1 / 4* = ± *2*

- 26 – 28 = -2 *Correction term* = -2
- 13 – 12 = 1 *Correction term* = 2
- 10 - 12 = -2 *Correction term* = -2
- Each correction requires a single bit, the total bits 23 + 3 = 26.
- Reconstruction

26	0	14	10
0	0	0	0
0	0	0	0
0	0	0	0

EZW Example (1):

- *T2 = ½ * T1 = ½ * 8 = 4*
- 6 > 4→ sp
- |-7| > 4 → sn
- 7 > 4→ sp
- 6 > 4 → sp
- 4 = 4 → sp
- 4 = 4 → sp
- |-4| = 4 → sn
- 2, -2 are coded as iz
- 4 = 4 → sp
- -3, -2, 0 are iz

*	6	*	*
-7	7	6	4
4	-4	4	-3
2	-2	-2	0

EZW Example (1):

- *T2 = ½ * T1 = ½ * 8 = 4*
- 6 > 4→ sp
- |-7| > 4 → sn
- 7 > 4→ sp
- 6 = 4 → sp
- 4 = 4 → sp
- 4 = 4 → sp
- |-4| = 4 → sn
- 2, -2 are coded as iz
- 4 = 4 → sp
- -3, -2, 0 are iz

*	***6***	*	*
-7	***7***	***6***	***4***
4	***-4***	***4***	-3
2	-2	-2	0

- *sp sn sp sp sp sp sn iz iz sp iz iz iz*
- Requires 26 bits
- Total bits = 26 + 26 = 52

EZW Example (1): Subordinate Pass

*	***6***	*	*
-7	***7***	***6***	***4***
4	***-4***	***4***	-3
2	-2	-2	0

- *The significant coefficient*

  1.5T2 = 1.5 * 4 = 6
- *Ls* = {26, 13, 10, 6, -7, 7, 6, 4, 4, -4, 4}
- reconstructed bands

*	***6***	*	*
-6	***6***	***6***	***6***
6	***-6***	***6***	-3
2	-2	-2	0

26	***6***	14	10
-6	***6***	***6***	***6***
6	***-6***	***6***	0
0	0	0	0

27	***7***	13	11
-7	***7***	***7***	***5***
5	***-5***	***5***	0
0	0	0	0