Lecture 13: Vector Quantization

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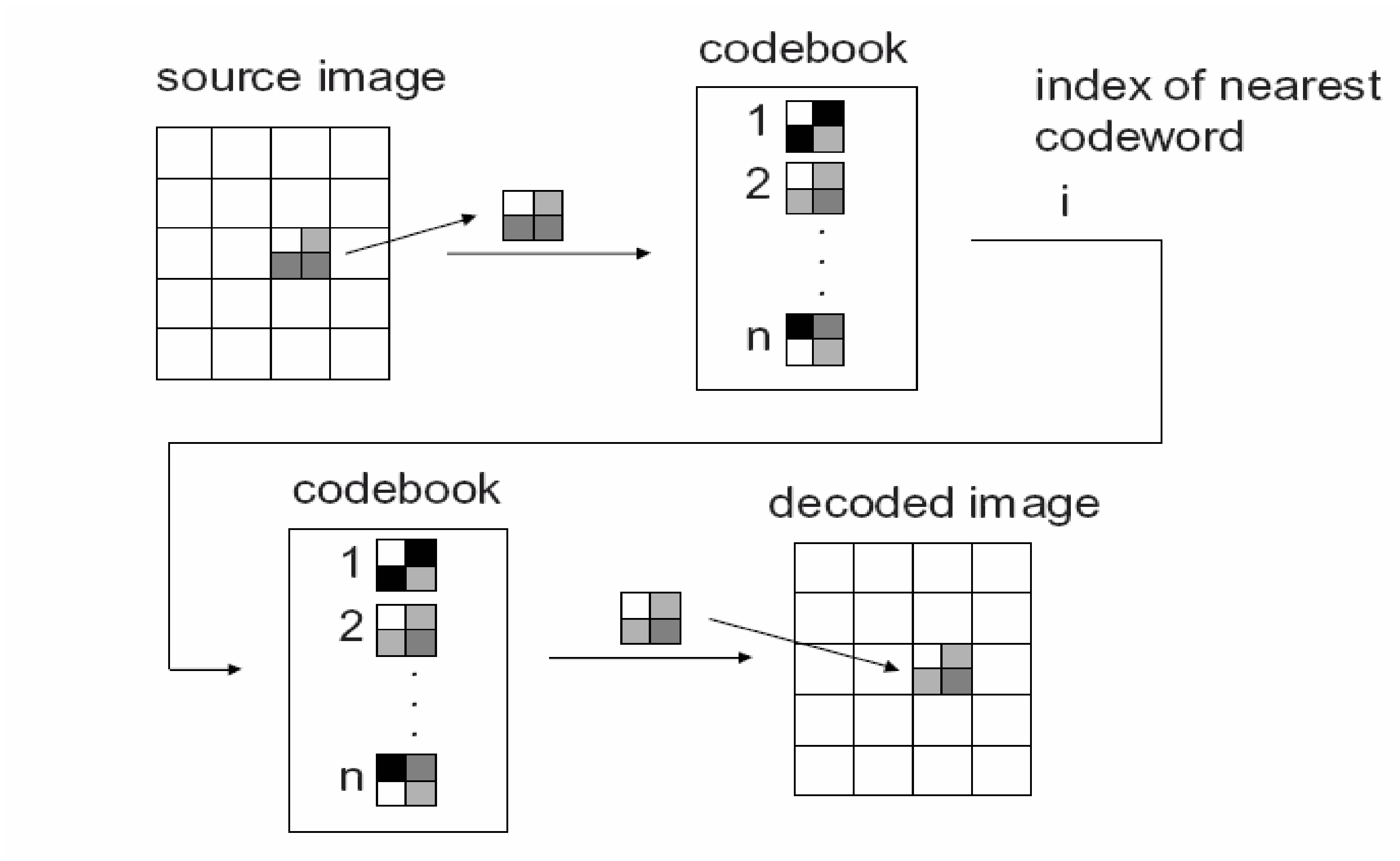
Vector Quantization



Vector

- An *a* x *b* block can be considered to be a vector of dimension *ab*.

$$\text{block} \quad \begin{array}{|c|c|} \hline w & x \\ \hline y & z \\ \hline \end{array} \quad = \quad (w,x,y,z) \text{ vector}$$

- Nearest means in terms of Euclidian distance or Euclidian squared distance. Both are equivalent.

$$\text{Distance} = \sqrt{(w_1 - w_2)^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$\text{Squared Distance} = (w_1 - w_2)^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

- Squared distance is easier to calculate.

Vector Quantization Facts

- The image is partitioned into *a* x *b* blocks.
- The codebook has *n* representative *a* x *b* blocks called codewords, each with an index.
- Compression with fixed length codes is

$$\frac{\log_2 n}{ab} \text{ bpp}$$

- Example: *a* = *b* = 4 and *n* = 1,024
  - compression is 10/16 = .63 bpp
  - compression ratio is 8 : .63 = 12.8 : 1
- Better compression can be achieved with entropy coding of indices.

Examples

4 x 4 blocks
.63 bpp

4 x 8 blocks
.31 bpp

8 x 8 blocks
.16 bpp

Codebook size = 1,024

Scalar vs. Vector Quantization

- Pixels within a block are correlated.
  - This tends to minimize the number of codewords needed to represent the vectors well.
- More flexibility:
  - Different size blocks.
  - Different size codebooks.

Scalar vs. Vector Quantization

- Encoding:
  - Scan the *a* x *b* blocks of the image. For eachblock, find the nearest codeword in the codebook and output its index.
  - This is a “Nearest neighbor search.”
- Decoding:
  - For each index, output the codeword with that index into the destination image.
  - This is a table lookup.

The Codebook

- Both encoder and decoder must have the same codebook.
- The codebook must be useful for many images and be stored somewhere.
- The codebook must be designed properly to be effective.
- Design requires a representative training set.
- *These are major drawbacks to VQ.*

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The Codebook Design Problem

- Input: A training set *X* of vectors of dimension *d* and a number *n*. (*d* = *a* x *b* and *n* is the number of codewords)
- Ouput: *n* codewords *c(0)*, *c(1)*, ... , *c(n-1)* that minimize the distortion:

$$D = \sum_{x \in X} \| x - c(index(x)) \|^2 \quad \text{sum of squared distances}$$

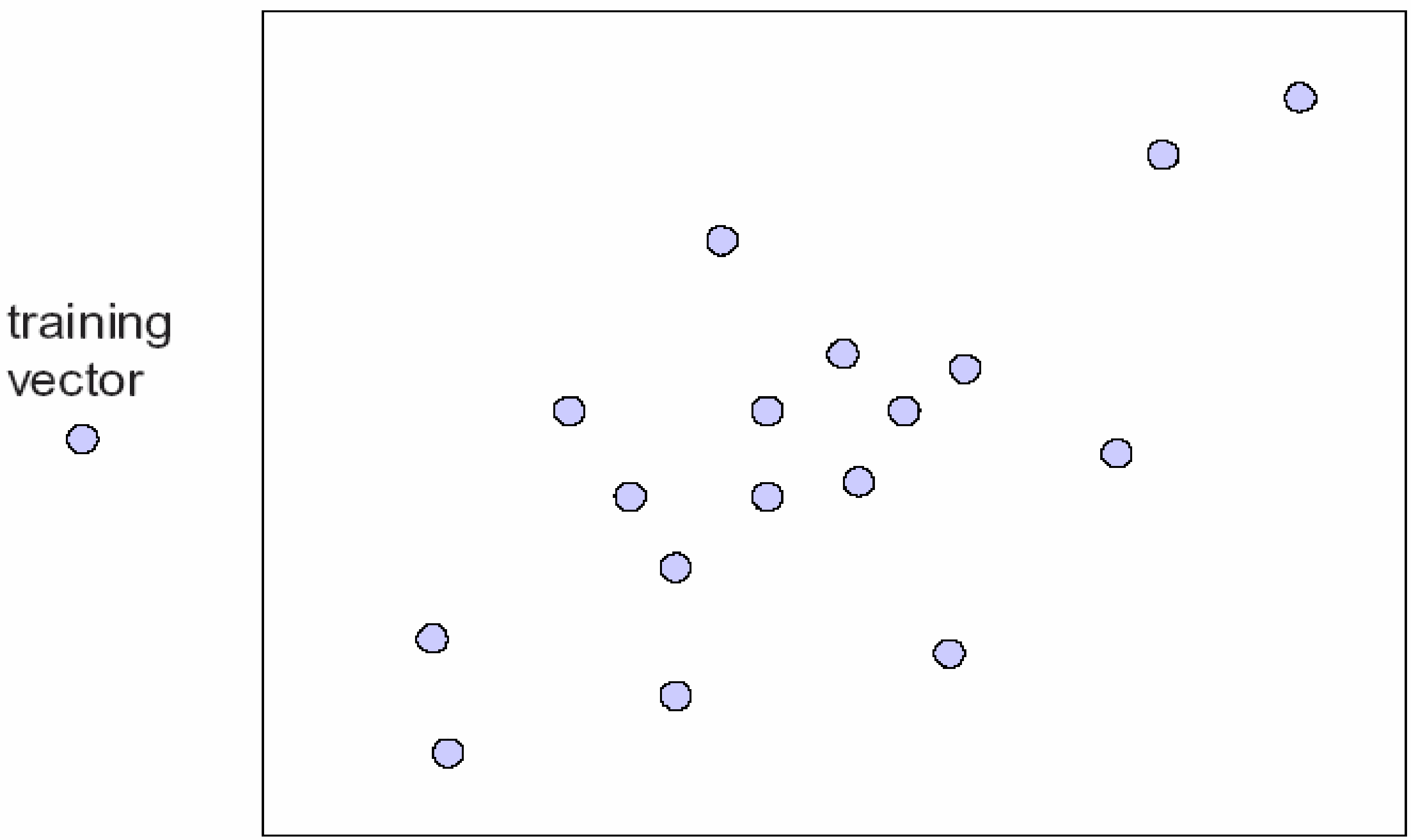
where *index(x)* is the index of the nearest codeword to *x*.

$$\|(x_0, x_1 \cdots, x_{d-1})\|^2 = x_0^{\,2} + x_1^{\,2} + \cdots + x_{d-1}^{\,2} \quad \text{squared norm}$$

Generalized Lloyd Algorithm (GLA)

- The Generalized Lloyd Algorithm (GLA) extends the Lloyd algorithm for scalars.
  - Also called LBG after inventors Linde, Buzo, Gray (1980).
  - It can be very slow for large training sets.

Example



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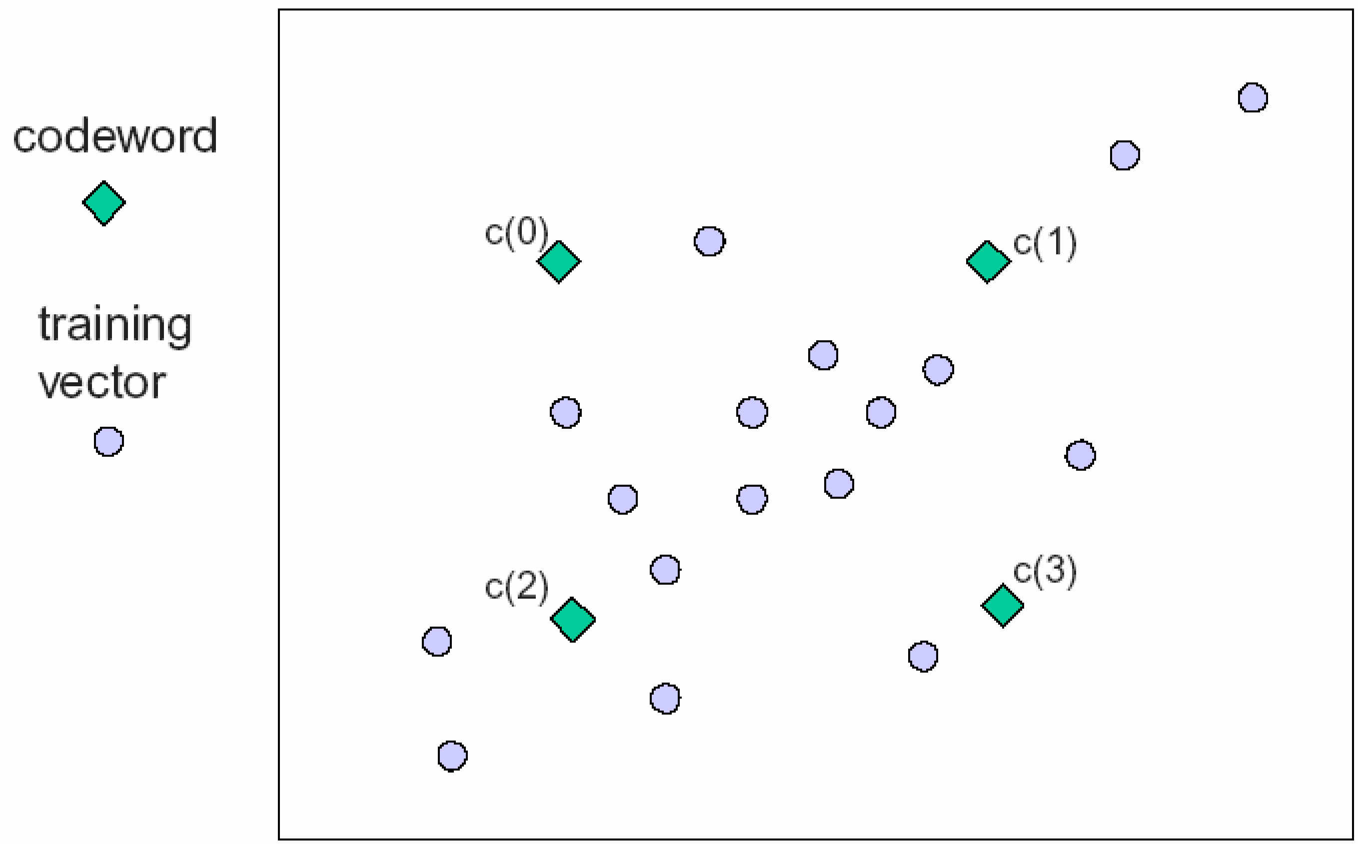

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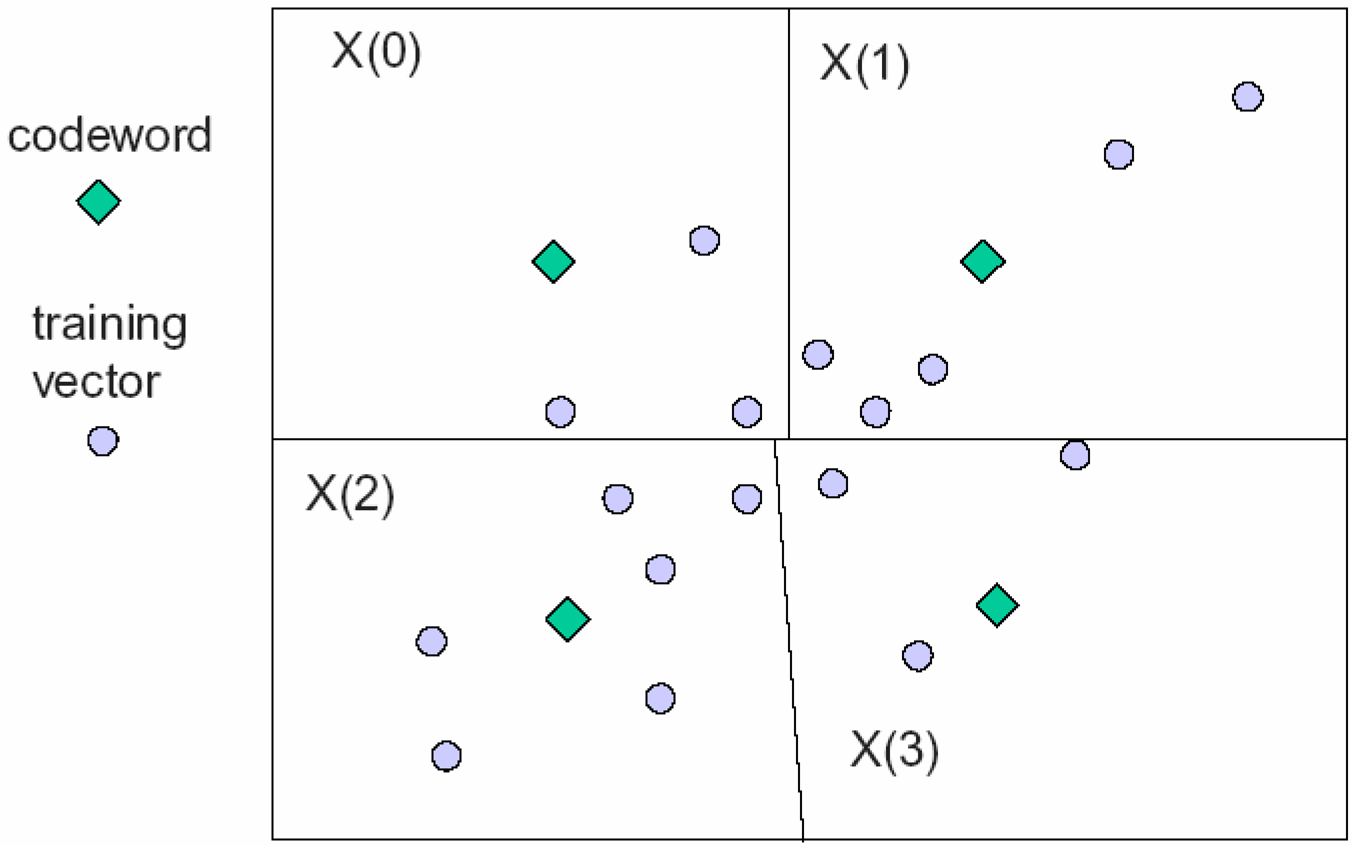

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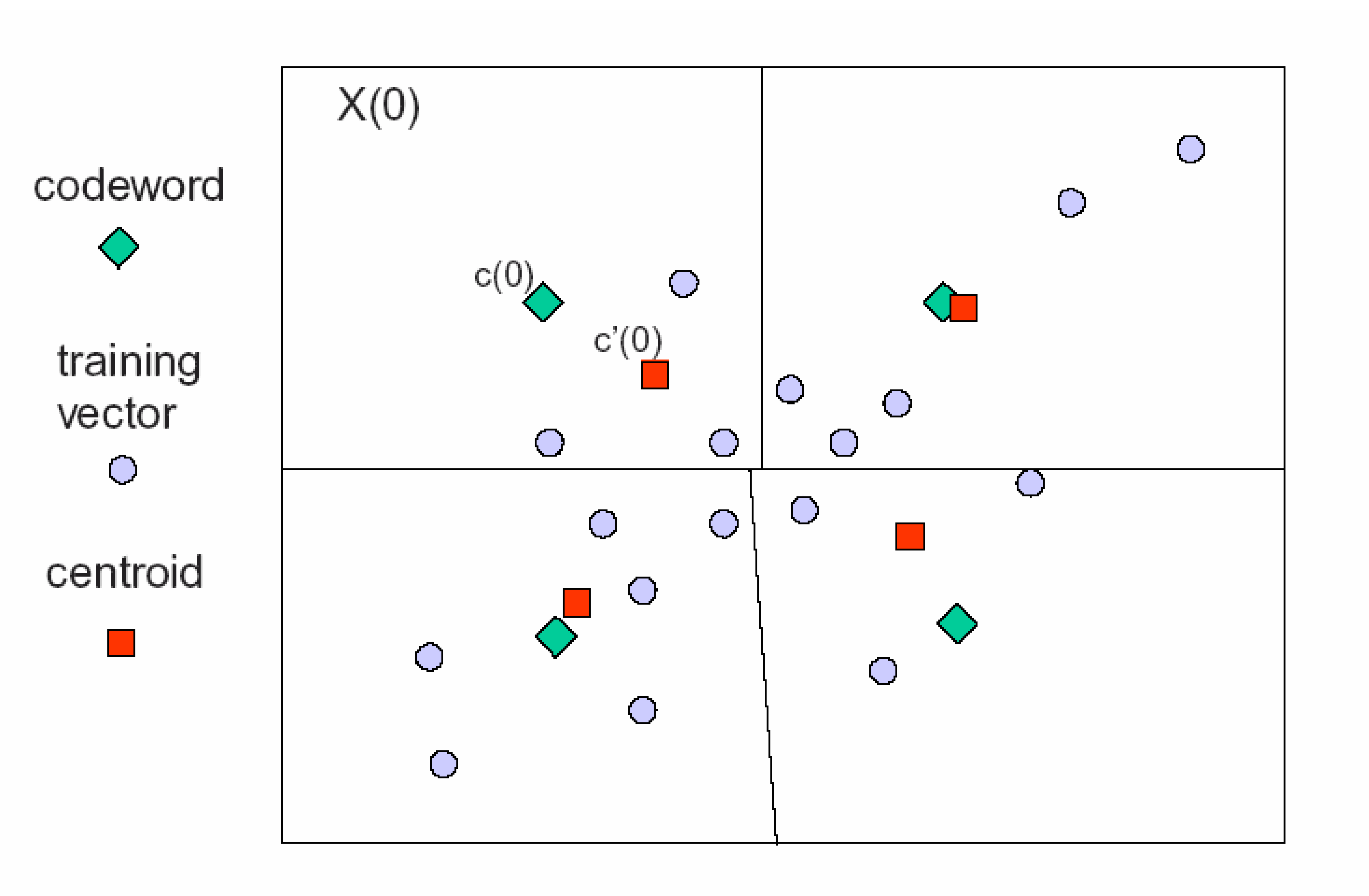

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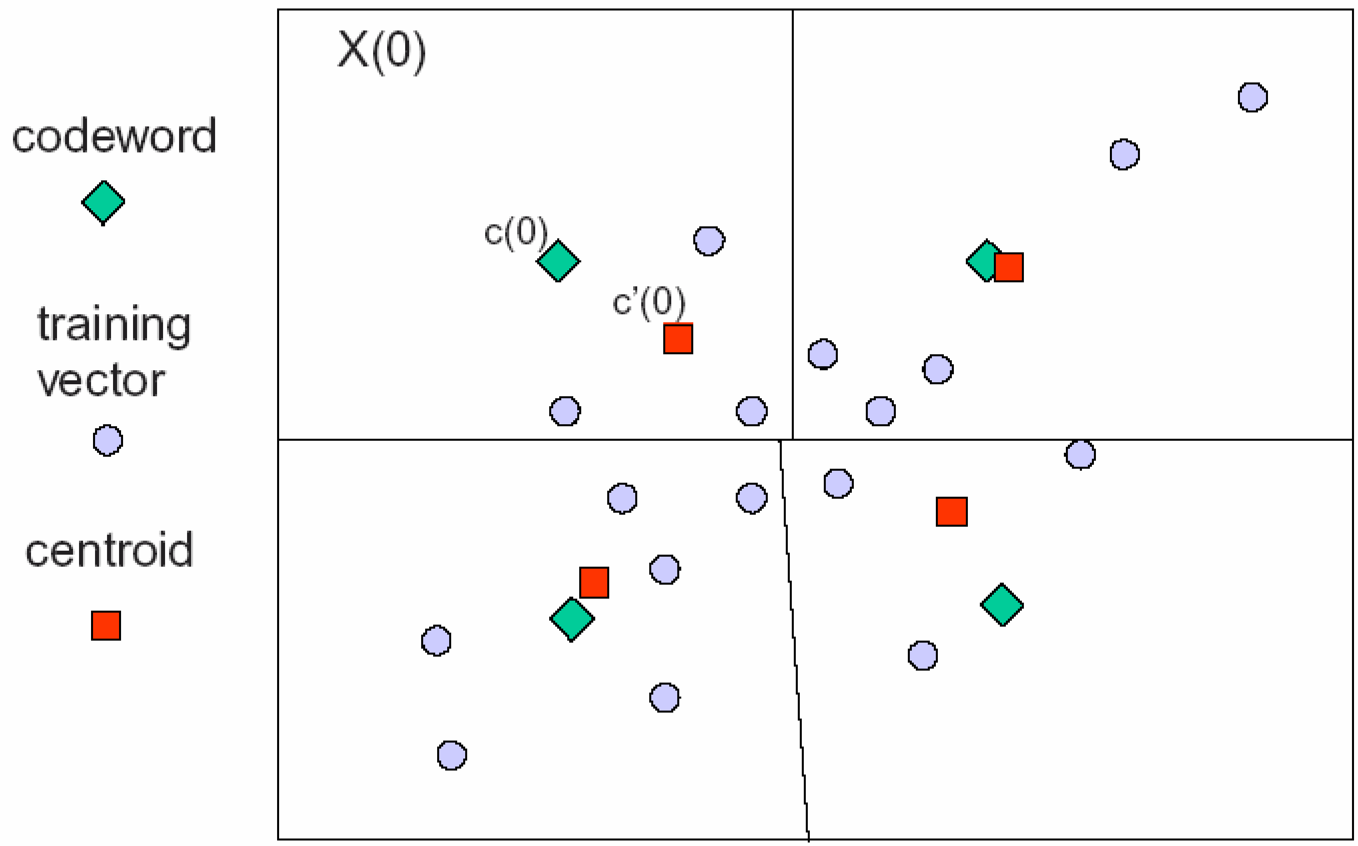

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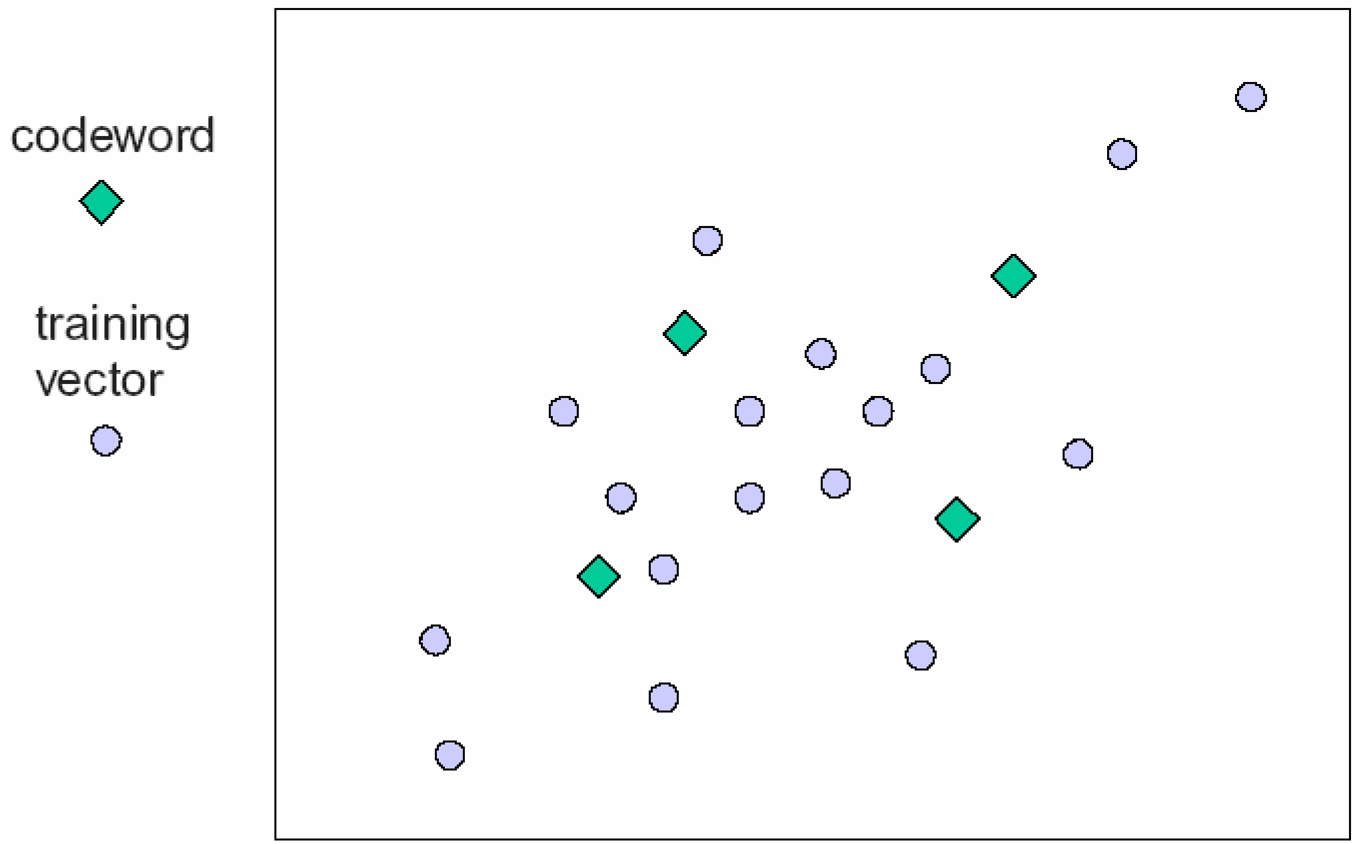

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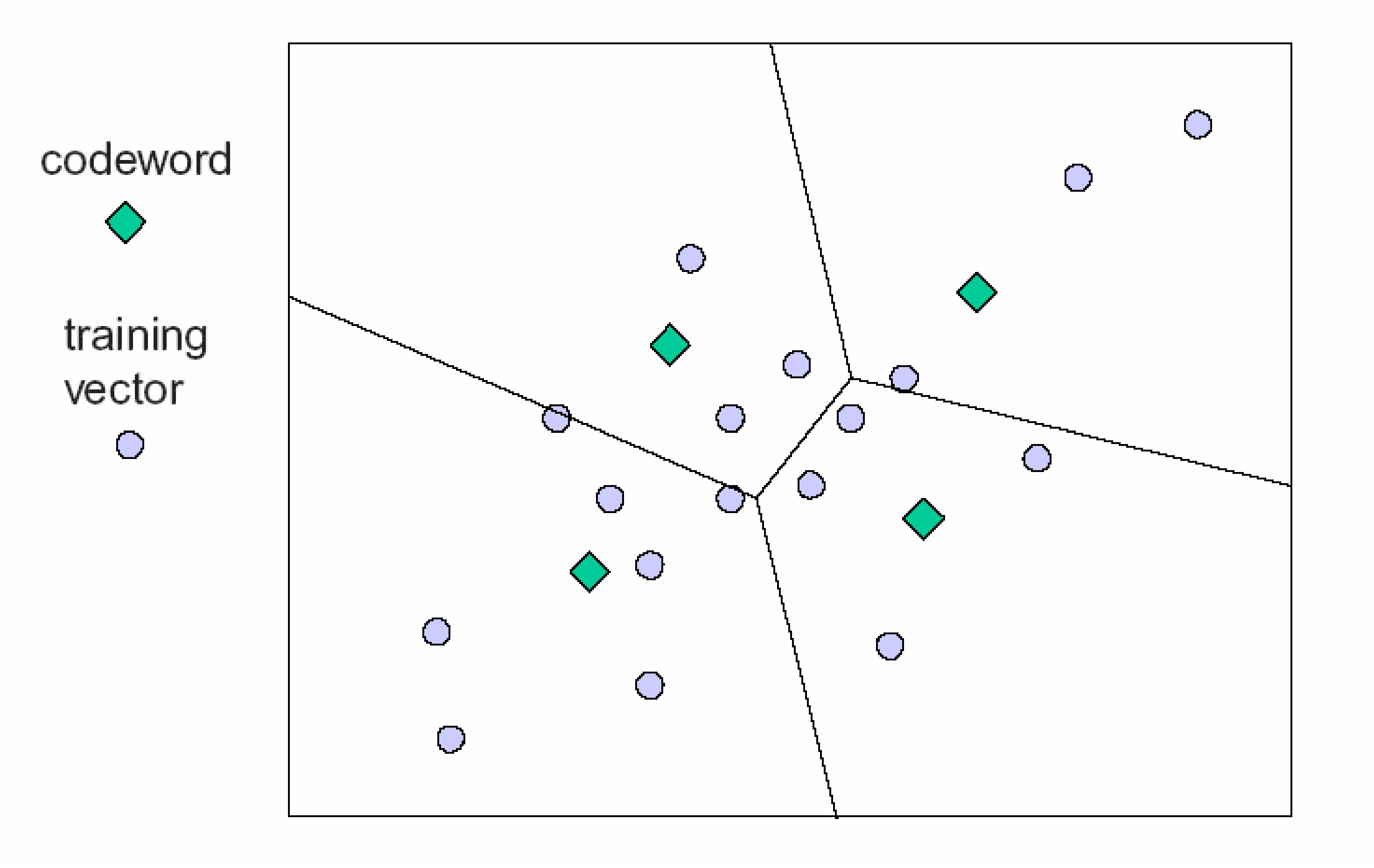

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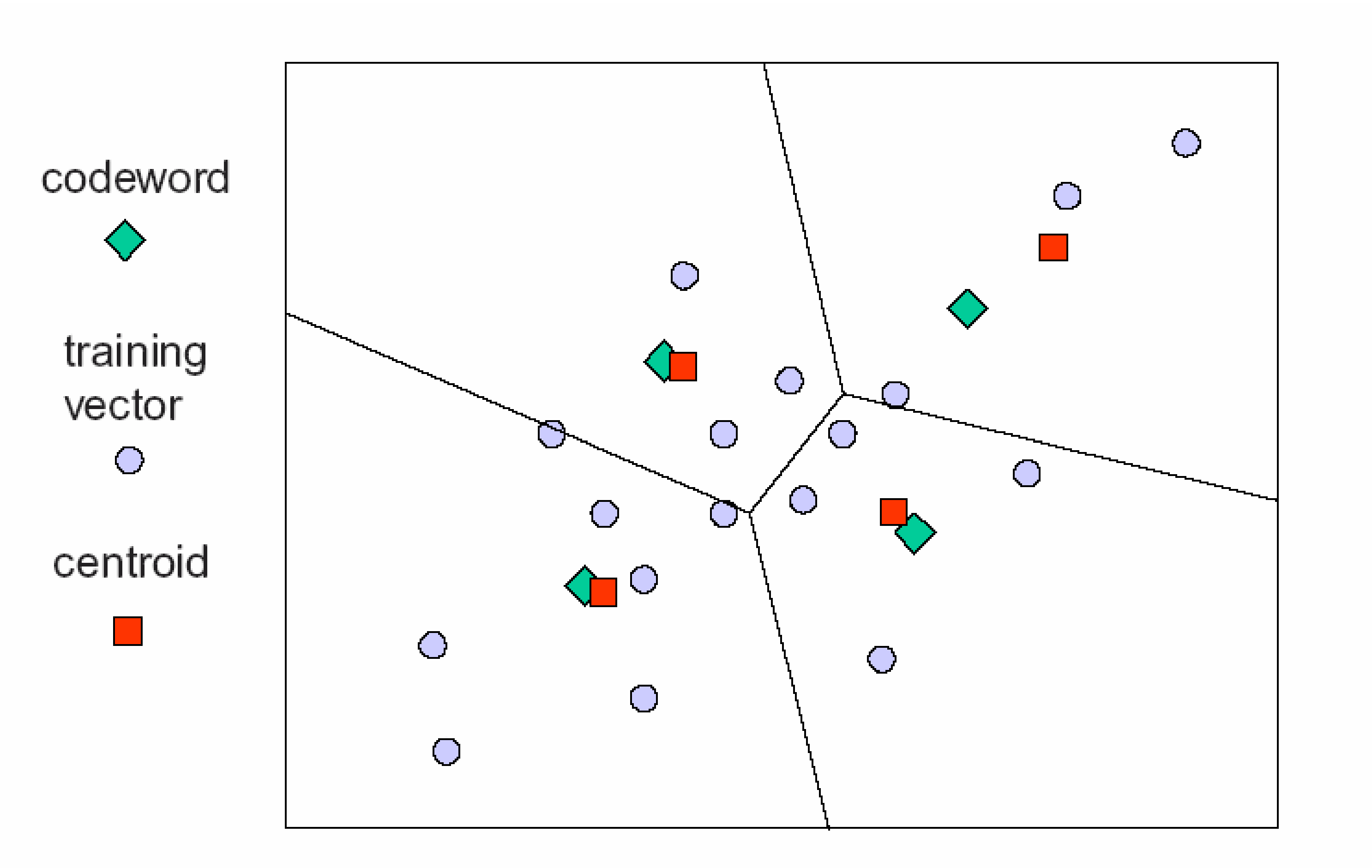

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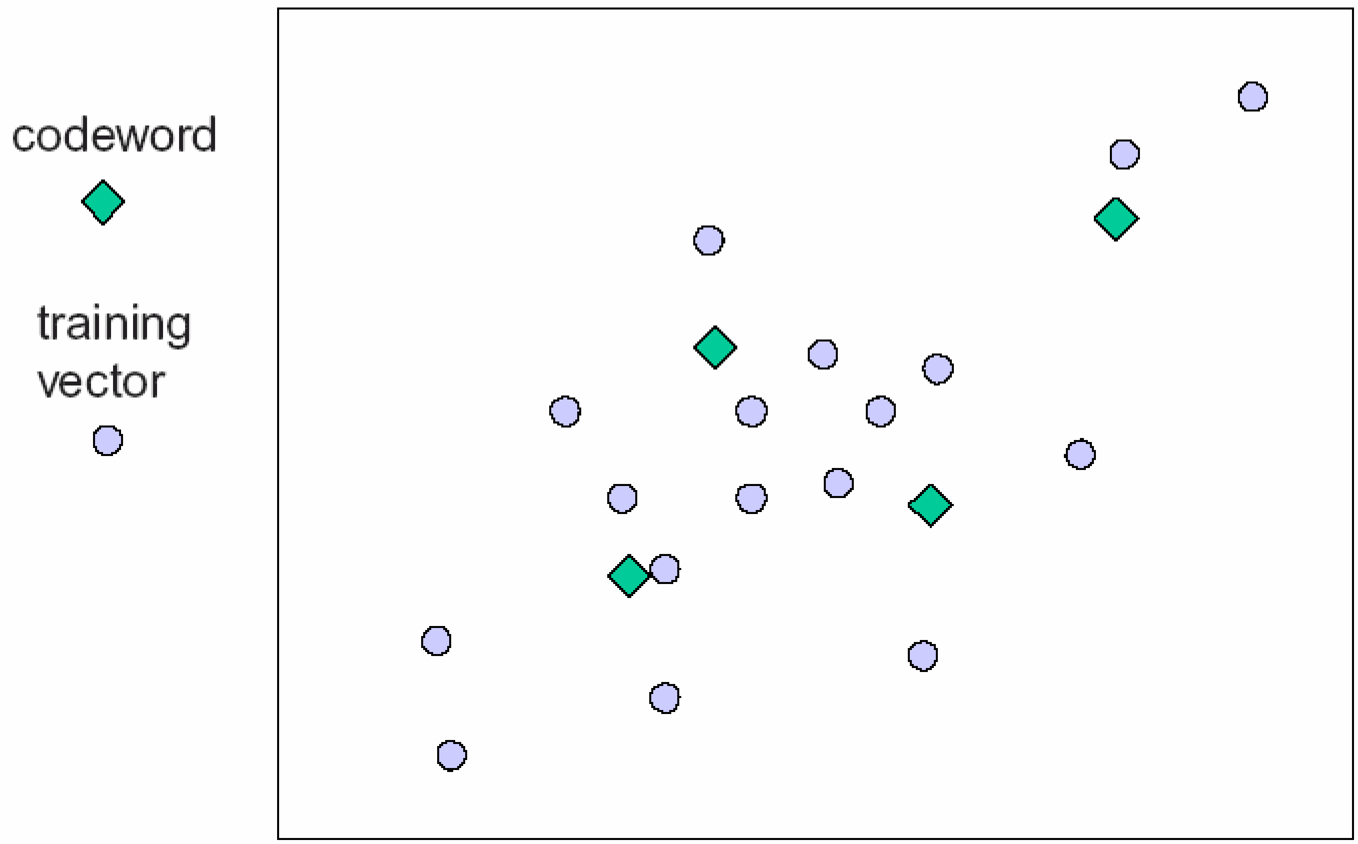

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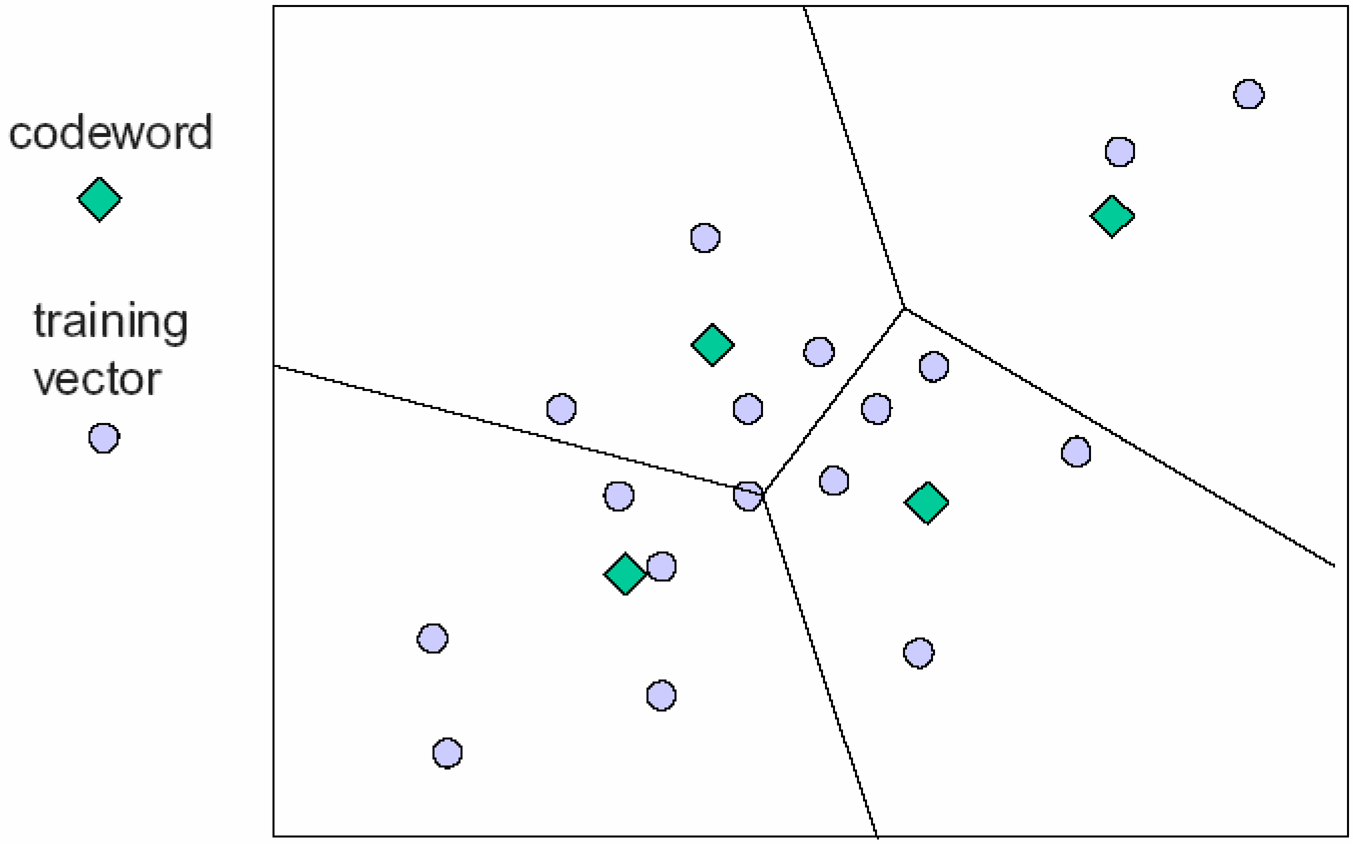

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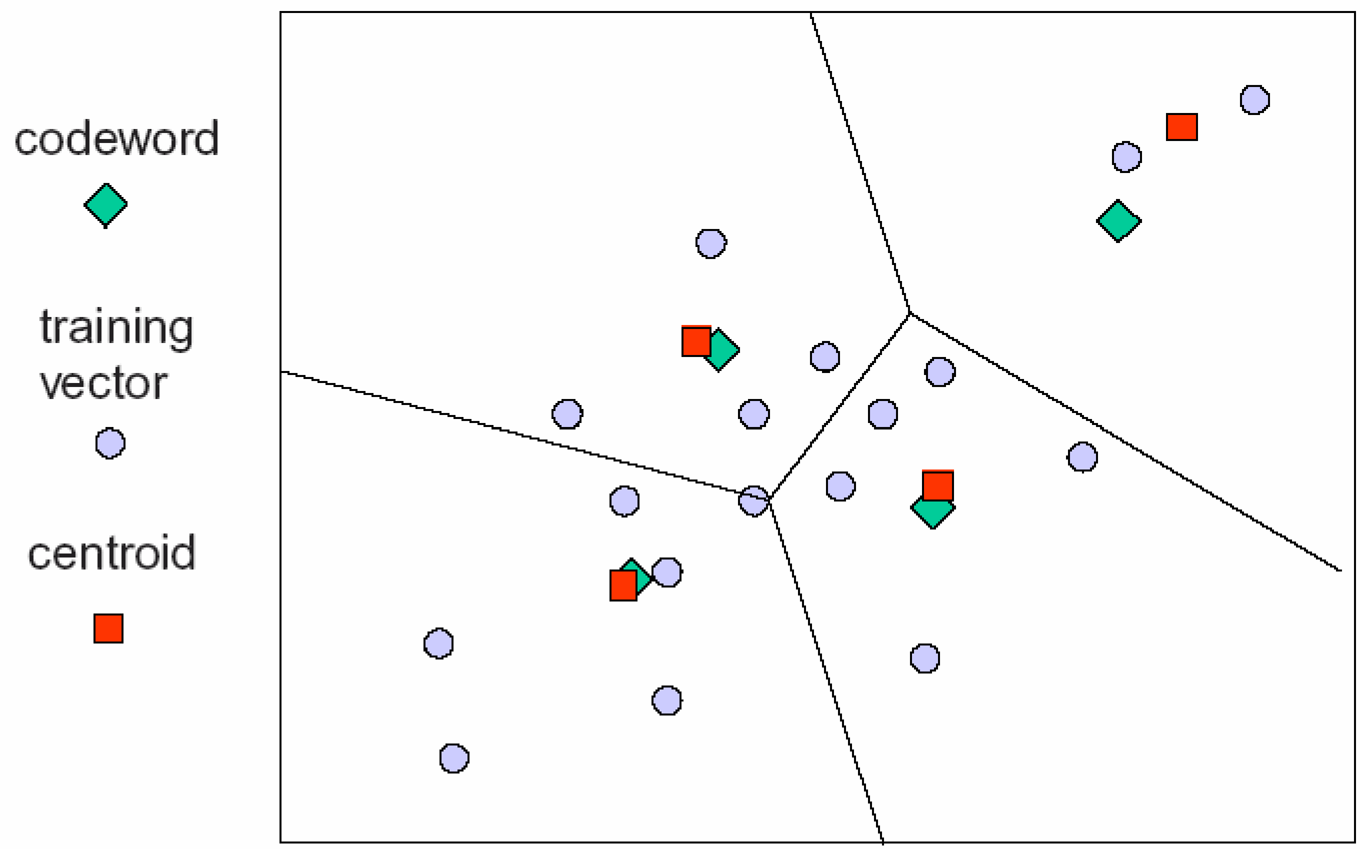

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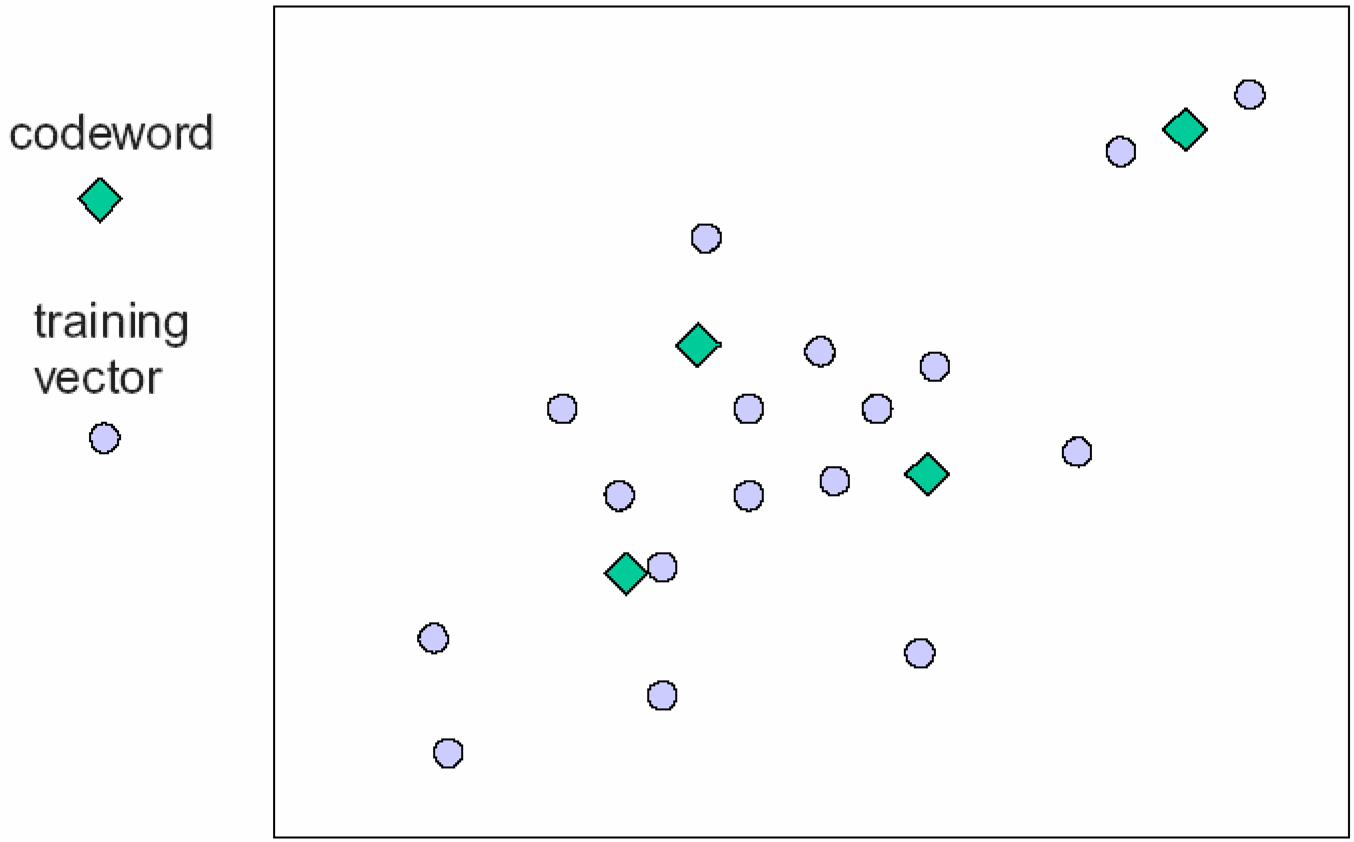


GLA Algorithm

Choose a training set **X** and small error tolerance $\varepsilon > 0$.
Choose start codewords c(0),c(1),...,c(n-1).
Compute X(j) := {x : x is the vector in **X** closest to c(j)}.
Compute distortion D for c(0),c(1),...,c(n-1).
Repeat
  Compute new codewords:

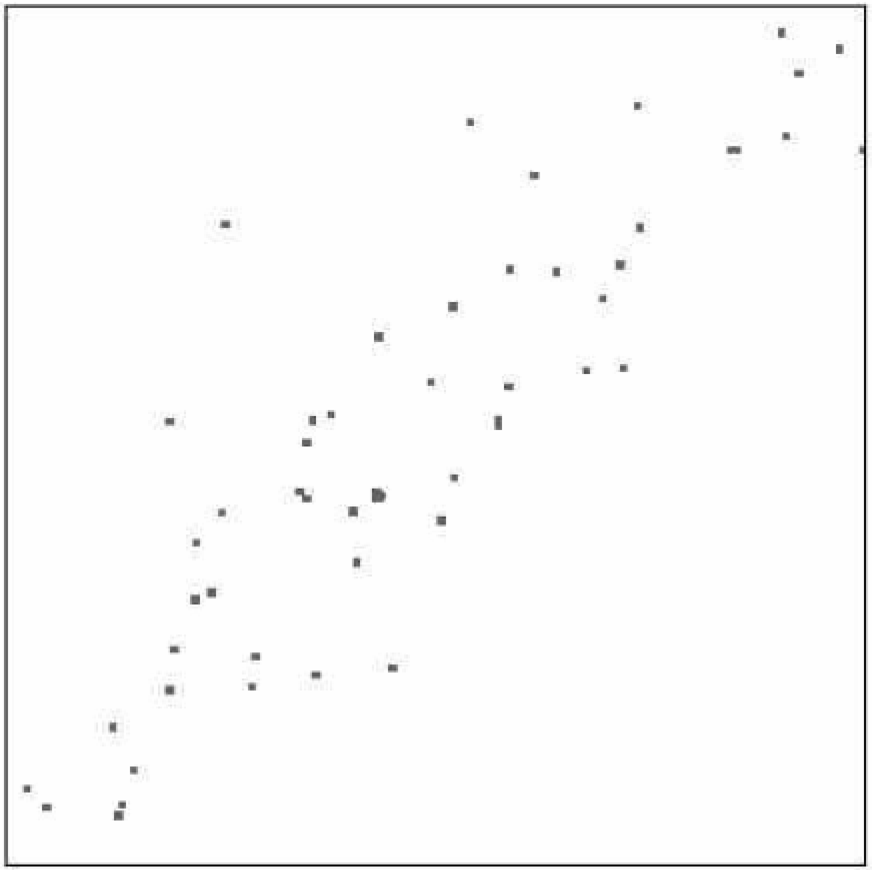
$$c'(j) := \mathrm{round}\left(\frac{1}{|X(j)|}\sum_{x \in X(j)} x\right) \qquad \text{(centroid)}$$

  Compute X'(j) = {x : x is the vector in **X** closest to c'(j)}.
  Compute distortion D' for c'(0),c'(1),...,c'(n-1).
  if $|(D - D')/D| < \varepsilon$ then quit,
  else c := c'; X := X', D := D'.
End{repeat}

Codebook

1 x 2 codewords

Note: codewords diagonally spread



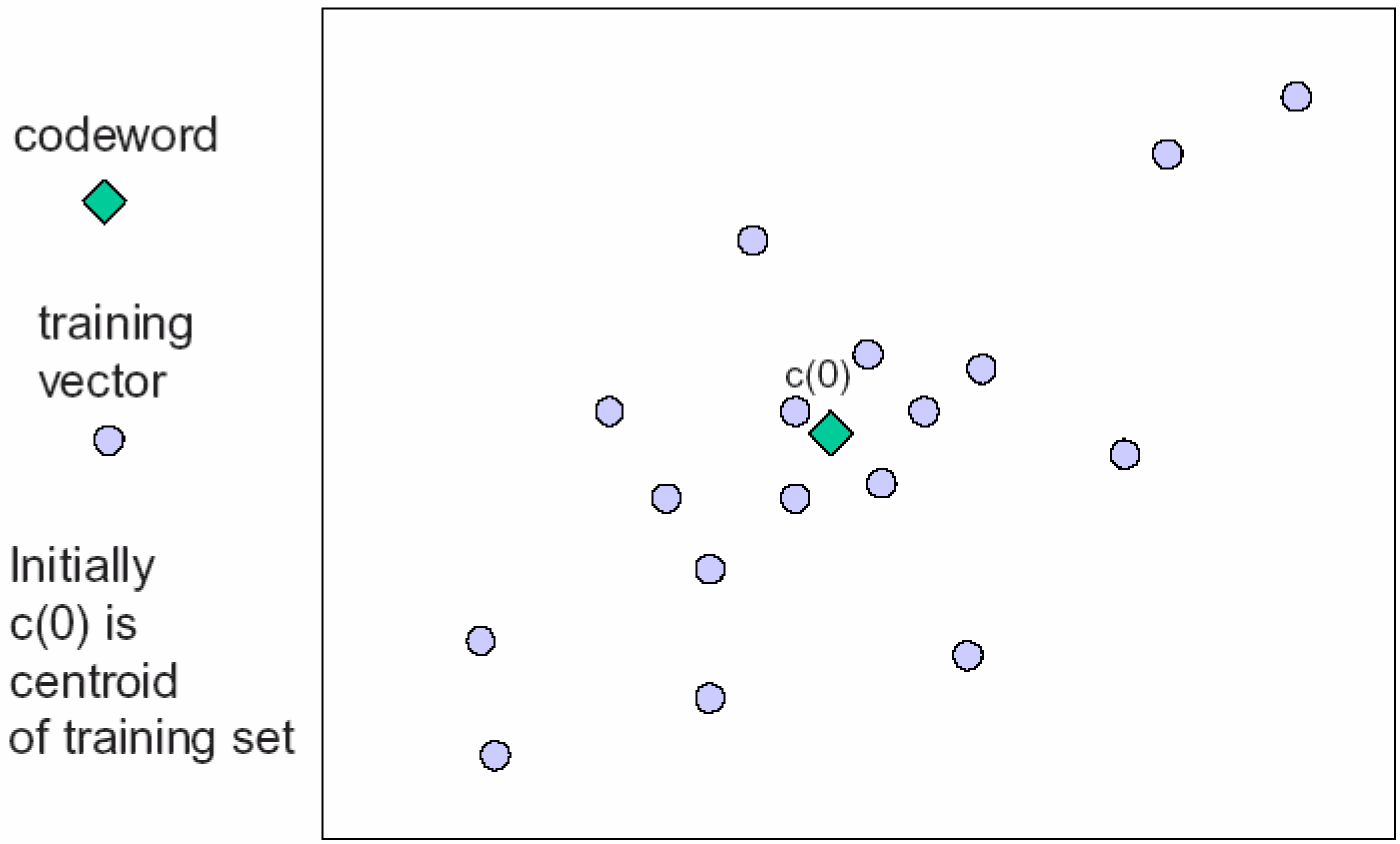
Codeword Splitting

- It is possible that a chosen codeword represents no training vectors, that is, X(j) is empty.
  - Splitting is an alternative codebook design algorithm that avoids this problem.
- Basic Idea
  - Select codeword c(j) with the greatest distortion.

$$D(j) = \sum_{x \in X(j)} \| x - c(j) \|^2$$

  - Split it into two codewords then do the GLA.

Example of Splitting



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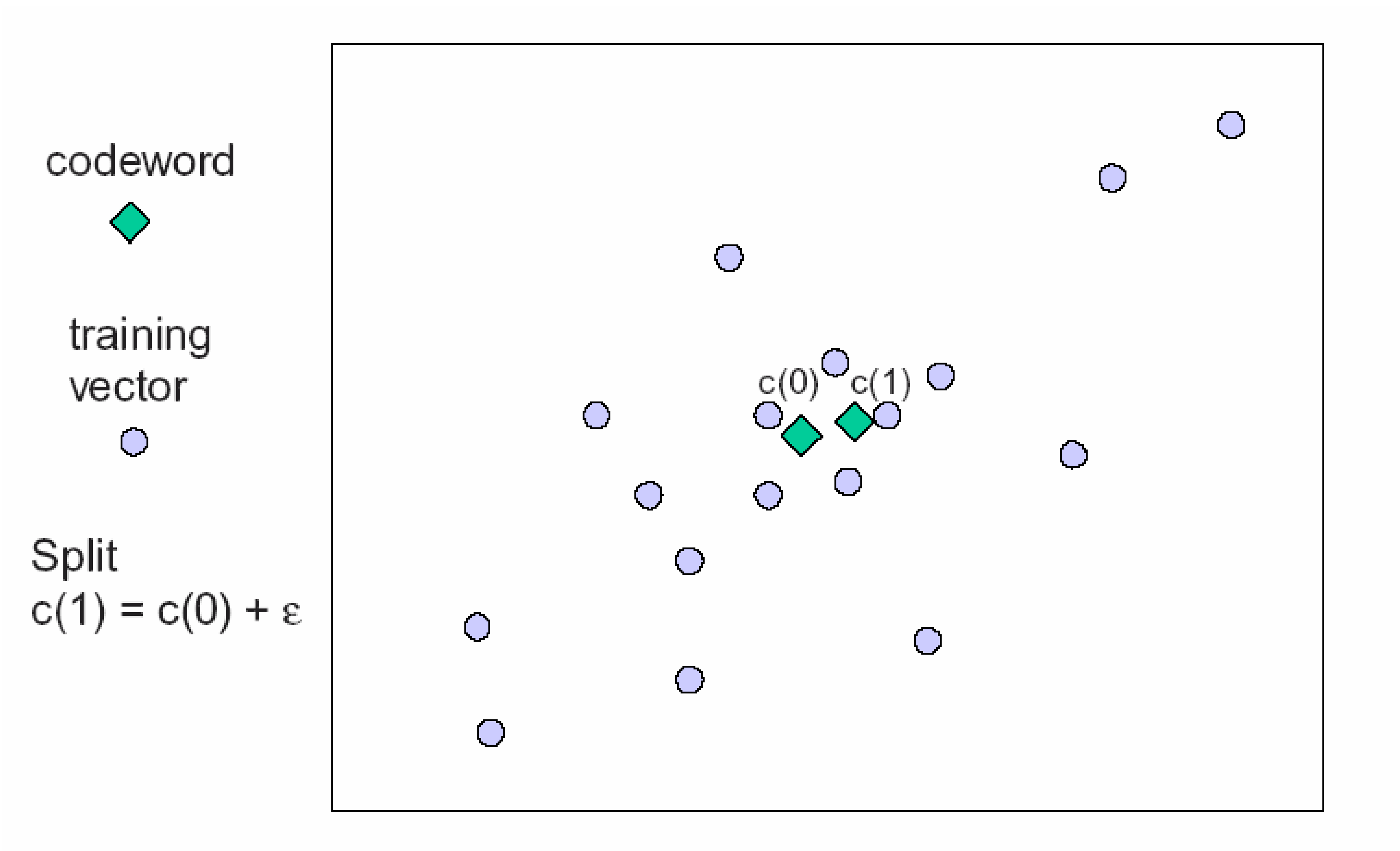

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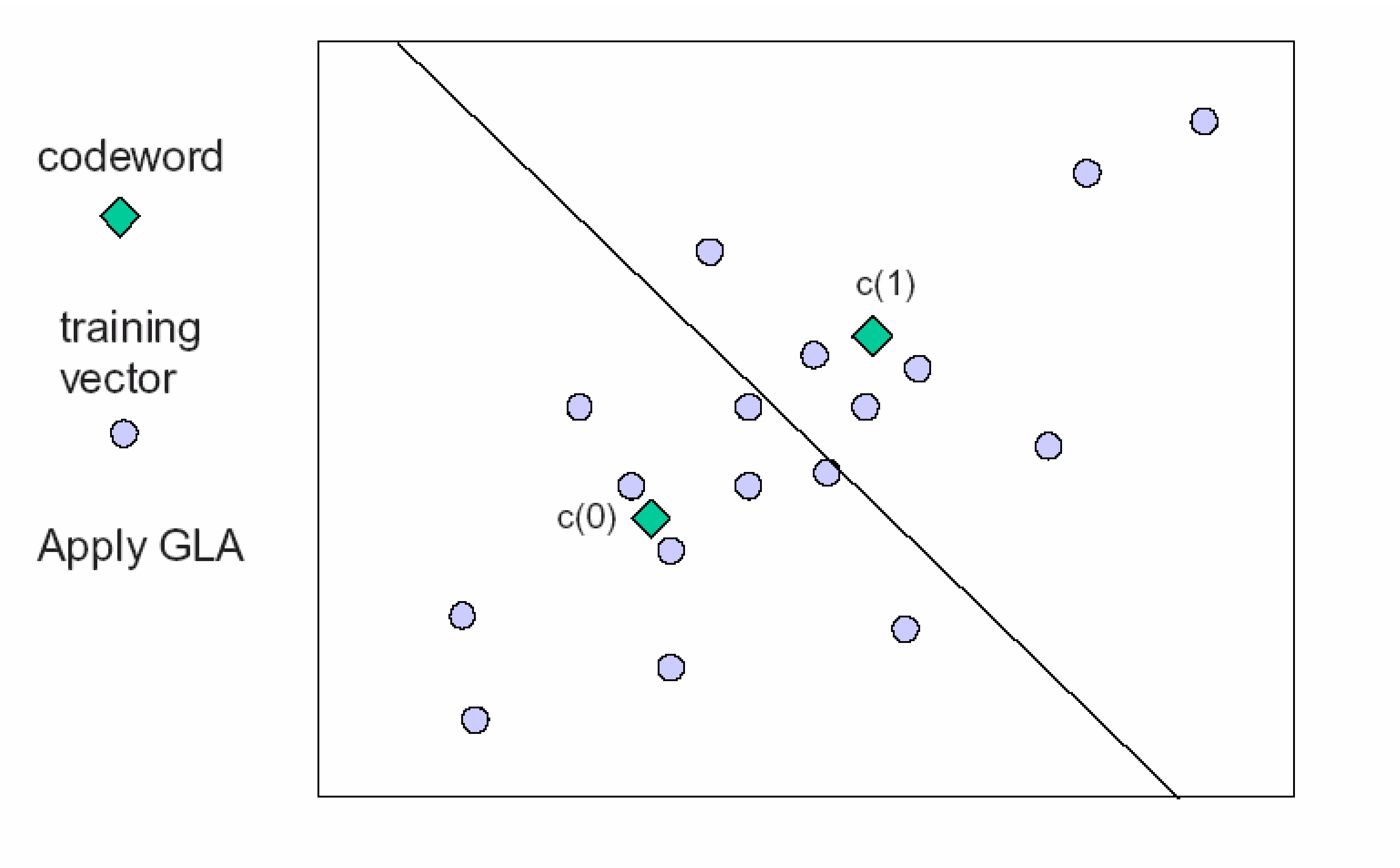
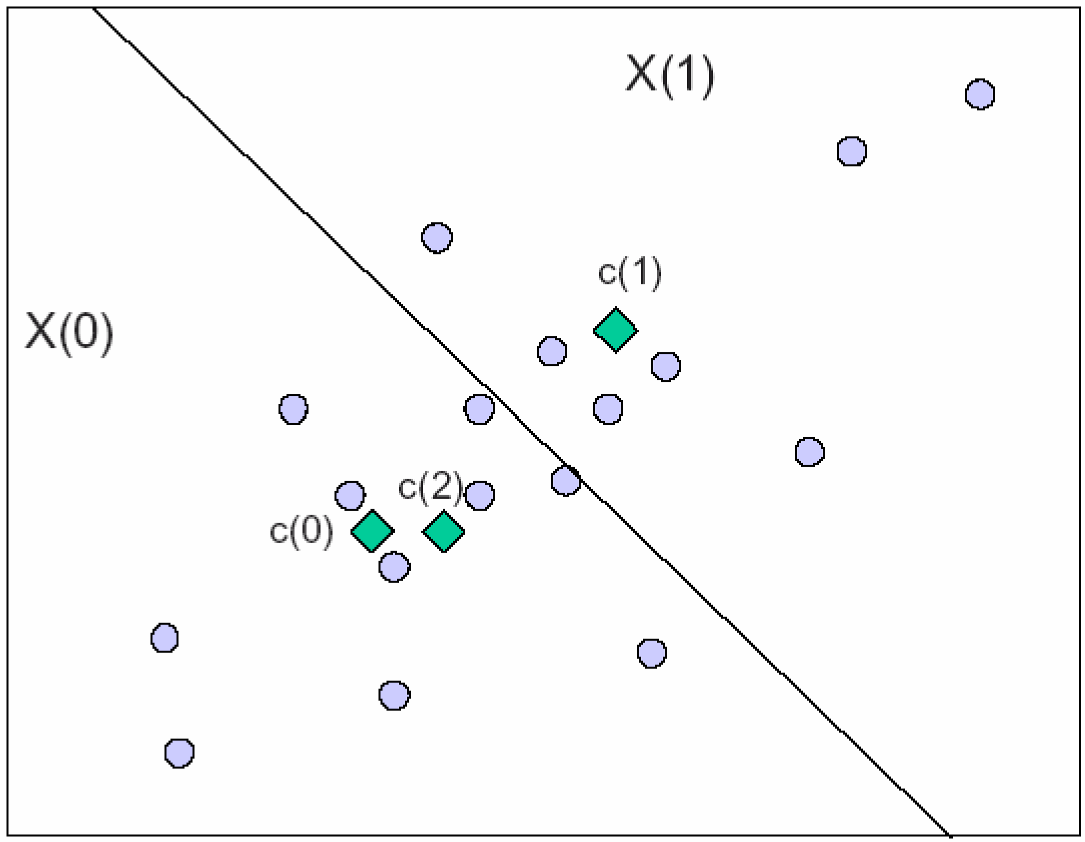


Example of Splitting

codeword

training vector

c(0) has max distortion so split it.



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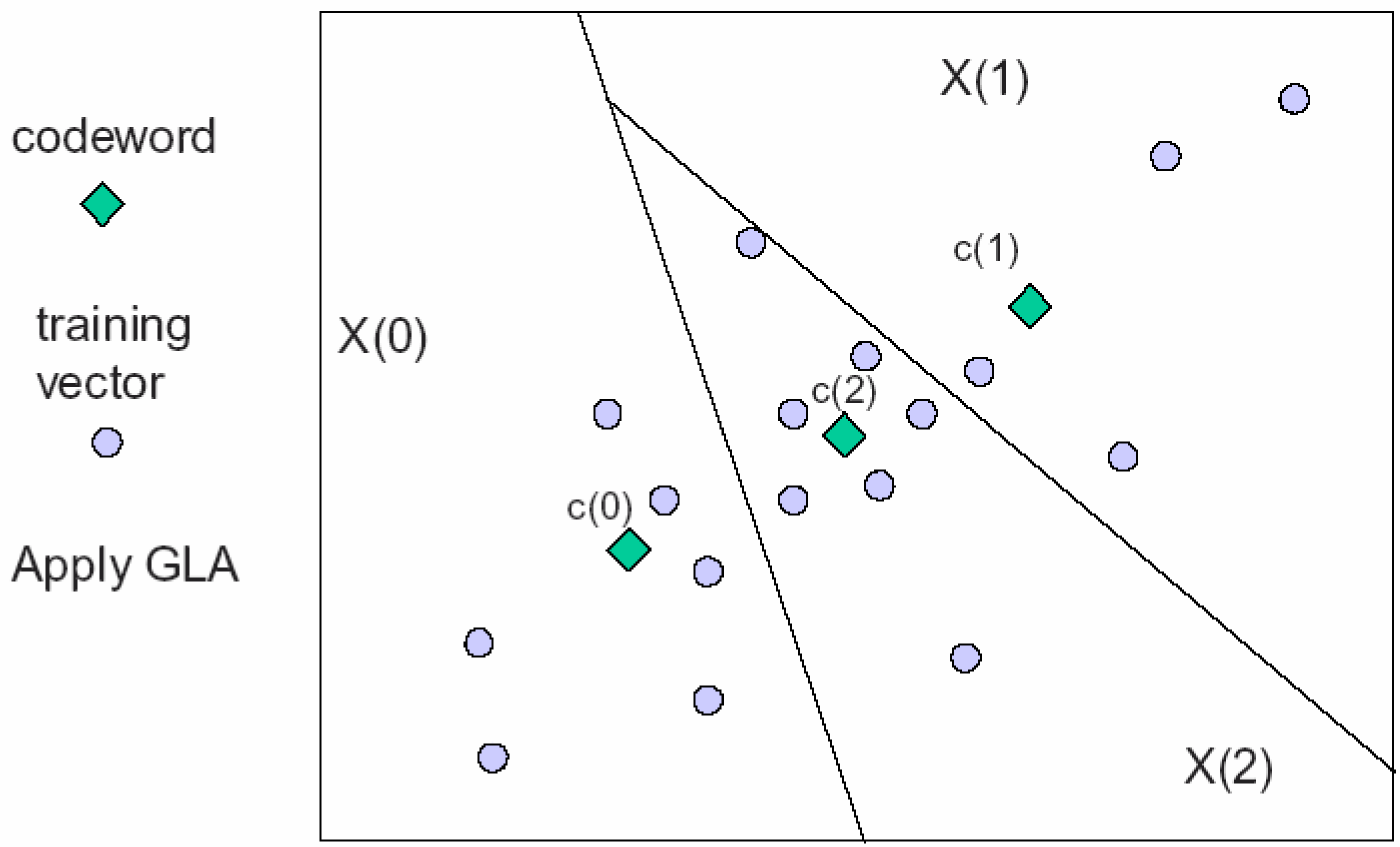

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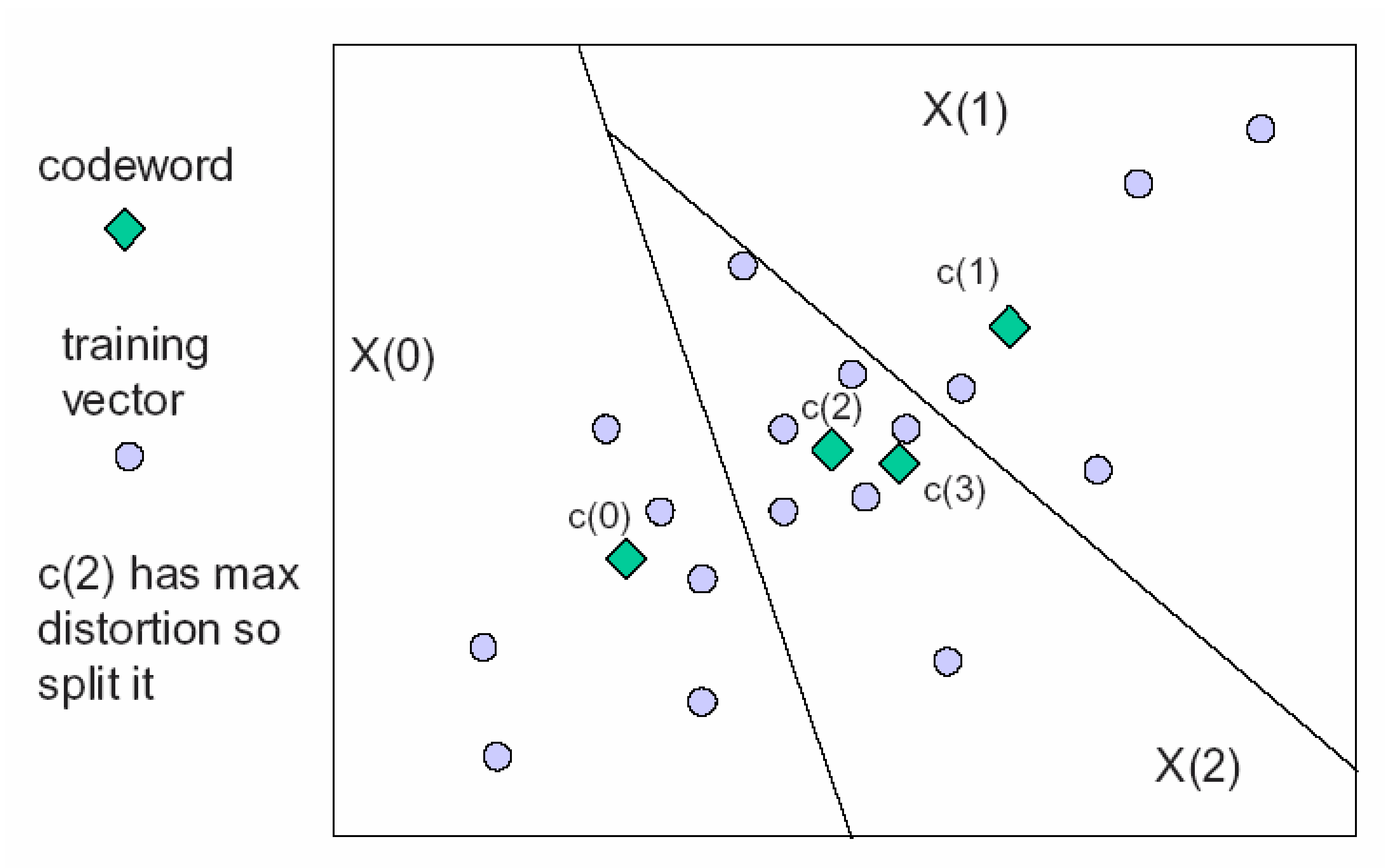

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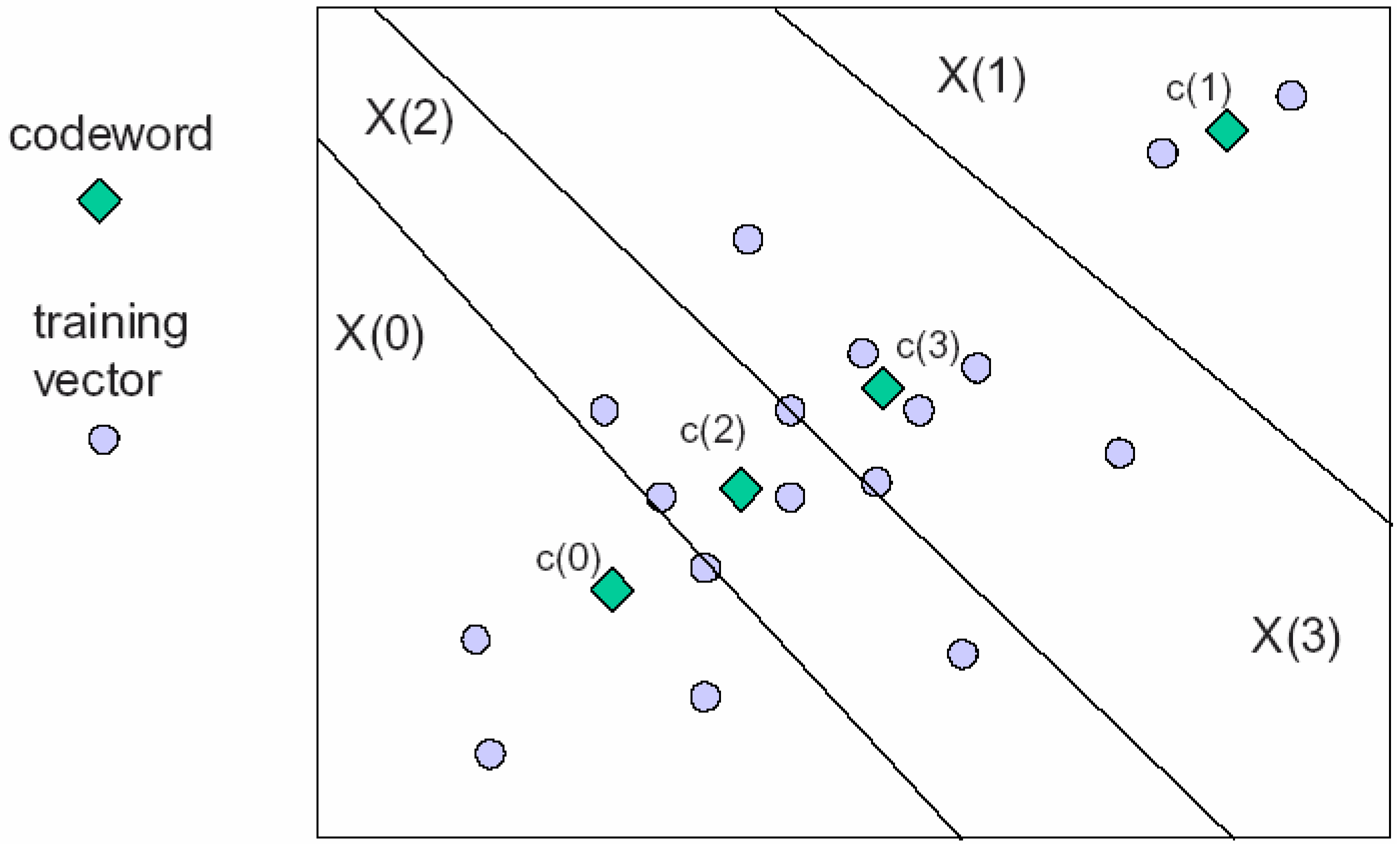

GLA Advices

- Time per iteration is dominated by the partitioning step, which is *m* nearest neighbor searches where *m* is the training set size.
  - Average time per iteration O(*m* log *n*) assuming is small.
- Training set size:
  - Training set should be at least 20 training vectors per code word to get reasonable performance.
  - Too small a training set results in "over training."
- Number of iterations can be large.