Capacitors

Capacitors are two terminal, passive energy storage devices. They store electrical potential energy in the form of an electric field or *charge* between two conducting surfaces separated by an insulator called a dielectric. Because an electrical insulator separates the plates, a capacitor cannot not conduct direct current. It does however, conduct alternating current.

Several schematic symbols for a capacitor are shown below. The symbol resembles the two conducting surfaces separated with a dielectric.

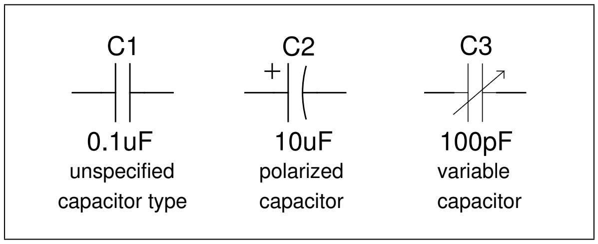


Figure 1: Capacitor Schematic Symbols

One of the main distinguishing characteristics of a capacitor is its dielectric. The dielectric material may be air, Mylar, polyester, mica or a variety of other materials. The dielectric affects many of the parameters of the capacitor such as its temperature stability, breakdown voltage and size. Some capacitors are polarized, such as electrolytic or tantalum capacitors. These capacitors require connections be made to it observing a particular polarity.

Capacitors are also differentiated as between being fixed or variable in value. In the air variable capacitor, there are conducting plates on a rotatable shaft that are sandwiched between plates on a fixed frame. Rotating the shaft changes the degree of overlap between the plates, effecting the capacitance value. The are also variable capacitors that use a thin plastic dielectric.

Below we see several different types of capacitors. From top left are two variable capacitors called "trimmer capacitors". These capacitors are generally small in value, 1-25pF, and are used at radio frequencies. Moving right, the next capacitor is a mica capacitor, so called because of its mica dielectric. It is a very low loss capacitor also useful at radio frequencies. The two twisted wires form what is known as as *gimmick capacitor*. It provides a *one-time* trimming capacitor that is set by clipping off a little of the two wires until the desired capacitance is achieved.

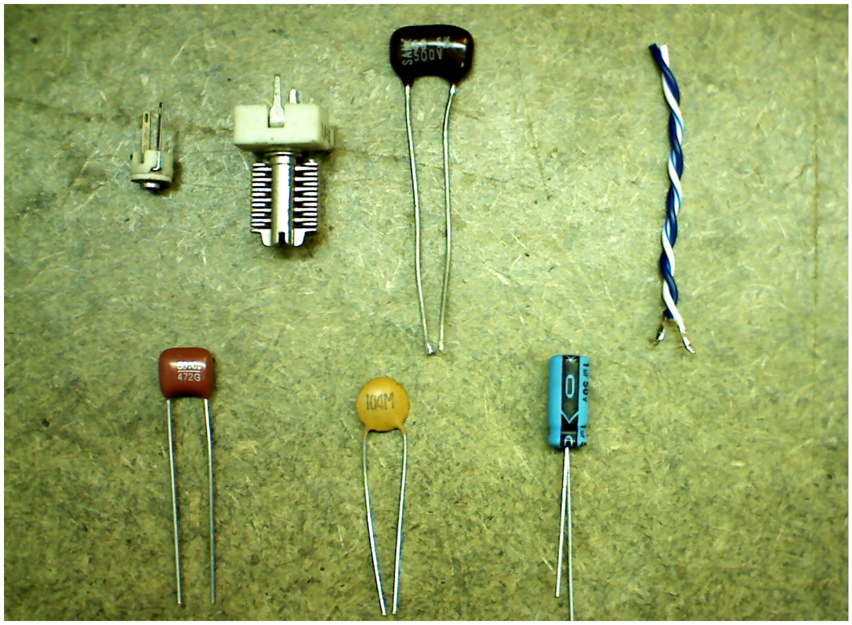


Figure 2: Various capacitors

On the bottom row starting from left, we have a Mylar capacitor, followed by a ceramic capacitor and an electrolytic capacitor. The Mylar cap is useful at lower frequencies and also has very low DC leakage. No dielectric is perfect so there is always some DC leakage in a capacitor.

The ceramic cap is a very general purpose capacitor and is useful from audio to high frequencies. The electrolytic capacitor on the far right is an example of a polarized capacitor. The black stripe indicates the negative terminal. Connecting the negative lead of an electrolytic capacitor to a higher potential than the positive lead, can cause permanent damage and possibly a rupture of the capacitor case.

The amount of capacitance available in a capacitor depends on its physical dimensions. For a parallel plate capacitor;

$$C = \epsilon A/d$$

where C is capacitance in Farads, A is surface area between the plates, ϵ is the permitivity of the dielectric and d is the distance between the plates. We often think of capacitors as parallel plate capacitors but they may be found in many physical configurations. However, the general meaning of this equation holds for other physical configurations. In other words, the closer the two surfaces and the larger their surface area is, the greater the capacitance.

In terms of charge and voltage, the capacitance is defined as;

$$C = Q/V$$

where Q is charge in Coulombs and V is the voltage existing between the plates; thus the units of capacitance are coulombs per volt.

Capacitors in Series and Parallel

From our work on reducing resistor network to a simplified form, we are often presented with parallel and series combinations of capacitors. The same techniques may be applied to capacitors to find an equivalent capacitor for a network of capacitors. However, unlike parallel resistors, parallel capacitors have an equivalent capacitance that is obtained by summing their values. Conversely, the equivalent capacitor to a set of series capacitors is the reciprocal of the sum of the capacitors.

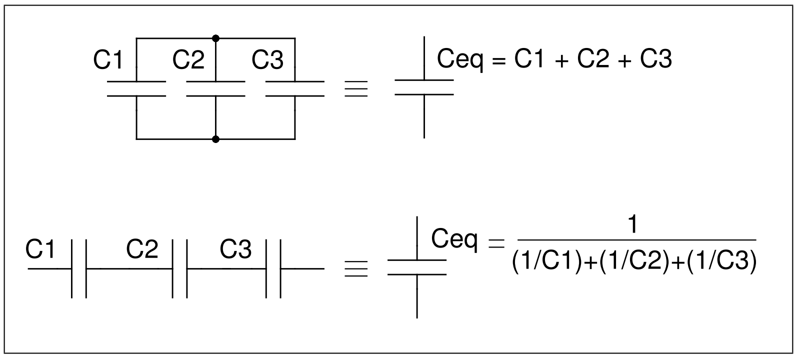


Figure 3: Capacitors in Series and Parallel

Capacitor Current-Voltage Relationship

The current-voltage relationship for the capacitor is;

$$i_C = C\frac{dv}{dt}$$

where i is the current through the capacitor and $\dfrac{dv}{dt}$ is the change in voltage per unit time. We could also say;

$$\frac{1}{C}i_C = \frac{dv}{dt}$$

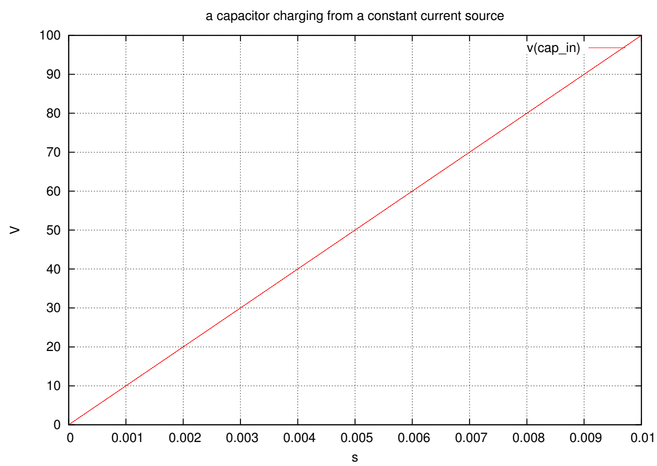
Note that the usual capital I and V has been exchanged for a lower case i and v. This is to emphasize the time varying nature of the current and voltage.

These equations above tell us several things very clearly. The first one makes clear that if the voltage is not changing, i.e.; $\frac{dv}{dt} = 0$ (a DC source) then no current is flowing. Therefore, a capacitor is an open circuit to DC currents.

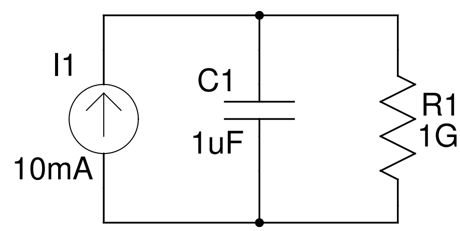
Secondly, if a constant current is applied to the capacitor, the change in voltage across its terminals with respect to time is constant, i.e., a straight line. A Spice simulation illustrates this well.

```
A capacitor charging from a constant current source

Iin gnd cap_in PULSE(0 10ma 0ms 1ns 1ns 25ms 50ms) ;10am current source
c1  cap_in    gnd       1uF
r1  cap_in    cap_in    1G ;resistor required for convergence

.control
  tran 1us 10ms
  plot v(cap_in)
  gnuplot cap_isrc_charging V(cap_in) xl 1u 10m ;make .eps for latex
  set noaskquit
.endc
.end
```

In this circuit, the 1G resistor is only required so that spice has a ground reference, otherwise it will have convergence errors. The resistor is so big, it may be ignored. Running this simulation yields:



(a) Spice Output



(b) Schematic

Figure 4: 10mA Constant Current Source Charging a 1uF Capacitor

Working the equation out yields:

$$\frac{dv}{dt} = \frac{1}{C} i_C$$
$$\frac{dv}{dt} = \frac{1}{1e^{-6}F} (10^{-3}A)$$
$$\frac{dv}{dt} = \frac{10V}{ms}$$

This make sense as the graphs shows a voltage ramp of 10 volts per millisecond.

Finally, we can see that the voltage across a capacitor cannot change instantaneously. For the voltage to change in zero time, then $i(c)$ would have to be infinite.

RC Circuit Behavior

When a capacitor is charged from a voltage source in series with a resistor, we can see that the voltage across the capacitor terminals charges differently. See the schematic below.

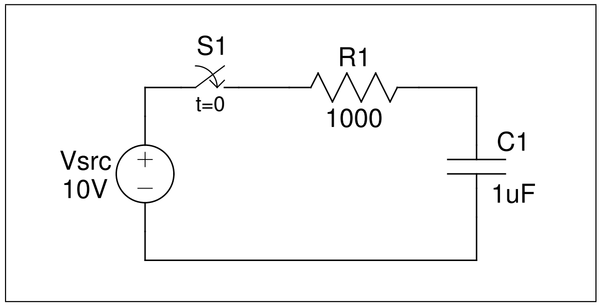


Figure 5: Circuit to Determine Capacitor Transient Response

Assuming that initially, there is no stored charge on the capacitor at $(t = 0)$, when the switch is closed, the charge on the battery begins to charge the capacitor through the resistor. As the voltage across the capacitor terminals charges to the source potential, the current flow asymptotically approaches zero. We say at this point the capacitor is *charged*. If we disconnect the capacitor from the circuit, we would find the voltage across it remains equal to the battery voltage.

For this particular circuit, it has been determined that the voltage across the capacitor is expressed by the equation:

$$V_c(t) = V_{src}(1 - e^{-t/RC})$$

This relationship can also be seen from a ngspice simulation as shown below.

```
RC network - charging

Vin vin 0 10.0 PULSE(0 10.0 1ms 1ns 1ns 25ms 50ms); 10V source
r1  vin      cap_in    1k
c1  cap_in   gnd       1uF

.control
  tran 100ns 10ms
  plot rc_sim V(vin) V(cap_in) xl 1u 8m
* gnuplot rc_sim V(vin) V(cap_in) xl 1u 8m ;make simout1.eps for latex
  set noaskquit
.endc

*measure the time difference between input reaching 0.01v to
*cap_in reaching 6.32...i.e, the RC time constant
.meas tran tdiff trig v(vin)    val=0.01 rise=1 td=500ps
+                targ v(cap_in) val=6.32 rise=1 td=500ns
.end
```

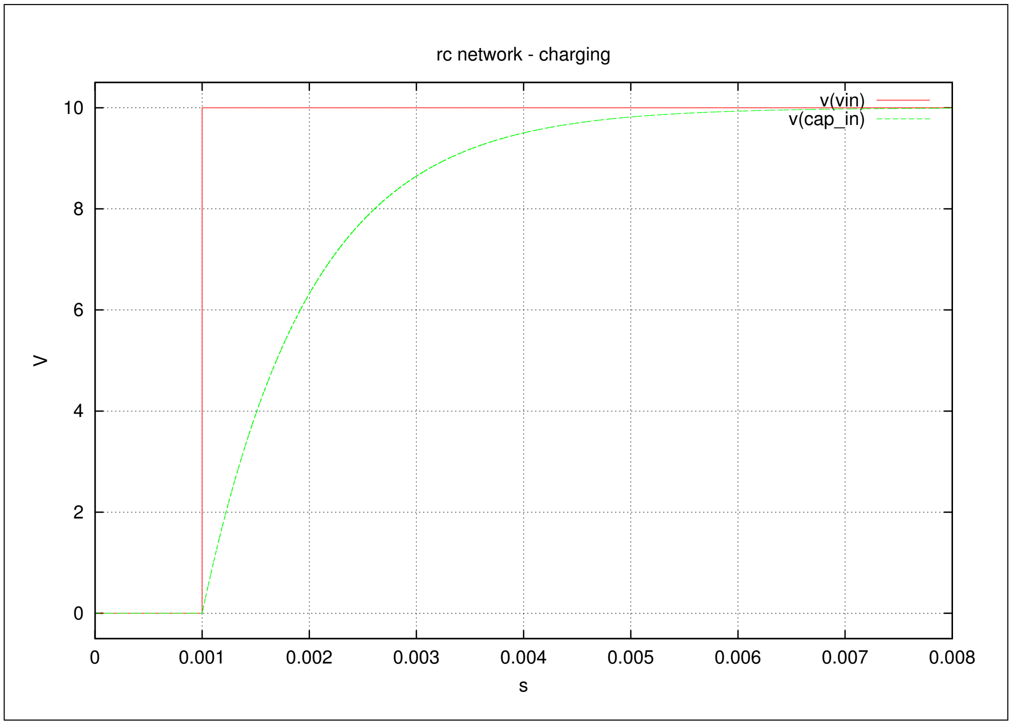


Figure 6: Capacitor charging through a resistor

Although there is no switch in this circuit, the voltage source is not turned on until .001 sec. We can clearly see the exponential charging curve.

If we choose t equal to the product of R and C, the capacitor voltage expression becomes:

$$
\begin{aligned}
V_c(t) &= V_{src}(1 - e^{-1}) \\
V_c(t) &= V_{src}(1 - 0.368) \\
V_c(t) &= 0.63V_{src}
\end{aligned}
$$

The product RC is called the *RC time constant* or τ and has units of seconds. When time t is equal to RC, the voltage across the capacitor will have reached 63% of final value of V_{src}. Our spice simulation found the time for the target voltage to reach 6.32 volts with the `.measure` statement. The value it found was:

```
Measurements for Transient Analysis
tdiff                   =  9.996728e-04
```

This is almost exactly 1ms, our time constant τ , as $(1000)10^{-6} = 10^{-3}$ seconds, or 1 millisecond

If we start with a capacitor charged to V_{src} and discharge it, we also see a different exponential behavior. For that circuit, without a source but with only a charged capacitor and resistor, the equation for the voltage across the capacitor is given by:

$$V_c(t) = V_{src}\left(e^{-t/RC}\right)$$

If we let t equal RC we get:

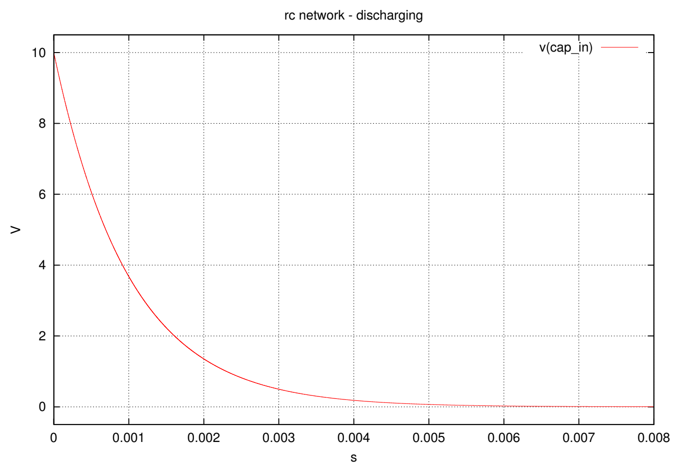
$$V_c(t) = 0.368V_{src}$$

So, the voltage across the cap after one time constant, τ, will be .368 times the initial voltage the capacitor was charged to.
Again, we can run an Spice simulation.

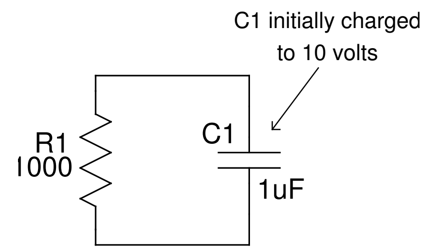
```
RC network - Discharging charged capacitor through a resistor

.ic v(cap_in)=10V ; cap initally charged to 10 volts
r1  cap_in    gnd    1k
c1  cap_in    gnd    1uF

.control
  tran 100ns 10ms
*  gnuplot rc_sim_disch V(cap_in) xl 1u 8m ;make .eps file for latex
  plot rc_sim_disch V(cap_in) xl 1u 8m
  set noaskquit
.endc

*measure the time difference between cap_in starting from 9.99v to
*reaching 3.68...i.e, the RC time constant
.meas tran tdiff trig v(cap_in) val=9.99 fall=1 td=1ps
+                targ v(cap_in) val=3.68 fall=1 td=1ps
.end
```

```
Measurements for Transient Analysis
tdiff=1.098612e-03
```

Again, almost exactly 1ms, our time constant, as $1000 \times 1 \times 10^{-6} = 1 \times 10^{-3}$



(a) Simulation of Capacitor discharging



(b) Schematic, Capacitor discharging

Figure 7: Capacitor discharging through a resistor

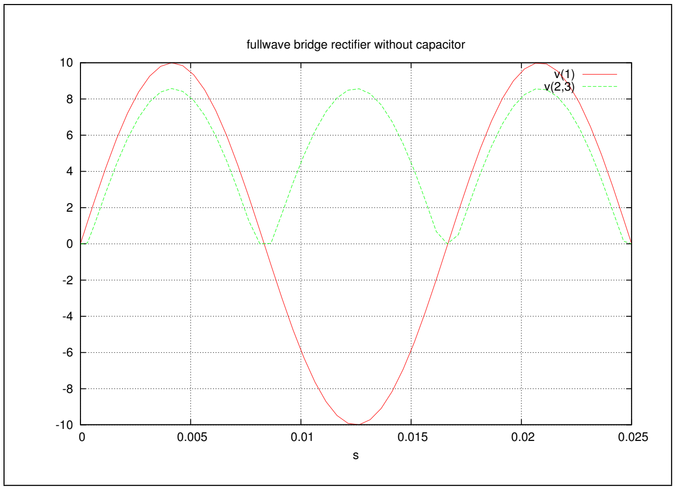
Capacitor Applications

Capacitors are used in many if not most electronic circuits. They primarily serve in to broad categories; as timing element and to provide selection or rejection of certain frequencies.

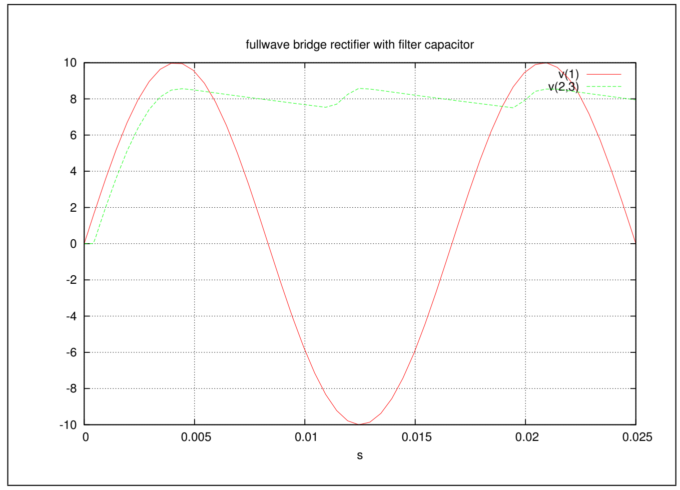
We have seen already how a capacitor can provide a time delay to an input voltage when used in conjunction with a resistor. The RC time constant τ allows calculation of a time period before a voltage waveform reaches a particular value. If we have an element that can make a *decision* or cause a switch to be actuated at this voltage then we can also assume that the delay from the onset of the applied voltage to the switch actuation will be given by τ.

Capacitors are frequently used in AC to DC power supplies. The input AC waveform is converted to a pulsating DC waveform that although is DC, is unusable to a device attached to it. The problem is that the DC waveform while always flowing in one direction and positive, pulsates between zero and a little over eight volts. However, we can use the energy storage properties of the capacitor to supply the current between the *humps* in the waveform. When the capacitor is used in this way, its also known as a *filter capacitor*. Essentially, its filtering out the big bumps in the DC waveform, providing a useful output voltage.

In the second waveform you can see the output voltage as the green trace. When the input voltage begins its downward path, the capacitor supplies current to the load resistor. As it does, we see the beginning of the exponential discharge curve in the output voltage. But before the voltage on the capacitor can drop too low, the next *hump* comes and both supplies current to the load resistor and charges the capacitor.



(a) Input and Output from Fullwave Rectifier



(b) Fullwave Rectifier with Filter Capacitor