Introduction to Functional Programming in Haskell
Outline

Why learn functional programming?

The essence of functional programming
   What is a function?
   Equational reasoning
   First-order vs. higher-order functions
   Lazy evaluation

How to functional program
   Functional programming workflow
   Data types
   Type classes
   Type-directed programming
   Haskell style
   Refactoring (bonus section)

Type inference
Outline

Why learn functional programming?

The essence of functional programming

How to functional program

Type inference
Why learn (pure) functional programming?

1. This course: strong correspondence of core concepts to PL theory
   - abstract syntax can be represented by algebraic data types
   - denotational semantics can be represented by functions

2. It will make you a better (imperative) programmer
   - forces you to think recursively and compositionally
   - forces you to minimize use of state

   …essential skills for solving big problems

3. It is the future!
   - more scalable and parallelizable (MapReduce)
   - functional features have been added to most mainstream languages
   - many cool new libraries built around functional paradigm
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What is a (pure) function?

A function is **pure** if:

- it always returns the same output for the same inputs
- it doesn’t do anything else — no “side effects”

In Haskell: whenever we say “function” we mean a **pure function**!
What are and aren’t functions?

Always functions:
- mathematical functions \( f(x) = x^2 + 2x + 3 \)
- encryption and compression algorithms

Usually not functions:
- C, Python, JavaScript, … “functions” (procedures)
- Java, C#, Ruby, … methods

Haskell only allows you to write (pure) functions!
Why procedures/methods aren’t functions

- output depends on environment
- may perform arbitrary side effects
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Type inference
Getting into the Haskell mindset

In Haskell, “=” means is not change to!

**Haskell**

```haskell
class Sum where
  sum :: [Int] -> Int
sum [] = 0
sum (x:xs) = x + sum xs
```

**Java**

```java
public int sum(List<Integer> xs) {
    int s = 0;
    for (int x : xs) {
        s = s + x;
    }
    return s;
}
```
Getting into the Haskell mindset

Quicksort in Haskell

```haskell
qsort :: Ord a => [a] -> [a]
qsort [] = []
qsort (x:xs) = qsort (filter (<= x) xs) ++ x : qsort (filter (> x) xs)
```

Quicksort in C

```c
void qsort(int low, int high) {
    int i = low, j = high;
    int pivot = numbers[low + (high-low)/2];
    while (i <= j) {
        while (numbers[i] < pivot) {
            i++;
        }
        while (numbers[j] > pivot) {
            j--;
        }
        if (i <= j) {
            swap(i, j);
            i++;
            j--;
        }
    }
    if (low < j) qsort(low, j);
    if (i < high) qsort(i, high);
}
void swap(int i, int j) {
    int temp = numbers[i];
    numbers[i] = numbers[j];
    numbers[j] = temp;
}
```
Referential transparency

An expression can be replaced by its **value** without changing the overall program behavior

$$\text{length } [1,2,3] + 4 \Rightarrow 3 + 4$$

**Corollary**: an expression can be replaced by **any expression** with the same value without changing program behavior

Supports **equational reasoning**

a.k.a. **referent**

what if **length** was a Java method?
Equational reasoning

Computation is just substitution!

Equations:

\[
\text{sum :: } [\text{Int}] \rightarrow \text{Int} \\
\text{sum } [] = 0 \\
\text{sum } (x:xs) = x + \text{sum } xs
\]

\[
\begin{align*}
\text{sum } [2,3,4] & \Rightarrow \text{sum } (2:(3:(4:[]))) \\
& \Rightarrow 2 + \text{sum } (3:(4:[])) \\
& \Rightarrow 2 + 3 + \text{sum } (4:[]) \\
& \Rightarrow 2 + 3 + 4 + \text{sum } [] \\
& \Rightarrow 2 + 3 + 4 + 0 \\
& \Rightarrow 9
\end{align*}
\]
Describing computations

**Function definition**: a list of **equations** that relate inputs to output
- matched top-to-bottom
- applied left-to-right

**Example: reversing a list**

**imperative view**: how do I rearrange the elements in the list? ✗

**functional view**: how is a list related to its reversal? ✓

```
reverse :: [a] -> [a]
reverse [] = []
reverse (x:xs) = reverse xs ++ [x]
```
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Type inference
First-order functions

Examples

- `cos :: Float -> Float`
- `even :: Int -> Bool`
- `length :: [a] -> Int`
Higher-order functions

Examples

- `map :: (a -> b) -> [a] -> [b]`
- `filter :: (a -> Bool) -> [a] -> [a]`
- `(.) :: (b -> c) -> (a -> b) -> a -> c`

The essence of functional programming
Higher-order functions as control structures

**map**: loop for doing something to each element in a list

\[
\text{map} :: (a \to b) \to [a] \to [b] \\
\text{map} f [] = [] \\
\text{map} f (x:xs) = f x : \text{map} f xs \\
\text{map} f [2,3,4,5] = [f 2, f 3, f 4, f 5] \\
\text{map} \text{even} [2,3,4,5] \\
= [\text{even} 2, \text{even} 3, \text{even} 4, \text{even} 5] \\
= [\text{True}, \text{False}, \text{True}, \text{False}] \\
\]

**fold**: loop for aggregating elements in a list

\[
\text{foldr} :: (a\to b\to b) \to b \to [a] \to b \\
\text{foldr} f y [] = y \\
\text{foldr} f y (x:xs) = f x (\text{foldr} f y xs) \\
\text{foldr} (+) 0 [2,3,4] = f 2 (f 3 (f 4 y)) \\
= (+) 2 ((+) 3 ((+) 4 0)) \\
= 2 + (3 + (4 + 0)) \\
= 9 \\
\]
Function composition

Can create new functions by **composing** existing functions

- *apply the second function, then apply the first*

Function composition

\[(.): (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c\]

\[(f . g) x = f (g x)\]

Types of existing functions

- `not :: Bool -> Bool`
- `succ :: Int -> Int`
- `even :: Int -> Bool`
- `head :: [a] -> a`
- `tail :: [a] -> [a]`

Definitions of new functions

- `plus2 = succ . succ`
- `odd = not . even`
- `second = head . tail`
- `drop2 = tail . tail`
Currying / partial application

In Haskell, functions that take multiple arguments are **implicitly higher order**

\[
\text{plus :: Int -> Int -> Int}
\]

\[
\text{increment :: Int -> Int}
\]

\[
\text{increment = plus 1}
\]

**Curried**

\[
\text{plus 2 3}
\]

**Uncurried**

\[
\text{plus (2,3)}
\]
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Lazy evaluation

In Haskell, expressions are reduced:

- only when needed
- at most once

Supports:

- infinite data structures
- separation of concerns

```haskell
nats :: [Int]
nats = 1 : map (+1) nats

fact :: Int -> Int
fact n = product (take n nats)

min3 :: [Int] -> [Int]
min3 = take 3 . sort
```

What is the running time of this function?

---

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  Type classes
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  Haskell style
  Refactoring (bonus section)

Type inference
FP workflow (simple)

Refactor

Define functions

Identify/define types

“obsessive compulsive refactoring disorder”
FP workflow (detailed)

1A. Data Description

1B. Data Examples

2. Function Description (Signature/Purpose/Header)

3. Functional Examples

4. Function Template

5. Code

6. Tests

7. Review & Refactor

Norman Ramsey, On Teaching “How to Design Programs”, ICFP’14
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Type inference
Algebraic data types

Data type definition
- introduces new **type** of value
- enumerates ways to **construct** values of this type

Definitions consists of …
- a **type name**
- a list of **data constructors** with **argument types**

Some example data types

```haskell
data Bool = True | False
data Nat = Zero | Succ Nat
data Tree = Node Int Tree Tree | Leaf Int```

Definition is **inductive**
- the arguments may **recursively** include the type being defined
- the constructors are the **only way** to build values of this type
Anatomy of a data type definition

```
data Expr = Lit Int |
            Plus Expr Expr
```

Example: $2 + 3 + 4 = \text{Plus (Lit 2) (Plus (Lit 3) (Lit 4))}$
FP data types vs. OO classes

Haskell

data Tree = Node Int Tree Tree  
  | Leaf

  • separation of type- and value-level
  • set of cases closed
  • set of operations open

Java

abstract class Tree { ... }
class Node extends Tree {
  int label;
  Tree left, right;
  ...
}
class Leaf extends Tree { ... }

  • merger of type- and value-level
  • set of cases open
  • set of operations closed

Extensibility of cases vs. operations = the “expression problem”
Type parameters

Like generics in Java

data List a = Nil
| Cons a (List a)

reference to type parameter

recursive reference to type

Specialized lists

type IntList = List Int
type CharList = List Char
type RaggedMatrix a = List (List a)
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Type inference
What is a type class?

1. an **interface** that is supported by many different types
2. a **set of types** that have a common behavior

```haskell
class Eq a where
    (==) :: a -> a -> Bool

class Show a where
    show :: a -> String

class Num a where
    (+) :: a -> a -> a
    (*) :: a -> a -> a
    negate :: a -> a
    ...
```

- types whose values can be compared for equality
- types whose values can be shown as strings
- types whose values can be manipulated like numbers
Type constraints

List elements can be of any type

\[
\begin{align*}
\text{length} & : \ [a] \rightarrow \text{Int} \\
\text{length} \; [] & = 0 \\
\text{length} \; (_:\!\!:\!\!xs) & = 1 + \text{length} \; xs
\end{align*}
\]

List elements must support equality!

\[
\begin{align*}
\text{elem} & : \text{Eq} \; a \implies a \rightarrow \text{[a]} \rightarrow \text{Bool} \\
\text{elem}_\_ \; [] & = \text{False} \\
\text{elem} \; y \; (_\!:\!\!:\!\!xs) & = x == y \; || \; \text{elem} \; y \; xs
\end{align*}
\]

use method ⇒ add type class constraint

class Eq a where
    (==) :: a -> a -> Bool
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Type inference
Tools for defining functions

Recursion and other functions

\[
\text{sum} :: [\text{Int}] \rightarrow \text{Int} \\
\text{sum} \; \text{xs} = \begin{cases} 
0 & \text{if \ null\ xs} \\
\text{head} \; \text{x} + \text{sum} \; (\text{tail} \; \text{xs}) & \text{else}
\end{cases}
\]

(1) case analysis

Pattern matching

\[
\begin{align*}
\text{sum} :: [\text{Int}] & \rightarrow \text{Int} \\
\text{sum} \; [] & = 0 \\
\text{sum} \; (x:xs) & = x + \text{sum} \; xs
\end{align*}
\]

(2) decomposition

Higher-order functions

\[
\text{sum} :: [\text{Int}] \rightarrow \text{Int} \\
\text{sum} = \text{foldr} \; (+) \; 0
\]

no recursion or variables needed!
What is type-directed programming?

Use the **type** of a function to help write its **body**
Type-directed programming

Basic goal: transform values of argument types into result type

If argument type is ...
• atomic type (e.g. Int, Char)
  • apply functions to it
• algebraic data type
  • use pattern matching
    • case analysis
    • decompose into parts
• function type
  • apply it to something

If result type is ...
• atomic type
  • output of another function
• algebraic data type
  • build with data constructor
• function type
  • function composition or partial application
  • build with lambda abstraction
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Type inference
Good Haskell style

Why it matters:

- layout is significant!
- eliminate misconceptions
- we care about *elegance*

Easy stuff:

- **use spaces!** (tabs cause layout errors)
- align patterns and guards

See style guides on course web page
Function application:
- is *just a space*
- associates to the left
- binds most strongly

\[
\begin{align*}
  f(x) & \quad f \ x \\
  (f \ x) \ y & \quad f \ x \ y \\
  (f \ x) + (g \ y) & \quad f \ x + g \ y
\end{align*}
\]

Use parentheses only to *override* this behavior:
- \( f (g \ x) \)
- \( f (x + y) \)
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Type inference
Refactoring in the FP workflow

Motivations:
- separate concerns
- promote reuse
- promote understandability
- gain insights

“obsessive compulsive refactoring disorder”
Refactoring relations

Semantics-preserving laws can prove with equational reasoning + induction

- Eta reduction:
  \[ \lambda x \rightarrow f \ x \equiv f \]

- Map–map fusion:
  \[ \text{map } f \ . \ \text{map } g \equiv \text{map } (f \ . \ g) \]

- Fold–map fusion:
  \[ \text{foldr } f \ b \ . \ \text{map } g \equiv \text{foldr } (f \ . \ g) \ b \]

“Algebra of computer programs”

John Backus, *Can Programming be Liberated from the von Neumann Style?*, ACM Turing Award Lecture, 1978
Strategy: systematic generalization

```
commas :: [String] -> [String]
commas [] = []
commas [x] = [x]
commas (x:xs) = x : ", " : commas xs
```

```
intersperse :: a -> [a] -> [a]
intersperse _ [] = []
intersperse _ [x] = [x]
intersperse s (x:xs) = x : s : intersperse s xs
```

```
seps :: String -> [String] -> [String]
seps _ [] = []
seps _ [x] = [x]
seps s (x:xs) = x : s : seps s xs
```

Introduce parameters for constants

Broaden the types

How to functional program
Strategy: abstract repeated templates

**abstract** (v): extract and make reusable (as a function)

```haskell
showResult :: Maybe Float -> String
showResult Nothing = "ERROR"
showResult (Just v) = show v

moveCommand :: Maybe Dir -> Command
moveCommand Nothing = Stay
moveCommand (Just d) = Move d

safeAdd :: Int -> Maybe Int -> Int
safeAdd x Nothing = x
safeAdd x (Just y) = x + y
```

Repeated structure:
- pattern match
- default value if **Nothing**
- apply function to contents if **Just**
Strategy: abstract repeated templates

Describe repeated structure in function

```haskell
maybe :: b -> (a -> b) -> Maybe a -> b
maybe b _ Nothing = b
maybe _ f (Just a) = f a
```

Reuse in implementations

```haskell
showResult = maybe "ERROR" show
moveCommand = maybe Stay Move
safeAdd x = maybe x (x+)
```
Refactoring data types

```haskell
data Expr = Var Name
    | Add Expr Expr
    | Sub Expr Expr
    | Mul Expr Expr

vars :: Expr -> [Name]
vars (Var x) = [x]
vars (Add l r) = vars l ++ vars r
vars (Sub l r) = vars l ++ vars r
vars (Mul l r) = vars l ++ vars r

eval :: Env -> Expr -> Int
eval m (Var x) = get x m
eval m (Add l r) = eval m l + eval m r
eval m (Sub l r) = eval m l - eval m r
eval m (Mul l r) = eval m l * eval m r
```
Refactoring data types

Factor out shared structure

```haskell
data Expr = Var Name
         | BinOp Op Expr Expr

data Op = Add | Sub | Mul

vars :: Expr -> [Name]
vars (Var x) = [x]
vars (BinOp _ l r) = vars l ++ vars r

eval :: Env -> Expr -> Int
eval m (Var x) = get x m
eval m (BinOp o l r) = op o (eval m l) (eval m r)
  where
    op Add = (+)
    op Sub = (-)
    op Mul = (*)
```
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Type inference
Type inference

How to perform type inference

If a literal, data constructor, or named function: write down the type – you’re done!

Otherwise:

1. pick an application $e_1 \ e_2$
2. recursively infer their types $e_1 : T_1$ and $e_2 : T_2$
3. $T_1$ should be a function type $T_1 = T_{\text{arg}} \rightarrow T_{\text{res}}$
4. unify $T_{\text{arg}} = ? T_2$, yielding type variable assignment $\sigma$
5. return $e_1 \ e_2 : \sigma T_{\text{res}}$ ($T_{\text{res}}$ with type variables substituted)

If any of these steps fails, it is a type error!
Exercises

Given

```haskell
data Maybe a = Nothing | Just a

gt :: Int -> Int -> Bool
not :: Bool -> Bool
map :: (a -> b) -> [a] -> [b]
even :: Int -> Bool
(.) :: (b -> c) -> (a -> b) -> a -> c
```

1. Just
2. not even 3
3. not (even 3)
4. not . even
5. even . not
6. map (Just . even)