Introduction to Functional Programming in Haskell
Outline

Why learn functional programming?

The essence of functional programming
   What is a function?
   Equational reasoning
   First-order vs. higher-order functions
   Lazy evaluation

How to functional program
   Functional programming workflow
   Data types
   Type classes
   Type-directed programming
   Haskell style
   Refactoring (bonus section)

Type inference
Outline

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The essence of functional programming

How to functional program

Type inference
Why learn (pure) functional programming?

1. This course: strong correspondence of core concepts to PL theory
   - abstract syntax can be represented by algebraic data types
   - denotational semantics can be represented by functions

2. It will make you a better (imperative) programmer
   - forces you to think recursively and compositionally
   - forces you to minimize use of state
   ...essential skills for solving big problems

3. It is the future!
   - more scalable and parallelizable (MapReduce)
   - functional features have been added to most mainstream languages
   - many cool new libraries built around functional paradigm
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What is a (pure) function?

A function is **pure** if:

- it always returns the same output for the same inputs
- it doesn’t do anything else — no “side effects”

In Haskell: whenever we say “function” we mean a **pure function**!
What are and aren’t functions?

Always functions:
• mathematical functions \( f(x) = x^2 + 2x + 3 \)
• encryption and compression algorithms

Usually not functions:
• C, Python, JavaScript, … “functions” (procedures)
• Java, C#, Ruby, … methods

Haskell only allows you to write (pure) functions!
Why procedures/methods aren’t functions

- output depends on environment
- may perform arbitrary side effects
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Getting into the Haskell mindset

In Haskell, “=” means *is not* change to!

**Haskell**

```haskell
sum :: [Int] -> Int
sum [] = 0
sum (x:xs) = x + sum xs
```

**Java**

```java
int sum(List<Int> xs) {
    int s = 0;
    for (int x : xs) {
        s = s + x;
    }
    return s;
}
```
Getting into the Haskell mindset

Quicksort in Haskell

```haskell
qsort :: Ord a => [a] -> [a]
qsort [] = []
qsort (x:xs) = qsort (filter (<= x) xs) ++ x : qsort (filter (> x) xs)
```

Quicksort in C

```c
void qsort(int low, int high) {
    int i = low, j = high;
    int pivot = numbers[low + (high-low)/2];

    while (i <= j) {
        while (numbers[i] < pivot) {
            i++;
        }
        while (numbers[j] > pivot) {
            j--;
        }
        if (i <= j) {
            swap(i, j);
            i++;  j--;
        }
    }

    if (low < j) qsort(low, j);
    if (i < high) qsort(i, high);
}
```
Referential transparency

An expression can be replaced by its value without changing the overall program behavior.

\[
\text{length} \ [1,2,3] + 4 \\
\Rightarrow \\
3 + 4
\]

What if \text{length} was a Java method?

**Corollary:** an expression can be replaced by any expression with the same value without changing program behavior.

Supports **equational reasoning**
Equational reasoning

Computation is just substitution!

```
sum :: [Int] -> Int
sum []    = 0
sum (x:xs) = x + sum xs
```

equations

```
sum [2,3,4]
⇒ sum (2:(3:(4:[])))
⇒ 2 + sum (3:(4:[]))
⇒ 2 + 3 + sum (4:[])
⇒ 2 + 3 + 4 + sum []
⇒ 2 + 3 + 4 + 0
⇒ 9
```
Describing computations

**Function definition**: a list of equations that relate inputs to output

- matched top-to-bottom
- applied left-to-right

---

**Example: reversing a list**

**imperative view**: how do I rearrange the elements in the list?

**functional view**: how is a list related to its reversal?

```haskell
reverse :: [a] -> [a]
reverse [] = []
reverse (x:xs) = reverse xs ++ [x]
```
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Type inference
First-order functions

Examples
- \( \text{cos} :: \text{Float} \rightarrow \text{Float} \)
- \( \text{even} :: \text{Int} \rightarrow \text{Bool} \)
- \( \text{length} :: [\text{a}] \rightarrow \text{Int} \)
Higher-order functions

Examples

- `map :: (a -> b) -> [a] -> [b]`
- `filter :: (a -> Bool) -> [a] -> [a]`
- `(.) :: (b -> c) -> (a -> b) -> a -> c`
Higher-order functions as control structures

**map**: loop for doing something to each element in a list

```haskell
map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x:xs) = f x : map f xs
```

- `map f [2,3,4,5] = [f 2, f 3, f 4, f 5]`
- `map even [2,3,4,5] = [even 2, even 3, even 4, even 5] = [True,False,True,False]`

**fold**: loop for aggregating elements in a list

```haskell
foldr :: (a->b->b) -> b -> [a] -> b
foldr f y [] = y
foldr f y (x:xs) = f x (foldr f y xs)
```

- `foldr (+) 0 [2,3,4] = (+) 2 ((+) 3 ((+) 4 0)) = 2 + (3 + (4 + 0)) = 9`
Function composition

Can create new functions by **composing** existing functions

- *apply the second function, then apply the first*

Function composition

\[
(\cdot) :: (b \to c) \to (a \to b) \to a \to c \\
(f \cdot g) x = f (g x)
\]

Types of existing functions

- `not :: Bool \to Bool`
- `succ :: Int \to Int`
- `even :: Int \to Bool`
- `head :: [a] \to a`
- `tail :: [a] \to [a]`

Definitions of new functions

- `plus2 = succ \cdot succ`
- `odd = not \cdot even`
- `second = head \cdot tail`
- `drop2 = tail \cdot tail`
Currying / partial application

In Haskell, functions that take multiple arguments are **implicitly higher order**

```
plus :: Int -> Int -> Int
```

**Curried**
```
plus :: Int -> Int -> Int
```
```
plus 2 3
```

**Uncurried**
```
plus :: (Int,Int) -> Int
```
```
plus (2,3)
```

```
increment :: Int -> Int
increment = plus 1
```

Haskell Curry
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Type inference
Lazy evaluation

In Haskell, expressions are reduced:

- only when needed
- at most once

Supports:

- infinite data structures
- separation of concerns

```
nats :: [Int]
nats = 1 : map (+1) nats

fact :: Int -> Int
fact n = product (take n nats)

min3 :: [Int] -> [Int]
min3 = take 3 . sort
```

What is the running time of this function?
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  Haskell style
  Refactoring (bonus section)

Type inference
“obsessive compulsive refactoring disorder”
FP workflow (detailed)

1A. Data Description

2. Function Description (Signature/Purpose/Header)

3. Functional Examples

4. Function Template

5. Code

6. Tests

7. Review & Refactor

names used in signature

signature guides template

write body

demands more

guide writing

overlooked cases

inputs

are also

validated by

Norman Ramsey, On Teaching “How to Design Programs”, ICFP’14
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Type inference
Algebraic data types

Data type definition
• introduces new type of value
• enumerates ways to construct values of this type

Some example data types

data Bool = True | False

data Nat = Zero | Succ Nat

data Tree = Node Int Tree Tree |
            Leaf Int

Definitions consists of …
• a type name
• a list of data constructors with argument types

Definition is inductive
• the arguments may recursively include the type being defined
• the constructors are the only way to build values of this type
Anatomy of a data type definition

```haskell
data Expr = Lit Int
           | Plus Expr Expr
```

Example: $2 + 3 + 4 \quad \text{Plus} \ (\text{Lit} \ 2) \ (\text{Plus} \ (\text{Lit} \ 3) \ (\text{Lit} \ 4))$
FP data types vs. OO classes

Haskell

```haskell
data Tree = Node Int Tree Tree
 | Leaf
```

- separation of type- and value-level
- set of cases closed
- set of operations open

Java

```java
abstract class Tree {
...
}
class Node extends Tree {
    int label;
    Tree left, right;
    ...
}
class Leaf extends Tree {
    ...
}
```

- merger of type- and value-level
- set of cases open
- set of operations closed

Extensibility of cases vs. operations = the “expression problem”
Type parameters

(Like generics in Java)

data List a = Nil | Cons a (List a)

Specialized lists

type IntList = List Int
type CharList = List Char
type RaggedMatrix a = List (List a)
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Type inference
What is a type class?

1. an **interface** that is supported by many different types
2. a **set of types** that have a common behavior

```haskell
class Eq a where
  (==) :: a -> a -> Bool

class Show a where
  show :: a -> String

class Num a where
  (+) :: a -> a -> a
  (*) :: a -> a -> a
  negate :: a -> a
  ...
```

**types whose values can be compared for equality**

**types whose values can be shown as strings**

**types whose values can be manipulated like numbers**
Type constraints

List elements can be of any type

\[
\text{length} :: [a] \rightarrow \text{Int} \\
\text{length} [] = 0 \\
\text{length} (_:xs) = 1 + \text{length} \hspace{1em} \text{xs}
\]

List elements must support equality!

\[
\text{elem} :: \text{Eq} \ a \Rightarrow \ a \rightarrow [a] \rightarrow \text{Bool} \\
\text{elem} \_ [] = \text{False} \\
\text{elem} \ y \ (x:xs) = x == y \text{ || elem} \ y \hspace{1em} \text{xs}
\]

use method ⇒ add type class constraint
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Type inference
Tools for defining functions

Recursion and other functions

\[
\text{sum} :: \ [\text{Int}] \rightarrow \text{Int} \\
\text{sum} \ xs = \begin{cases} 
0 & \text{if null } xs \\
\text{head } xs + \text{sum} \ (\text{tail } xs) & \text{otherwise}
\end{cases}
\]

Pattern matching

\[
\text{sum} :: \ [\text{Int}] \rightarrow \text{Int} \\
\text{sum} \ [] = 0 \\
\text{sum} \ (x:xs) = x + \text{sum} \ xs
\]

(1) case analysis

(2) decomposition

Higher-order functions

\[
\text{sum} :: \ [\text{Int}] \rightarrow \text{Int} \\
\text{sum} = \text{foldr} \ (+) \ 0
\]

no recursion or variables needed!
What is type-directed programming?

Use the **type** of a function to help write its **body**
Type-directed programming

Basic goal: transform values of argument types into result type

If argument type is ...  
- atomic type (e.g. Int, Char)  
  - apply functions to it
- algebraic data type  
  - use pattern matching  
    - case analysis  
    - decompose into parts
- function type  
  - apply it to something

If result type is ...  
- atomic type  
  - output of another function
- algebraic data type  
  - build with data constructor
- function type  
  - function composition or partial application  
  - build with lambda abstraction
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Type inference
Good Haskell style

Why it matters:
• layout is significant!
• eliminate misconceptions
• we care about elegance

Easy stuff:
• use spaces! (tabs cause layout errors)
• align patterns and guards

See style guides on course web page
Function application:
- is *just a space*
- associates to the left
- binds most strongly

\[ f(x) \quad f(x) \]
\[ (f \ x) \ y \quad f \ x \ y \]
\[ (f \ x) + (g \ y) \quad f \ x + g \ y \]

Use parentheses only to *override* this behavior:
- \( f (g \ x) \)
- \( f (x + y) \)
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Type inference
Refactoring in the FP workflow

Motivations:
- separate concerns
- promote reuse
- promote understandability
- gain insights

“obsessive compulsive refactoring disorder”
Refactoring relations

Semantics-preserving laws prove with equational reasoning and/or induction

- Eta reduction:
  \[ \lambda x \to f \, x \equiv f \]

- Map–map fusion:
  \[ \text{map } f \cdot \text{map } g \equiv \text{map } (f \cdot g) \]

- Fold–map fusion:
  \[ \text{foldr } f \, b \cdot \text{map } g \equiv \text{foldr } (f \cdot g) \, b \]

“Algebra of computer programs”

John Backus, *Can Programming be Liberated from the von Neumann Style?*, ACM Turing Award Lecture, 1978
Strategy: systematic generalization

**commas** :: [String] -> [String]
commas [] = []
commas [x] = [x]
commas (x:xs) = x : ", " : commas xs

**seps** :: String -> [String] -> [String]
seps _ [] = []
seps _ [x] = [x]
seps s (x:xs) = x : s : seps s xs

**intersperse** :: a -> [a] -> [a]
intersperse _ [] = []
intersperse _ [x] = [x]
intersperse s (x:xs) = x : s : intersperse s xs

Introduce parameters for constants

Broaden the types

How to functional program
Strategy: abstract repeated templates

**abstract** (v): extract and make reusable (as a function)

```
showResult :: Maybe Float -> String
showResult Nothing = "ERROR"
showResult (Just v) = show v

moveCommand :: Maybe Dir -> Command
moveCommand Nothing = Stay
moveCommand (Just d) = Move d

safeAdd :: Int -> Maybe Int -> Int
safeAdd x Nothing = x
safeAdd x (Just y) = x + y
```
Strategy: abstract repeated templates

Describe repeated structure in function

```haskell
maybe :: b -> (a -> b) -> Maybe a -> b
maybe b _ Nothing = b
maybe _ f (Just a) = f a
```

Reuse in implementations

```haskell
showResult = maybe "ERROR" show
moveCommand = maybe Stay Move
safeAdd x = maybe x (x+)
```
Refactoring data types

data Expr = Var Name
       | Add Expr Expr
       | Sub Expr Expr
       | Mul Expr Expr

vars :: Expr -> [Name]
vars (Var x) = [x]
vars (Add l r) = vars l ++ vars r
vars (Sub l r) = vars l ++ vars r
vars (Mul l r) = vars l ++ vars r

eval :: Env -> Expr -> Int
eval m (Var x) = get x m
eval m (Add l r) = eval m l + eval m r
eval m (Sub l r) = eval m l - eval m r
eval m (Mul l r) = eval m l * eval m r
Refactoring data types

Factor out shared structure

data Expr = Var Name 
  | BinOp Op Expr Expr 

data Op = Add | Sub | Mul 

vars :: Expr -> [Name] 
vars (Var x) = [x] 
vars (BinOp _ l r) = vars l ++ vars r 

eval :: Env -> Expr -> Int 
eval m (Var x) = get x m 
eval m (BinOp o l r) = op o (eval m l) (eval m r) 
  where 
    op Add = (+) 
    op Sub = (-) 
    op Mul = (*)
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Type inference
How to perform type inference

If a literal, data constructor, or named function: write down the type – you’re done!

Otherwise:

1. pick an application $e_1 e_2$
2. recursively infer their types $e_1 : T_1$ and $e_2 : T_2$
3. $T_1$ should be a function type $T_1 = T_{\text{arg}} \rightarrow T_{\text{res}}$
4. unify $T_{\text{arg}} =? T_2$, yielding type variable assignment $\sigma$
5. return $e_1 e_2 : \sigma T_{\text{res}}$ (with type variables substituted)

If any of these steps fails, it is a type error!
Exercises

Given

data Maybe a = Nothing | Just a

1. Just
2. not even 3
3. not (even 3)
4. not . even
5. even . not
6. map (Just . even)