SECTION 3: BJT AMPLIFIERS
BJT Amplifier Circuits
In this section of the course, we will look at three BJT amplifiers, with a focus on the following two circuits:

- **Common-Emitter Amplifier:**
  - High voltage gain
  - An amplifier

- **Emitter-Follower Amplifier:**
  - Near unity gain
  - A buffer
BJT Amplifier Circuits

- Recall the two functional pieces of a BJT amplifier:
  - **Bias network**
    - Sets the DC operating point of the transistor
    - Ensures the BJT remains in the forward-active region
  
  - **Signal path**
    - Sets the gain of the amplifier circuit

- Significant overlap between the two parts
BJT Amplifier Biasing
BJT Amplifier Biasing

- To function as an amplifier, a transistor must be biased in the *forward-active region*.

- DC operating point set by the *bias network*:
  - Resistors and power supply voltages
  - Sets the transistor’s *DC terminal voltages and currents* – its DC bias

- How a transistor is *biased* determines:
  - Small-signal characteristics
  - Small-signal model parameters
  - How it will behave as an amplifier
Voltage Transfer Characteristic

- BJT amplifier biased in the middle of its linear region
- Slope of the large-signal transfer characteristic gives the amplifier gain
  - Negative slope – gain is inverting
  - Small input signals yield larger output signals
  - Slope is nearly linear in this region
BJT Biasing – Four-Resistor Bias Circuit

- **Four-resistor bias circuit:**
  - Commonly-used for both *common-emitter* amplifiers and *emitter-followers*
  - Single power supply or bipolar supply

- Provides *nearly-\(\beta\)-independent biasing*
  - \(\beta\) is often unknown and may be variable
  - DC operating point stays nearly constant as \(\beta\) changes

- Analyze the bias circuit by replacing the transistor with its large-signal model
Analysis of the Four-Resistor Bias Circuit

- To analyze the bias circuit, replace the transistor with its large-signal model.
- First, simplify by replacing the base network with its Thevenin equivalent.

\[
V_{BB} = V_{CC} \frac{R_{B2}}{R_{B1} + R_{B2}}, \quad R_B = \frac{R_{B1}R_{B2}}{R_{B1} + R_{B2}}
\]
Analysis of the Four-Resistor Bias Circuit

- Replace the transistor with its large-signal model
- Apply KVL around the B-E loop:
  \[ V_{BB} - I_B R_B - V_{BE} - I_E R_E = 0 \]
- Express \( I_E \) in terms of \( I_B \), then solve for \( I_B \):
  \[ I_E = (\beta + 1) I_B \]
  \[ V_{BB} - I_B R_B - V_{BE} - (\beta + 1) I_B R_E = 0 \]
  \[ I_B = \frac{V_{BB} - V_{BE}}{R_B + (\beta + 1) R_E} \]
Analysis of the Four-Resistor Bias Circuit

- Get $I_C$ and $I_E$ from $I_B$
  
  \[
  I_C = \beta I_B \\
  I_E = (\beta + 1)I_B
  \]

- Use currents to calculate terminal voltages:
  
  \[
  V_C = V_{CC} - I_C R_C \\
  V_E = I_E R_E = V_B - V_{BE} \\
  V_B = V_{BB} - I_B R_B = V_E + V_{BE}
  \]

- Verify that the transistor is biased in the forward active region:
  
  \[V_{BE} > 0\]  
  \[V_{BC} < 0\]

- Next, we’ll use the DC operating point to determine small-signal model parameters and a small-signal equivalent circuit
Design of the Four-Resistor Bias Network

- Design the bias network for bias current that is independent of $\beta$ and temperature variation

- $\beta$ independence
  - Changes in base current do not affect bias current
  - Current in the base bias resistors ($R_{B1}$ and $R_{B2}$) must be much larger than the base current
    \[ I_{R_{B1,2}} \gg I_B \]
  - Set resistive divider current in same order of magnitude as bias current
    \[ 0.1I_E \leq I_{R_{B1,2}} \leq I_E \]

- Temperature independence
  - Changes in B-E voltage with temperature do not affect bias current
  - Set base voltage much larger than B-E voltage
    \[ V_B \gg V_{BE} \]
**Rule of thumb for designing a bias network:**

- Select $R_{B1}$ and $R_{B2}$ to conduct approximately $1/10$ of the desired bias current.
  \[
  \frac{V_{CC}}{R_{B1} + R_{B2}} \approx 0.1I_E
  \]

- Set $V_{BB}$ to approximately $1/3$ of the supply voltage.
  \[
  V_{BB} \approx \frac{V_{CC}}{3}
  \]
  \[
  \frac{R_{B2}}{R_{B1} + R_{B2}} \approx \frac{1}{3}
  \]

- Select $R_C$ to drop approximately $1/3$ of the supply voltage.
  \[
  R_C \approx \frac{V_{CC}/3}{I_C}
  \]
Rule of thumb for designing a bias network (continued):

- Determine $R_E$ to provide the desired bias current

$$R_E = \frac{V_{BB} - V_{BE}}{I_E} - \frac{R_B}{\beta + 1}$$

- This configuration provides approximately:
  - $V_{CC}/3$ across $R_C$
  - $V_{CC}/3$ across the C-B junction
  - $V_{CC}/3$ at the base (or, roughly, across $R_E$)

- $v_o$ can swing approximately:
  - $V_{CC}/3$ in the positive direction, before cutoff
  - $V_{CC}/3$ in the negative direction, before saturation
Bias Circuit Design - Example

- Design the bias network to provide $I_E = 1 \ mA$
- Set $R_{B1}$ and $R_{B2}$ to conduct approximately $0.1I_E$

\[
\frac{V_{CC}}{R_{B1} + R_{B2}} = 0.1I_E = 100 \ \mu A
\]

\[
R_{B1} + R_{B2} = \frac{15 \ V}{100 \ \mu A} = 150 \ k\Omega
\]

- Determine $R_{B1}$ and $R_{B2}$ to set $V_{BB} \approx V_{CC}/3$

\[
\frac{R_{B2}}{R_{B1} + R_{B2}} = 1/3
\]

\[
R_{B1} = 2R_{B2}
\]

\[
R_{B1} = 100 \ k\Omega
\]

\[
R_{B2} = 50 \ k\Omega
\]
Bias Circuit Design - Example

- Select $R_C$ to drop approximately $1/3$ of the supply voltage

$$R_C = \frac{V_{CC}}{3} = \frac{5 \text{ V}}{1 \text{ mA}}$$

$$R_C = 5 \text{ k}\Omega$$

- Finally, determine $R_E$ to provide the desired bias current

$$R_E = \frac{V_{BB} - V_{BE}}{I_E} - \frac{R_B}{\beta + 1}$$

$$R_E = \frac{(5 \text{ V} - 700 \text{ mV})}{1 \text{ mA}} - \frac{(100 \text{ k}\Omega || 50 \text{ k}\Omega)}{101}$$

$$R_E = 4.3 \text{ k}\Omega - 330 \Omega$$

$$R_E = 3.97 \text{ k}\Omega$$

$\beta = 100$
Common-Emitter Amplifier
Common-Emitter Amplifier

- Common-emitter amplifier
- All capacitors are *AC-coupling/DC blocking capacitors*
  - Open at DC
  - Shorts at signal frequencies
  - Isolate transistor bias from source/load
- Called *common*-emitter, because emitter is connected to common – i.e., ground or a power supply
  - $C_E$ is a small-signal short to ground
  - Emitter is at small-signal ground
Common-Emitter Amplifier

- Analyze the amplifier to find:
  - DC operating point
  - Small-signal voltage gain

- Large-signal (DC) equivalent circuit:
  - Replace all capacitors with open circuits
  - Simplify the base bias network
  - Replace the transistor with its large-signal model
As we have seen, base current is given by
\[
I_B = \frac{V_{BB} - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{4.1 \text{ V} - 700 \text{ mV}}{3.38 \text{ k}\Omega + 201 \cdot 680 \Omega} = 24.3 \mu\text{A}
\]

Use \(I_B\) to get \(I_C\) and \(I_E\)

\[
I_C = \beta I_B = 200 \cdot 24.3 \mu\text{A} = 4.9 \text{ mA}
\]

\[
I_E = (\beta + 1)I_B = 201 \cdot 24.3 \mu\text{A} = 4.9 \text{ mA}
\]

Next, use currents to determine terminal voltages

\[
V_B = V_{BB} - I_B R_B = 4.1 \text{ V} - 24.3 \mu\text{A} \cdot 3.38 \text{ k}\Omega = 4.02 \text{ V}
\]

\[
V_C = V_{CC} - I_C R_C = 12 \text{ V} - 4.9 \text{ mA} \cdot 820 \Omega = 7.98 \text{ V}
\]

\[
V_E = I_E R_E = 4.9 \text{ mA} \cdot 680 \Omega = 3.33 \text{ V}
\]
The complete DC operating point:

\[
\begin{align*}
I_B &= 24.3 \, \mu A \\
I_C &= 4.9 \, mA \\
I_E &= 4.9 \, mA \\
V_B &= 4.02 \, V \\
V_C &= 7.98 \, V \\
V_E &= 3.33 \, V
\end{align*}
\]

Use the operating point to determine small-signal model parameters

\[
\begin{align*}
\beta &= \frac{I_C}{V_{th}} = \frac{4.9 \, mA}{26 \, mV} = 188 \, mS \\
\beta &= \frac{V_{th}}{I_C} = 1.06 \, k\Omega \\
\beta &= \frac{r_{\pi}}{g_m} = 200 \cdot \frac{1.06 \, k\Omega}{I_C} = 5.28 \, \Omega
\end{align*}
\]
The DC operating point allowed us to determine the small-signal model for the transistor.

Next, create the small-signal equivalent circuit for the amplifier and perform a small-signal analysis.

Small signal analysis:

1. Replace all AC coupling capacitors with shorts
   - Large enough to look like shorts at signal frequencies

2. Connect all DC supply voltages to ground
   - From a small-signal perspective these are all constant voltages
   - Small-signal ground

3. Replace the transistor with its small-signal model
C-E Amplifier – Small-Signal Analysis

- **Small-signal equivalent circuit**
  - Use to determine small-signal voltage gain

- **Emitter connected to ground**
  - Emitter capacitor, $C_E$, is a small-signal short

- **$R_B$** is in parallel with $r_\pi$, and both connect to ground

- **$R_C$** is in parallel with $R_L$, and both connect to ground

- **Input voltage, $v_i(t)$**, appears across $r_\pi$ and is the same as $v_{be}$

- The transistor is a **transconductance** device
  - Input voltage, $v_{be}$, creates output current, $i_c$
Determine the small-signal voltage gain:

$$A_v = \frac{v_o}{v_i}$$  \hspace{1cm} (1)

The input is applied across the B-E junction, so

$$v_{be} = v_i$$  \hspace{1cm} (2)

The output is the collector current applied across the output resistance

$$v_o = -i_c R_o = -g_m v_{be} R_o$$  \hspace{1cm} (3)

where $R_o$ is the total resistance seen by the collector:

$$R_o = R_C || R_L = \frac{R_C R_L}{R_C + R_L}$$  \hspace{1cm} (4)
Substituting (4) and (2) into (3), we have

\[ v_o = -g_m v_i R_o = -v_i \cdot g_m (R_C \parallel R_L) \]  

The amplifier gain:

\[ A_v = \frac{v_o}{v_i} = -g_m (R_C \parallel R_L) = -g_m R_o \]  

This is the gain for any common-emitter amplifier

\[ A_v = -g_m R_o \]

The negative sign indicates that the amplifier has inverting gain
C-E Amplifier – Small-Signal Analysis

- For this circuit, the output resistance is
  \[ R_o = R_C || R_L = 820 \, \Omega || 1 \, k\Omega = 451 \, \Omega \]

- The gain is
  \[ A_v = -188 \, mS \cdot 451 \, \Omega = -84.7 \]

- The output for a 50 mV_{pp}, 100 kHz input:
C-E Amplifier – Dynamic Range

- **Dynamic range**
  - Range of input or output signal for which the transistor remains in the *forward-active region*
  - The amplifier’s *linear range*

- For forward-active bias:
  - B-C junction must remain reverse biased
    \[ v_{BC} < 0 \]
  - Total collector voltage must remain above the base voltage
    \[ v_C > v_B \]
  - Collector cannot enter the cutoff region
    \[ I_C > 0, v_C < V_{CC} \]
C-E Amplifier – Dynamic Range

- **Optimal collector bias**
  - DC collector voltage halfway between the base voltage and supply
    \[ V_C = \frac{(V_{CC} + V_B)}{2} \]
  - Output can swing positive and negative equal amounts

- Then, the output dynamic range is
  \[ v_{opp} < (V_{CC} - V_B) \]

- The input is smaller than the output by the gain factor, so
  \[ v_{ipp} < \frac{(V_{CC} - V_B)}{A_v} \]

- Here,
  \[ v_{opp} < 8 \text{ V} \quad \text{and} \quad v_{ipp} < 94.5 \text{ mV} \]
C-E Amplifier – Gain from $v_s(t)$

- If, instead, we want gain from other side of source resistance, $v_s$ to $v_o$, we must account for source loading
  - Cascade of gain from $v_s$ to $v_i$ with gain from $v_i$ to $v_o$
    \[
    A_v = \frac{v_o}{v_s} = \frac{v_i}{v_s} \cdot \frac{v_o}{v_i}
    \]

- Voltage division from $v_s$ to $v_i$
  \[
  \frac{v_i}{v_s} = \frac{R_B || r_\pi}{R_s + R_B || r_\pi} = \frac{R_i}{R_s + R_i}
  \]

- Overall gain is now
  \[
  A_v = -\frac{R_i}{R_s + R_i} g_m R_o
  \]
Input resistance is an important property of any amplifier. For the C-E amplifier,

\[ R_i = R_B || r_\pi \]

\[ R_i = R_B || \frac{\beta}{g_m} \]

\[ R_i = R_B || \frac{\beta V_{th}}{I_C} \]

Dependent on \( \beta \) and \( I_C \)
We used the hybrid-$\pi$ model for small-signal analysis

Could also use the T-model

Result is the same:

\[ v_o = -i_c R_o = -g_m v_{be} R_o \]

\[ v_o = -g_m R_o \cdot v_i \]

\[ A_v = \frac{v_o}{v_i} = -g_m R_o \]
C-E Amplifier – Gain

\[ A_v = -g_m R_o \]

- C-E gain is **determined by** \( g_m \) **and** \( R_o \)
  - Select \( R_o \) (\( R_C \)) for desired gain
  - Transconductance is proportional to bias current
    \[ g_m = \frac{I_C}{V_{th}} \]
    - Therefore, **gain is proportional to bias current**
  - Transconductance is inversely proportional to temperature
    \[ g_m = \frac{I_C q}{kT} \]
    - Therefore, **gain is inversely proportional to temperature**
Emitter Degeneration
The C-E amplifier we have looked at so far had its emitter grounded (small-signal ground)

- Due to bypass capacitor, $C_E$, around $R_E$

What if we remove $C_E$?

- Or add another emitter resistor not bypassed by $C_E$

**Emitter degeneration**
Now, $R_E$ is included in the small signal equivalent circuit
- Emitter is no longer connected to small-signal ground

Analysis will be simplified if we use the T-model
- Usually the case whenever we have emitter resistance
- $R_E$ will be in series with $r_e$ from the model
The output is still given by

\[ v_o = -i_C R_o = -g_m v_{be} R_o \]

But, now, \( v_{be} \) is the portion of \( v_i \) that appears across \( r_e \)

\[ v_{be} = v_i \frac{r_e}{r_e + R_E} \]

\[ v_{be} = v_i \frac{\alpha}{g_m} = v_i \frac{\alpha}{\alpha + g_m R_E} \]

The output is

\[ v_o = v_i \left( -g_m R_o \frac{\alpha}{\alpha + g_m R_E} \right) \]
Emitter Degeneration – Gain

- Rearranging the expression for the output gives the gain
  \[ A_v = -g_m R_o \frac{\alpha}{\alpha + g_m R_E} \]

- Recognizing that \( \alpha \approx 1 \), we can simplify
  \[ A_v \approx -\frac{g_m R_o}{1 + g_m R_E} \]

- **Emitter degeneration reduces the gain by a factor of** \( 1 + g_m R_E \)

- If \( R_E \gg r_e \), then \( g_m R_E \gg 1 \), and
  \[ A_v \approx -\frac{R_o}{R_E} \]
Emitter Degeneration – Transconductance

\[ A_v \approx - \frac{g_m R_o}{1 + g_m R_E} \]

- We can rewrite the gain as
  \[ A_v = -G_m R_o \]

- \( G_m \) is the **effective transconductance of the amplifier**
  \[ G_m = \frac{g_m}{1 + g_m R_E} \]

- **Emitter degeneration reduces the transconductance by a factor of** \((1 + g_m R_E)\)
  - This is why we see a reduction in gain by the same factor
Emitter Degeneration – Input Resistance

- By definition, the input resistance (at \( v_i \)) is given by

\[
R_i = \frac{v_i}{i_b}
\]

- Base current is related to emitter current

\[
i_b = \frac{i_e}{\beta + 1}
\]

- Emitter current is

\[
i_e = \frac{v_i}{(r_e + R_E)}
\]

- Substituting into the previous expressions gives

\[
R_i = \frac{v_i}{i_b} = \frac{v_i}{\frac{v_i}{(\beta + 1)(r_e + R_E)}} = (\beta + 1)(r_e + R_E)
\]
Resistance Reflection Rule

\[ R_i = (\beta + 1)(r_e + R_E) \]

- **Resistance reflection rule:**
  
  The resistance seen looking into the base is \((\beta + 1)\) times the total resistance at the emitter.

- Equally applicable when \(R_E = 0\):
  \[ R_i = (\beta + 1)r_e = r_\pi \]

- **Base input resistance,** \(r_\pi\), **is the reflected emitter resistance**
  \[ r_\pi = (\beta + 1)r_e \]
Emitter Degeneration – Negative Feedback

- **Without** emitter degeneration, any increase in $v_i$ appears as $v_{be}$

- Think through what happens when $v_i$ increases *with* emitter degeneration:
  - $v_{be}$ *does* increase
  - $i_c$ and $i_e$ increase
  - Voltage drop across $R_E$ increases, driving $v_e$ up
  - Increasing $v_e$ reduces the amount of the $v_i$ increase that appears as $v_{be}$

- This is **negative feedback**
  - Increasing output, $i_c$ or $i_e$, subtracted from the input:
    \[ v_{be} = v_i - i_e R_E \]

- Similar to negative feedback in opamp circuits
  - Feedback reduces gain
  - In the limit, gain is set by a resistor ratio
Emitter Degeneration – Example

- Determine the gain of the C-E amplifier with emitter degeneration
  - DC circuit is the same as before, but now only part of $R_E$ is bypassed
- DC operating point unchanged:
  - $I_B = 24.3 \mu A$  $V_B = 4.02 V$
  - $I_C = 4.9 mA$  $V_C = 7.98 V$
  - $I_E = 4. mA$  $V_E = 3.33 V$
- Small-signal model parameters unchanged:
  - $g_m = 188 mS$
  - $r_\pi = 1.06 k\Omega$
  - $r_e = 5.28 \Omega$
Gain is given by

\[ A_v = -G_m R_o \]

where

\[ G_m = \frac{g_m}{(1 + g_m R_E)} = \frac{188 \text{ } mS}{19.8} = 9.5 \text{ } mS \]

so

\[ A_v = -9.5 \text{ } mS \cdot 451 \Omega \]

\[ A_v = -4.3 \]

Note the reduction in gain due to the emitter degeneration

\[ A_v = -\frac{g_m R_o}{(1 + g_m R_E)} = -\frac{84.7}{19.8} = -4.3 \]

Also note that we can roughly approximate the gain as

\[ A_v \approx -\frac{R_o}{R_E} = -\frac{451 \Omega}{100 \Omega} = -4.5 \]
Emitter Follower
Buffering

- In previous classes, you have learned about **loading effects**
  - Signal attenuation between output/input resistances of cascaded stages

- Inter-stage **buffers** can reduce attenuation due to loading
  - High input resistance, low output resistance
  - Unity gain

- We can use BJTs as buffers
  - *Emitter follower*
Emitter-Follower

- **Emitter-follower amplifier**
  - Input applied to the base
  - Output at the emitter
  - Emitter *follows* the base

- Also called a *common-collector* amplifier (CC)
  - Collector is connected to small-signal ground
Emitter-Follower

- Similar to opamp unity-gain buffer
  - Near-unity gain
  - High $R_i$, low $R_o$
  - Buffers source impedance
  - Reduces attenuation due to loading

- We will now analyze the emitter-follower
  - Large-signal analysis is the same as for the CE amplifier
  - Perform a small-signal analysis to determine voltage gain
Emitter Follower – Small-Signal Analysis

- Replace the BJT with small-signal model
  - Emitter resistance, so use T-model
  - Short coupling caps
  - DC voltages connect to ground
  - Simplify parallel resistances
First, determine gain from $v_i$ to $v_o$

$$A_v = \frac{v_o}{v_i}$$

Applying voltage division gives the output

$$v_o = v_i \frac{R_E \| R_L}{(R_E \| R_L + r_e)}$$

Rearrange to get the gain

$$A_v = \frac{R_E \| R_L}{(R_E \| R_L + r_e)}$$

- Clearly, $A_v < 1$
- But, for $R_E \| R_L \gg r_e$, $A_v \approx 1$
Emitter Follower – Input Resistance

- The emitter follower’s input resistance is defined as
  \[ R_i = \frac{v_i}{i_b} \]
  where
  \[ i_b = \frac{i_e}{\beta + 1} = \frac{v_i}{(\beta + 1)(r_e + R_E || R_L)} \]

- The input resistance is
  \[ R_i = (\beta + 1)(r_e + R_E || R_L) \]

- \((\beta + 1)\) times larger than the total resistance at the emitter
  - The reflected emitter resistance
  - Typically a very large input resistance, as we would expect from a circuit used as a buffer

- Note that this is the resistance at the base
  - In parallel with \(R_{BB}\)
Emitter Follower – Output Resistance

- To determine output resistance, set the input to zero
  - First, consider the case where the input is applied directly to the base (i.e., \( R_S = 0 \))
    - Set \( v_i \) to zero – ground the base
- For now, ignore \( R_E \)
  - In parallel with what we will calculate as \( R_o \)
- The output resistance is simply \( r_e \)
  \[
  R_o = r_e
  \]
- Recall the expression for \( r_e \)
  \[
  r_e = \frac{V_{th}}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}
  \]
  - Typically a small resistance, as expected from a circuit used as a buffer
    - Increasing bias current decreases \( r_e \) and \( R_o \)
Emitter Follower – Output Resistance

Next, determine $R_o$ for non-zero source resistance, $R_s \neq 0$

- Set $v_s$ to zero – ground the source

By definition

$$R_o = -\frac{v_o}{i_e}$$

- Emitter current is

$$i_e = (\beta + 1)i_b$$

- KVL around the B-E loop

$$v_o = -i_b(R_s||R_{BB}) - i_e r_e = -\frac{i_e}{\beta + 1}(R_s||R_{BB}) - i_e r_e$$

$$v_o = -i_e \left( \frac{(R_s||R_{BB})}{\beta + 1} + r_e \right)$$

- $R_o$ now includes all resistance at the base, reflected to the emitter:

$$R_o = \frac{(R_s||R_{BB})}{\beta + 1} + r_e = \frac{(R_s||R_{BB})}{\beta + 1} + \frac{r_\pi}{\beta + 1}$$
The input and output resistance of the emitter follower illustrate two versions of the \textit{resistance reflection rule}

**Version 1:**
- \textit{Resistance seen at the base is the total resistance at the emitter increased by a factor of} $(\beta + 1)$

$$R_b = (\beta + 1)[r_e + R_E]$$

**Version 2:**
- \textit{Resistance seen at the emitter is the total resistance at the base reduced by a factor of} $(\beta + 1)$

$$R_e = \frac{(r_\pi + R_B)}{(\beta + 1)}$$
Common-Base Amplifier
The third BJT amplifier configuration we will look at is the **common-base amplifier**

- Input applied to the emitter
- Output taken from the collector
- Base is connected to small-signal ground
- By far the least common of the three amplifiers
Common-Base Amplifier – Gain

- There is emitter resistance, so use the T-model for small-signal analysis.

- The output is given by
  \[ v_o = -i_c R_C || R_L \]
  \[ v_o = -\alpha i_e R_C || R_L \]
  where
  \[ i_e = -\frac{v_i}{r_e} \]
  so
  \[ v_o = v_i \frac{\alpha}{r_e} R_C || R_L = v_i g_m R_C || R_L \]

- Common-base voltage gain is
  \[ A_v = g_m R_C || R_L \]
Common-Base – Input Resistance

- $R_i$ is the parallel combination of the resistance connected to the emitter and the resistance looking into the emitter:

$$R_i = R_E || r_e = R_E || \frac{\alpha}{g_m}$$

- Note that typically, $r_e$ is quite small, so:

$$R_i \approx r_e \approx \frac{1}{g_m}$$
If we neglect the transistor’s output resistance, \( r_o \), the common-base output resistance is

\[
R_o = R_C
\]

Entirely determined by the collector resistor
Common-Base Amplifier

- Low input resistance

\[ R_i = r_e \approx \frac{1}{g_m} \]

- For \( R_s \gg r_e \), there will be significant attenuation from \( v_s \) to \( v_i \)

\[ v_i \ll v_s \]

- The overall gain from \( v_s \) to \( v_o \) may be small

- Typically only useful in certain applications:
  - Low source resistance
    - E.g., amplifiers driven by cables
    - \( R_i \) matched to \( Z_0 \) (e.g. 50 \( \Omega \) or 75 \( \Omega \)) to avoid reflections
  - Current buffers
    - E.g., in *cascode* amplifiers
Transistors as Switches
Transistors as Switches

- Our focus in this course is the use of transistors for designing *linear amplifiers*
  - Output is a scaled version of the input
- Transistors can be used also be used as *nonlinear switches*
  - Either *on* or *off* (closed or open)
  - Fundamental building block of *digital integrated circuits*
    - Microprocessors have *billions* of transistors (MOSFETS) used as switches
  - Useful for switching large amounts of current, e.g.,
    - Controlling a mechanical device (e.g., pump, heater, motor) with a microcontroller
    - Power inverters
Saturation/Cutoff Region Models

- Transistors used as **amplifiers** must stay in the **forward active** region.
- Transistors used as **switches** operate alternately in the **saturation** (closed) and **cutoff** (open) regions.
- Equivalent circuit models:

**Saturation Region (on):**
- Both junctions forward biased.
- Small, nearly constant $V_{CE}$
  
  $V_{CE, sat} \approx 200 \text{ mV}$

**Cutoff Region (off):**
- Both junctions reverse biased.
- Open circuits between all terminals.
Using a BJT as a Switch - Example

- Let’s say we want to turn resistance heater on and off using a microcontroller
  - Heater may require amperes of current
  - Microcontroller output may be limited to tens of mA

- Control a BJT switch with the microcontroller output
  - Low-current control signal from the microcontroller
    - Base resistor limits output current
  - BJT switches the large current required by the heater
Using a BJT as a Switch - Example

- When the $\mu$-controller’s output is low (0 V)
  - $V_{BE} = 0 \text{ V}$
  - Transistor is in the cutoff region
  - Switch is off
  - No current flows
  - The heater is off
Using a BJT as a Switch - Example

- When the \( \mu \)-controller’s output is high (3.3 V)
  - \( V_{BE} \approx 700 \text{ mV} \)
  - \( V_{CE} = V_{CE,sat} \approx 200 \text{ mV} \)
  - Transistor is saturated
  - Switch and heater are on

- Collector/heater current:
  \[
  I_C = \frac{48 \text{ V} - 200 \text{ mV}}{45 \ \Omega} = 1.1 \text{ A}
  \]

- Heater power:
  \[
  P_h = I_C^2 \cdot R_h
  \]
  \[
  P_h = (1.1 \text{ A})^2 \cdot 45 \ \Omega
  \]
  \[
  P_h = 50.8 \text{ W}
  \]
Using a BJT as a Switch - Example

- Microcontroller output current (base current):
  \[ I_B = \frac{3.3 \, V - V_{BE}}{R_B} \]

- Select \( R_B \) to limit base current
  - Let’s say \( I_{B,max} = 20 \, mA \)
  \[ R_B \geq \frac{3.3 \, V - 700 \, mV}{I_{B,max}} = \frac{2.6 \, V}{20 \, mA} \]

  \[ R_B \geq 130 \, \Omega \]

- Typically choose \( R_B \) to keep \( I_B \) well below \( I_{B,max} \)