MOSFET Amplifier Circuits
In this section of the course, we will look at three MOSFET amplifiers, with a focus on the following two circuits:

- **Common-Source Amplifier:**
  - High voltage gain
  - An amplifier

- **Source-Follower Amplifier:**
  - Near unity gain
  - A buffer
MOSFET Amplifier Biasing
MOSFET Amplifier Biasing

- To function as an amplifier, a MOSFET must be biased in the *saturation region*
- DC operating point set by the *bias network*
  - Resistors and power supply voltages
  - Sets the transistor’s *DC terminal voltages and currents* – its DC bias
- How a transistor is *biased* determines:
  - Small-signal characteristics
  - Small-signal model parameters
  - How it will behave as an amplifier
Voltage Transfer Characteristic

- MOSFET amplifier biased in the middle of its saturation region
- Slope of the large-signal transfer characteristic gives the amplifier gain
  - Negative slope – gain is inverting
  - Small input signals yield larger output signals
  - Slope is nearly linear in this region
MOSFET Biasing – Four-Resistor Bias Circuit

- We can use a similar four-resistor bias network for MOSFET amplifiers

- Commonly-used for both *common-source* amplifiers and *source-followers*
  - Single power supply or bipolar supply

- Stable biasing over device parameter variations
  - Insensitive to variations in $V_t$, $k'_n$, $\frac{W}{L}$
Analysis of the Four-Resistor Bias Circuit

- Since $I_G = 0$, gate voltage is simply set by the voltage divider

\[ V_G = V_{DD} \frac{R_{G2}}{R_{G1} + R_{G2}} \]

- Drain current is given by

\[ I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2 = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (V_{GS} - V_t)^2 \]

\[ I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (V_G - V_S - V_t)^2 = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (V_G - I_D R_S - V_t)^2 \]

- After some rearranging, we arrive at a quadratic equation, which we can solve for $I_D$:

\[ R_S^2 I_D^2 - \left[ 2 R_S (V_G - V_t) + \frac{1}{2 k'_n \left( \frac{W}{L} \right)} \right] I_D + (V_G - V_t)^2 = 0 \]
Four-Resistor Bias Circuit – Example

- Determine terminal voltages and drain current for the following circuit
- Gate voltage:
  \[ V_G = 12 \, V \cdot \frac{30 \, k\Omega}{50 \, k\Omega + 30 \, k\Omega} = 4.5 \, V \]

- Drain current:
  \[ I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (V_G - V_S - V_t)^2 \]
  \[ I_D = 1 \frac{mA}{V^2} \left( 4.5 \, V - I_D \cdot 8 \, k\Omega - 700 \, mV \right)^2 \]
  \[ I_D = 1 \frac{mA}{V^2} \left( -8 \, k\Omega \cdot I_D + 3.8 \, V \right)^2 \]

\[
\frac{1}{V^2} \left( 64e6 \cdot I_D^2 - 60.8e3 \cdot I_D + 14.44 \right) - I_D = 0
\]
\[
64e6 \cdot I_D^2 - 61.8e3 \cdot I_D + 14.44 = 0
\]
Four-Resistor Bias Circuit – Example

\[64e6 \cdot I_D^2 - 61.8e3 \cdot I_D + 14.44 = 0\]

- Solving the quadratic equation for \( I_D \) gives
  \[I_D = 569 \ \mu A \text{ or } I_D = 396 \ \mu A\]

- For \( I_D = 569 \ \mu A \)
  \[V_S = I_D R_S = 569 \ \mu A \cdot 8 \ k\Omega = 4.55 \ V\]
  \[V_{GS} = -50 \ mV < V_t\]
  - The transistor would be cut-off, so this is not a valid solution

- DC operating point:
  \[I_D = 396 \ \mu A\]
  \[V_S = 396 \ \mu A \cdot 8 \ k\Omega = 3.17 \ V\]
  \[V_{GS} = 1.33 \ V\]
  \[V_{OV} = 630 \ mV\]
  \[V_D = V_{DD} - I_D R_D = 8.04 \ V\]
To design a bias network to provide a desired drain current:

- Select $R_D$ and $R_S$ to each drop approximately one third of the supply voltage
  - That will leave approximately one third of the supply voltage across $V_{DS}$
- Calculate the required $V_{OV}$, $V_{GS}$, and $V_G$
- Select the voltage divider resistors at the gate to provide the required gate voltage
Bias Circuit Design - Example

- Design the bias network to provide $I_D = 800 \, \mu A$
- Calculate $R_D$ and $R_S$ to each drop $V_{DD}/3$

$$R_D = R_S = \frac{V_{DD}/3}{I_D} = \frac{5 \, V}{800 \, \mu A} = 6.25 \, k\Omega$$

- The required overdrive voltage is

$$V_{OV} = \sqrt{\frac{2I_D}{k'_n \left( \frac{W}{L} \right)}} = \sqrt{\frac{1.6 \, mA}{1 \, \frac{mA}{V^2}}} = 1.26 \, V$$

- The gate-source voltage

$$V_{GS} = V_{OV} + V_t = 1.26 \, V + 800 \, mV$$

$$V_{GS} = 2.06 \, V$$
Determine the required gate voltage

\[ V_G = V_S + V_{GS} = I_D R_S + V_{GS} \]

\[ V_G = 800 \mu A \cdot 6.25 \, k\Omega + 2.06 \, V \]

\[ V_G = 7.06 \, V \]

Finally, select \( R_{G1} \) and \( R_{G2} \) to provide the required \( V_G \)

\[ R_{G1} = 100 \, k\Omega \]

\[ R_{G2} = 89 \, k\Omega \]
Common-Source Amplifier
Common-Source Amplifier

- Common-source amplifier
- All capacitors are **AC-coupling/DC blocking capacitors**
  - Open at DC
  - Shorts at signal frequencies
  - Isolate transistor bias from source/load
- Called *common*-source, because source is connected to common – i.e., ground or a power supply
  - $C_S$ is a small-signal short to ground
  - Source is at small-signal ground

\[
V_t = 1.6 \, V \quad k'_n \left(\frac{W}{L}\right) = 170 \left(\frac{mA}{V^2}\right)
\]
Common-Source Amplifier

- Analyze the amplifier to find:
  - DC operating point
  - Small-signal voltage gain

- DC operating point:
  - The gate voltage is given by

\[ V_G = V_{DD} \frac{R_{G2}}{R_{G1} + R_{G2}} \]

\[ V_G = 12 \text{ V} \frac{115 \text{ k}\Omega}{100 \text{ k}\Omega + 115 \text{ k}\Omega} \]

\[ V_G = 6.4 \text{ V} \]
C-S Amplifier – Large-Signal Analysis

- Drain current is given by
  \[ I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2 = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (V_G - I_D R_S - V_t)^2 \]

- As we have seen, solving for \( I_D \) results in the following quadratic
  \[
  R_S^2 I_D^2 - \left[ 2R_S(V_G - V_t) + \frac{1}{2} k'_n \left( \frac{W}{L} \right) \right] I_D + (V_G - V_t)^2 = 0
  \]

  \[ 6.4e3 \cdot I_D^2 - 779.8 \cdot I_D + 23.0 = 0 \]

- This has two solutions
  \[ I_D = 72 \ mA \quad \text{or} \quad I_D = 51 \ mA \]

- The first solution would put the transistor in cutoff, so \( I_D = 51 \ mA \)
Use the drain current to determine terminal voltages

\[ V_D = V_{DD} - I_D R_D \]

\[ V_D = 12 V - 51 mA \cdot 80 \Omega = 7.95 V \]

\[ V_S = I_D R_S = 51 mA \cdot 80 \Omega \]

\[ V_S = 4.05 V \]

The complete DC operating point:

\[ V_G = 6.42 V \quad I_D = 51 mA \]

\[ V_{GS} = 2.37 V \quad V_D = 7.95 V \]

\[ V_{OV} = 0.77 V \quad V_S = 4.05 V \]
C-S Amplifier – Small-Signal Analysis

- The DC operating point allows us to determine the transconductance for the transistor’s small-signal model

\[ g_m = k'_n \left( \frac{W}{L} \right) V_{OV} = 170 \frac{mA}{V^2} \cdot 0.77 V = 131 \, mS \]

- Next, create the **small-signal equivalent circuit** for the amplifier and perform a **small-signal analysis**:

  1. Replace all AC coupling capacitors with shorts
     - Large enough to look like shorts at signal frequencies
  2. Connect all DC supply voltages to ground
     - From a small-signal perspective these are all constant voltages
     - Small-signal ground
  3. Replace the transistor with its small-signal model
C-S Amplifier – Small-Signal Analysis

- Small-signal equivalent circuit
  - Use to determine small-signal voltage gain

- Source is connected to small signal ground through $C_S$
- $R_{G1}$ and $R_{G2}$ appear in parallel at the gate

$$R_i = R_{G1} || R_{G2} = 53.5 \text{ k}\Omega$$

- $R_D$ and $R_L$ are in parallel at the output

$$R_o = R_D || R_L = 74 \text{ } \Omega$$

- Input voltage, $v_i(t)$, is the gate-source voltage, $v_{gs}$
C-S Amplifier – Small-Signal Analysis

- **Determine the small-signal voltage gain:**
  \[ A_v = \frac{v_o}{v_i} \]  
  \[ (1) \]

- **The input is applied across the G-S junction, so**
  \[ v_i = v_{gs} \]  
  \[ (2) \]

- **The output is the drain current applied across the output resistance**
  \[ v_o = -i_d R_o = -g_m v_{gs} R_o \]  
  \[ (3) \]
Substituting (3) and (2) into (1) gives the gain:

$$A_v = \frac{v_o}{v_i} = -\frac{g_m v_{gs} R_o}{v_{gs}} = -g_m R_o$$

This is the gain for any common-source amplifier

$$A_v = -g_m R_o$$

The negative sign indicates that the amplifier has inverting gain.
For this circuit, the gain (from $v_i$ to $v_o$) is

$$A_v = \frac{v_o}{v_i} = -131 \text{ mS} \cdot 74 \text{ }\Omega = -9.7$$

For the gain from $v_s$ to $v_o$, account for attenuation due to source loading

$$A_v = \frac{v_o}{v_s} = \frac{v_i}{v_s} \cdot \frac{v_o}{v_i} = \frac{R_i}{R_s + R_i} \cdot (-g_mR_o)$$

Here,

$$A_v = \frac{v_o}{v_s} = \frac{53.5 \text{ k}\Omega}{500 \text{ }\Omega + 53.5 \text{ k}\Omega} \cdot (-9.7) = -9.6$$
The output for a 200 mV_{pp}, 100 kHz input:
C-S Amplifier – Dynamic Range

- **Dynamic range**
  - Range of input or output signal for which the transistor remains in the **saturation region**
  - The amplifier’s **linear range**

- For saturation bias:
  - D-S voltage must remain greater than the overdrive voltage
    \[
    v_{DS} > V_{OV}
    \]
  - G-S voltage must remain greater than the threshold voltage
    \[
    v_{GS} > V_t
    \]
Gate resistance is infinite, so amplifier input resistance is

\[ R_i = R_{G1} || R_{G2} \]

Output resistance is the drain resistance:

\[ R_o = R_D \]

Or, if accounting for channel-length modulation:

\[ R_o = R_D || r_o \]
C-S Amplifier – Gain

\[ A_v = -g_m R_o \]

- C-S gain is **determined by** \( g_m \) **and** \( R_o \)
  - Select \( R_o \) (\( R_D \)) and set \( g_m \) for desired gain
  - Transconductance is proportional to the square root of bias current

\[ g_m = \sqrt{k'_n \left( \frac{W}{L} \right) I_D} \]

- Therefore, **gain is proportional to the square root of bias current**
Source Degeneration
The C-S amplifier we have looked at so far had its source grounded (small-signal ground)

- Due to bypass capacitor, $C_S$, around $R_S$

What if we remove $C_S$?
- Or add another source resistor not bypassed by $C_S$

*Source degeneration*
Now, $R_S$ is included in the small signal equivalent circuit
- Source is no longer connected to small-signal ground

Analysis will be simplified if we use the T-model
- Usually the case whenever we have source resistance
- $R_S$ will be in series with resistance in the model
The output is still given by

$$v_o = -i_d R_o = -g_m v_{gs} R_o$$

But, now, $v_{gs}$ is the portion of $v_i$ that appears across the $1/g_m$ resistance

$$v_{gs} = v_i \frac{1/g_m}{1/g_m + R_s}$$

$$v_{gs} = v_i \frac{1}{1 + g_m R_s}$$

The output is

$$v_o = v_i \left(-g_m R_o \frac{1}{1 + g_m R_s}\right)$$
Source Degeneration – Gain

- Rearranging the expression for the output gives the gain

\[ A_v = -\frac{g_m R_o}{1 + g_m R_S} \]

- **Source degeneration reduces the gain by a factor of** \((1 + g_m R_S)\)

- If \(R_S \gg 1/g_m\), then \(g_m R_S \gg 1\), and

\[ A_v = -\frac{R_o}{R_S} \]
Source Degeneration – Transconductance

\[ A_v = -\frac{g_m R_o}{1 + g_m R_S} \]

- We can rewrite the gain as
  \[ A_v = -G_m R_o \]

- \( G_m \) is the **effective transconductance of the amplifier**
  \[ G_m = \frac{g_m}{1 + g_m R_S} \]

- **Source degeneration reduces the transconductance by a factor of** \( (1 + g_m R_S) \)
  - This is why we see a reduction in gain by the same factor
Source Follower
Source-Follower

- **Source-follower amplifier**
  - Input applied to the gate
  - Output at the source
  - Source *follows* the gate
- Also called a **common-drain** amplifier (CD)
  - Drain is connected to small-signal ground
Replace the MOSFET with small-signal model

- Source resistance, so use T-model
- Short coupling caps
- DC voltages connect to ground
- Simplify parallel resistances
Determine the gain from $v_i$ to $v_o$

$$A_v = \frac{v_o}{v_i}$$

Applying voltage division gives the output

$$v_o = v_i \frac{R_S||R_L}{(R_S||R_L + \frac{1}{g_m})}$$

Rearrange to get the gain

$$A_v = \frac{R_S||R_L}{(R_S||R_L + \frac{1}{g_m})}$$

- Clearly, $A_v < 1$
- But, for $R_S||R_L \gg 1/g_m$, $A_v \approx 1$
Gate resistance is infinite, so amplifier input resistance is

\[ R_i = R_{G1} \parallel R_{G2} \]

The output resistance is the source resistance in parallel with \( 1/g_m \):

\[ R_o = R_S \parallel \frac{1}{g_m} \]
Common-Gate Amplifier
The third MOSFET amplifier configuration we will look at is the *common-gate amplifier*:

- Input applied to the source
- Output taken from the drain
- Gate is connected to small-signal ground
- The least common of the three amplifiers
Common-Gate Amplifier – Gain

- There is source resistance, so use the T-model for small-signal analysis.

- The output is given by
  \[ v_o = -i_d R_D || R_L \]
  \[ v_o = -g_m v_{gs} R_D || R_L \]
  \[ v_o = g_m v_i R_D || R_L \]

- Common-gate voltage gain is
  \[ A_v = g_m R_D || R_L \]
- $R_i$ is the parallel combination of the resistance connected to the source and the resistance looking into the source

$$R_i = R_S \| \frac{1}{g_m}$$

- If $1/g_m \ll R_S$, then

$$R_i \approx \frac{1}{g_m}$$
Common-Gate – Output Resistance

- If we neglect the transistor’s output resistance, $r_O$, the common-gate output resistance is
  
  $$R_O = R_D$$

- Entirely determined by the drain resistor
Common-Gate Amplifier

- Low input resistance
  \[ R_i \approx \frac{1}{g_m} \]
  - For \( R_{os} \gg \frac{1}{g_m} \), there will be significant attenuation from \( v_s \) to \( v_i \)
    \[ v_i \ll v_s \]
  - The overall gain from \( v_s \) to \( v_o \) may be small

- Like the common-base amplifier, useful in specific applications:
  - Low source resistance
    - E.g., amplifiers driven by cables
    - \( R_i \) matched to \( Z_0 \) (e.g. 50 Ω or 75 Ω) to avoid reflections
  - Current buffers
    - E.g., in \textit{cascode} amplifiers
MOSFETs as Switches
MOSFETs as Switches

- MOSFETs used as **switches** operate alternately in the **triode** (closed) and **cutoff** (open) regions
- Equivalent circuit models:

**Triode Region (ON):**
- \( V_{GS} > V_t \)
  - \( V_{GS} = V_{DD} \)
- Switch is on
- \( I_D \geq 0 \)
- \( V_{DS} = I_D r_{DS} < V_{OV} \)

**Cutoff Region (OFF):**
- \( V_{GS} < V_t \)
  - \( V_{GS} = 0 \)
- Switch is off
- \( I_D = 0 \)
- \( V_{DS} = V_{DD} \)
Using a MOSFET as a Switch - Example

- Turn resistance heater on and off using a microcontroller
- Heater may require amperes of current
- Microcontroller output may be limited to tens of mA

- Control a MOSFET switch with the microcontroller output
  - Low-current control signal from the microcontroller
    - Gate draws no DC current
  - MOSFET switches the large current required by the heater

\[
k_n' \frac{W}{L} = 1.2 \frac{A}{V^2}
\]

\[
V_t = 2 \, V
\]
Using a MOSFET as a Switch - Example

- When the $\mu$-controller’s output is low (0 V)
  - $V_{GS} = 0 \text{ V}$
  - Transistor is in the cutoff region
  - Switch is off
  - No current flows
  - The heater is off
Using a MOSFET as a Switch - Example

- When the $\mu$-controller’s output is high (3.3 V)
  - $V_{GS} = 3.3 \, V$, $V_{OV} = 1.3 \, V$
  - Transistor is in triode
  - Switch and heater are on

- Drain current in triode is:
  $$I_D = k'_n \frac{W}{L} \left[ V_{OV} - \frac{1}{2} V_{DS} \right] V_{DS}$$

  $$I_D = k'_n \frac{W}{L} \left[ V_{OV} - \frac{1}{2} (48 \, V - I_D R_h) \right] (48 \, V - I_D R_h)$$

- Can solve the above quadratic, or, assuming $V_{DS}$ is small, approximate switch on-resistance as:
  $$r_{DS} \approx \frac{1}{k'_n \frac{W}{L} V_{OV}} = \frac{1}{1.2 \frac{A}{V^2} \cdot 1.3 \, V} = 641 \, m\Omega$$
Using a MOSFET as a Switch - Example

- Voltage division gives approximate drain voltage

\[ V_D = 48 \, V \cdot \frac{r_{DS}}{R_h + r_{DS}} = 48 \, V \cdot \frac{641 \, m\Omega}{45 \, \Omega + 641 \, m\Omega} \]

\[ V_D = 674 \, mV \]

- Drain current is approximately

\[ I_D = \frac{48 \, V}{R_h + r_{DS}} = \frac{48 \, V}{45 \, \Omega + 641 \, m\Omega} \]

\[ I_D = 1.05 \, A \]

- Heater power is

\[ P_h = I_D^2 R_h = (1.05 \, A)^2 \cdot 45 \, \Omega \]

\[ P_h = 49.75 \, W \]