

SECTION 5: STRUCTURED PROGRAMMING IN PYTHON

ENGR 103 – Introduction to Engineering Computing

Conditional Statements

- `if` statements
- Logical and relational operators
- `if...else` statements

The `if` Statement

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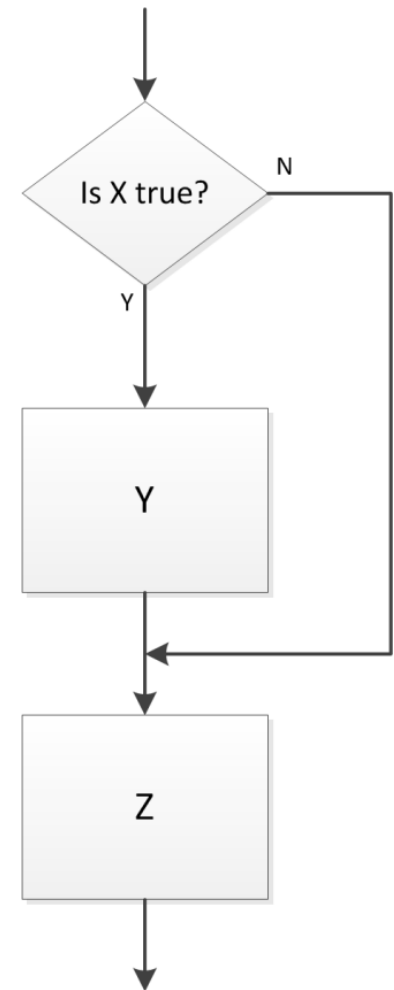
- We've already seen the ***if structure***
 - ▣ If X is true, do Y, if not, don't do Y
 - ▣ In either case, then proceed to do Z

- In Python:

```
if condition:  
    statements  
    ⋮
```

- ***Statements*** are executed ***if condition*** is ***True***
 - ▣ Statement block defined by ***indenting*** those lines of code
- ***Condition*** is a ***logical expression***
 - ▣ Boolean - either True or False
 - ▣ Makes use of ***logical and relational operators***
- May use a ***single line*** for a single statement:

```
if condition: statement
```



Logical and Relational Operators

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Operator	Relationship or Logical Operation	Example
==	Equal to	$x == b$
!=	Not equal to	$k != 0$
<	Less than	$t < 12$
>	Greater than	$a > -5$
<=	Less than or equal to	$7 <= f$
>=	Greater than or equal to	$(4+r/6) >= 2$
and	AND – both expressions must evaluate to true for result to be true	$(t > 0) \text{ and } (c == 5)$
or	OR – either expression must evaluate to true for result to be true	$(p > 1) \text{ or } (m > 3)$
not	NOT– negates the logical value of an expression	$\text{not } (b < 4*g)$

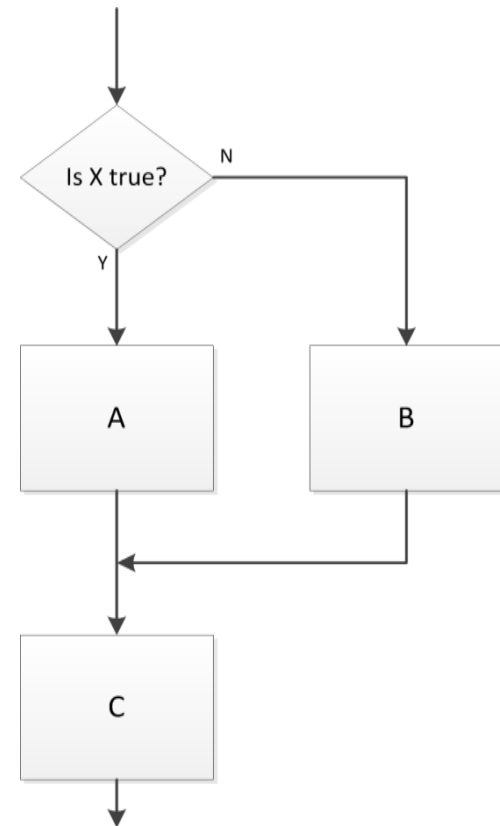
The `if...else` Structure

5

- The ***if ... else structure***
 - ▣ Perform one process if a condition is true
 - ▣ Perform another if it is false
- In Python:

```
if condition:  
    statements1  
else:  
    statements2
```

- Note that `if` and `else` code blocks are defined by ***indents***

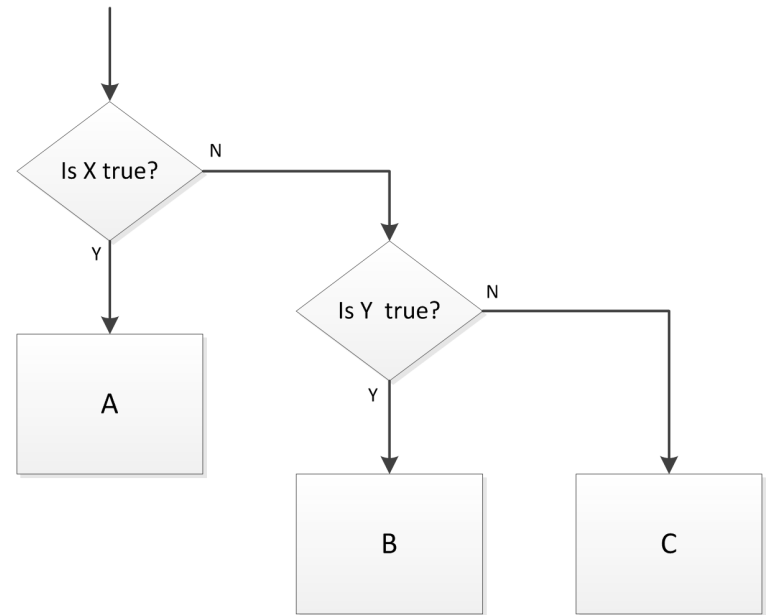


The `if...elif...else` Structure

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- The ***if ... elif ... else structure***
 - ▣ If a condition evaluates as false, check another condition
 - ▣ May have an arbitrary number of ***elif*** statements
- In Python:

```
if condition1:  
    statements1  
elif condition2:  
    statements2  
else:  
    statements3
```



The `if...else`, `if...elif...else` Structures

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□ Some examples:

```
9
10  if (t >= 0) and (p > 8):
11      x = p**2 * t
12      y = 3*q + p
13  else:
14      x = 0
15      y = q + p**2
16
```

```
17
18  if x == 0:
19      f = 2*np.pi
20  elif x <= -1:
21      f = np.pi/4
22  elif (y != 436) or (x > 18):
23      f = 0
24  else:
25      f = 2*np.pi/3
26
```

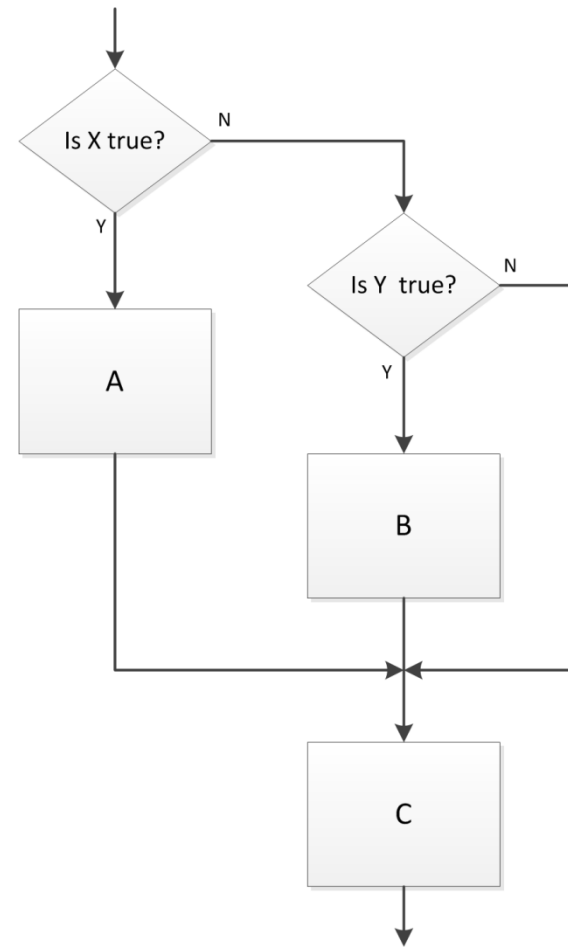
- Note that code blocks are defined by indents
 - Each line must have the same indent - use the Tab key
 - Meaningful whitespace is a distinguishing characteristic of Python
 - Other languages use brackets or *end* statements

The `if...elif` Structure

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- We can have an `if` statement without an `else`
- Similarly, an `if...elif` structure need not have an `else`
- In Python:

```
if condition1:  
    statements1:  
elif condition2:  
    statements2
```



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while Loops

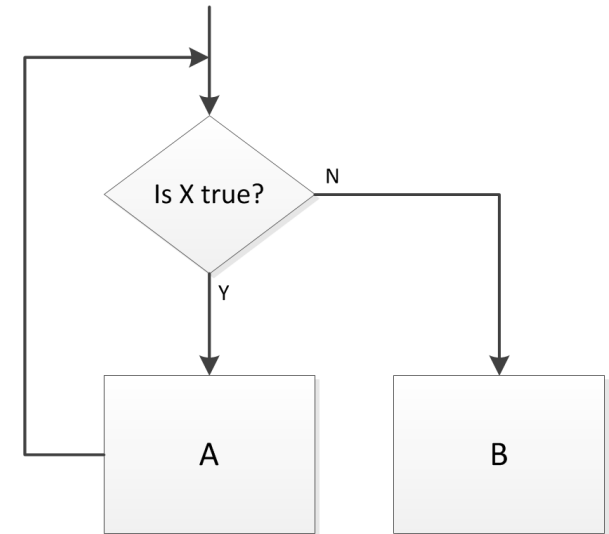
The while loop

10

- The ***while loop***
 - ▣ *While X is true, do A*
 - ▣ Once X becomes false, proceed to B
- In Python:

```
while condition:  
    statements  
    ⋮
```

- *Statements* are executed as long as *condition* remains true
 - ▣ *Condition* is a ***logical expression***
- ***Whitespace*** (indent) defines while block



while Loop – Example 1

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- Consider the following while loop example
 - ▣ Repeatedly increment x by 7 as long as x is less than or equal to 30
 - ▣ Value of x is displayed on each iteration

```
7 # increment a number by 7 until it exceeds 30
8
9 x = 12
10
11 while x <= 30:
12     x = x + 7
13     print(x)
14
```

- x values displayed: 19, 26, 33
- x gets incremented beyond 30
 - ▣ All loop code is executed as long as the condition was true at the ***start of the loop***

The break Statement

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- Let's say we don't want x to increment beyond 30
 - ▣ Add a conditional break statement to the loop

```
18     # increment a number by 7 and exit the loop before it exceeds 30
19     x = 12
20
21     while x <= 30:
22         if (x+7)>30:
23             break
24         x = x + 7
25         print(x)
```

- `break` statement causes loop exit before executing all code
- Now, if $(x+7) > 30$, the program will break out of the loop and continue with the next line of code
- x values displayed: 19, 26
- For nested loops, a `break` statement breaks out of the current loop level only

while Loop – Example 1

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- The previous example could be simplified by modifying the while condition, and not using a break at all

```
30     # or, change the while condition so that x will not increment beyond 30
31
32     x = 12;
33
34     while (x+7) <= 30:
35         x = x + 7
36         print(x)
37
```

- Now the result is the same as with the break statement
 - ▣ x values displayed: 19, 26
- This is not always the case
 - ▣ The break statement can be very useful
 - ▣ May want to break based on a condition other than the loop condition
- break works with both while and for loops

while Loop – Example 2

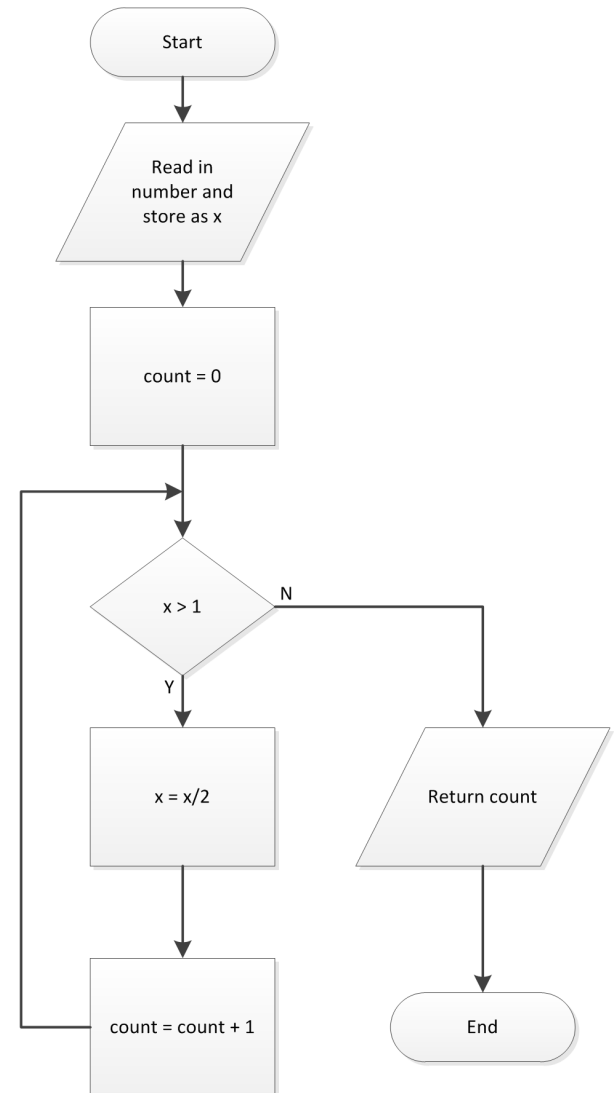
14

- Next, let's revisit the while loop examples from Section 4
- Use `input()` to prompt for input
- Use `print()` to return the result

```
39 # determine how many times a number
40 # must be halved to get a result <= 1
41
42 x = input('Enter a number: ');
43 x = float(x)
44
45 count = 0;
46
47 while x > 1:
48     x = x/2
49     count = count + 1
50
51 print('count = {:d}'.format(count))
```

```
Enter a number: 130
count = 8
```

```
In [42]:
```



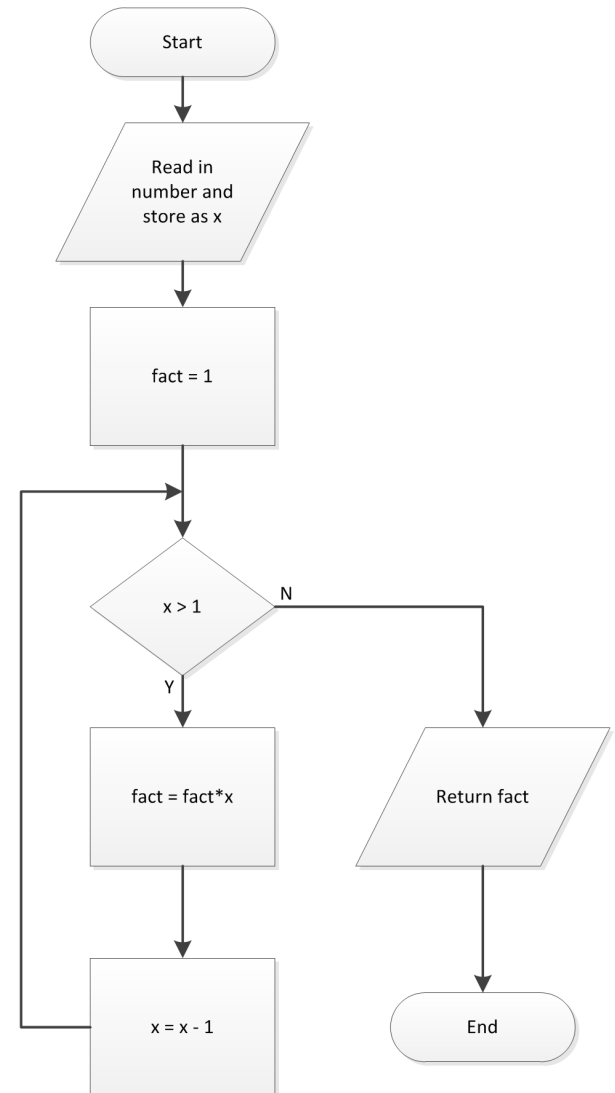
while Loop – Example 3

15

- Here, we use a `while` loop to calculate the factorial value of a specified number

```
54 # calculate factorial(x)
55
56 x = input('Enter an integer: ')
57 x = xin = float(x)
58
59 fact = 1
60
61 while x > 1:
62     fact = fact*x
63     x = x - 1
64
65
66 print('\nfact({}) = {}'.format(xin, fact))
```

```
Enter an integer: 12
fact(12.0) = 479001600.0
In [52]:
```



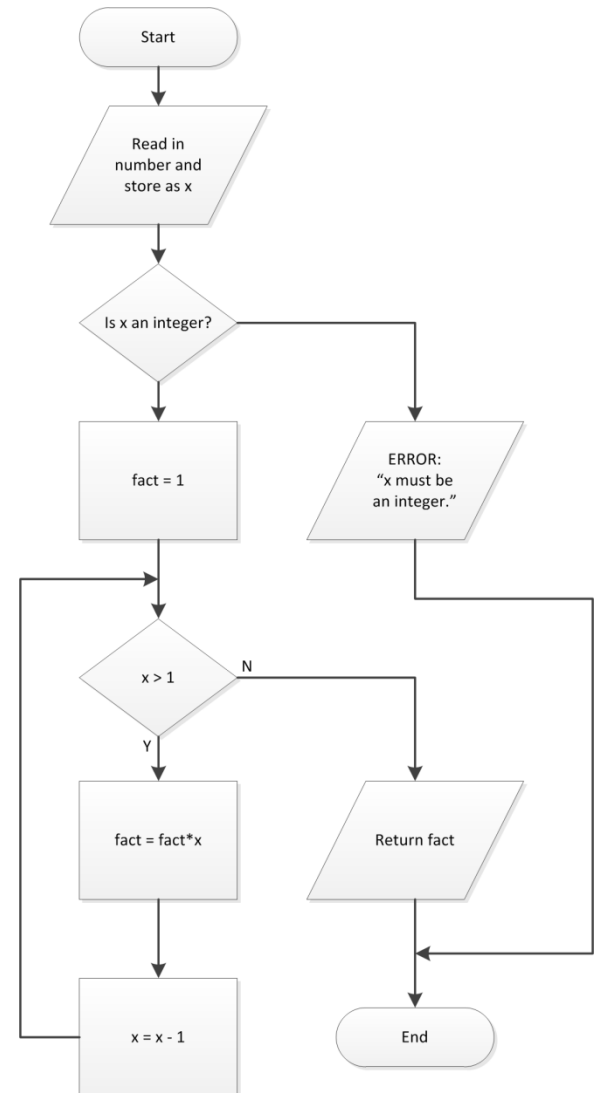
while Loop – Example 3

16

- Add error checking to ensure that x is an integer
- One way to check if x is an integer:

```
69 # calculate factorial(x)
70 # with error checking for integer input
71
72 x = input('Enter an integer: ')
73 x = float(x)
74
75 # check if x is an integer
76 if x != int(x):
77     raise Exception('ERROR: x must be an integer.')
78
79 fact = 1
80
81 while x > 1:
82     fact = fact*x
83     x = x - 1
84
85 print('\nfact({:d}) = {:d}'.format(xin, fact))
```

```
Enter an integer: 11.5
Traceback (most recent call last):
  File "C:\Users\webbky\Box\KWebb\Classes\ENGR102_103\Notes\Python\
    raise Exception('ERROR: x must be an integer.')
Exception: ERROR: x must be an integer.
```



while Loop – Example 3

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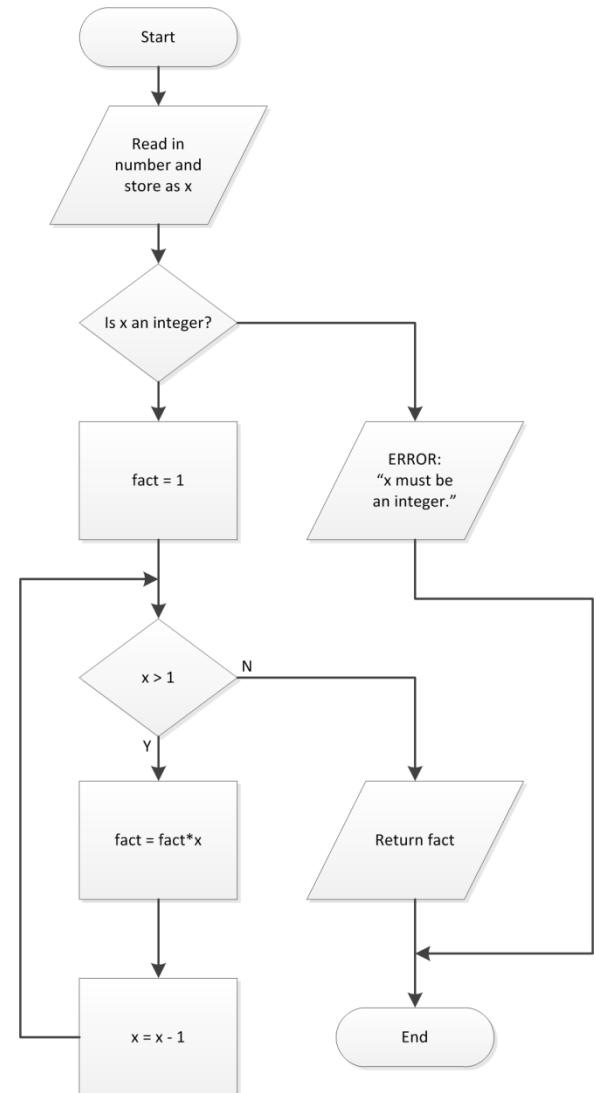
- Another possible method for checking if x is an integer:

```
88 # calculate factorial(x)
89 # alternative way to check for an integer input
90
91 x = input('Enter an integer: ')
92 x = float(x)
93
94 # check if x is an integer
95 if (x - np.floor(x)) != 0:
96     raise Exception('ERROR: x must be an integer.')
97
98 fact = 1
99
100 while x > 1:
101     fact = fact*x
102     x = x - 1
103
104 print('\nfact({:d}) = {:d}'.format(xin, fact))
```

```
Enter an integer: 20.3
Traceback (most recent call last):
```

```
File "C:\Users\webbky\Box\KWebb\Classes\ENGR102_103\Notes\Python\
raise Exception('ERROR: x must be an integer.')
```

```
Exception: ERROR: x must be an integer.
```



Infinite Loops

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- A loop that never terminates is an ***infinite loop***
- Often, this unintentional
 - ▣ Coding error
- Other times infinite loops are intentional
 - ▣ E.g., microcontroller in a control system
- A while loop will never terminate if the while condition is always true
 - ▣ By definition, True is always true:

```
while True:  
    statements repeat infinitely
```

while True

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- The `while True` syntax can be used in conjunction with a `break` statement, e.g.:
- Useful for multiple break conditions
- Control over break point
- Could also modify the while condition

```
43 while True:
44     iter = iter + 1      # increment iteration index
45
46     xrold = xr          # store previous estimate for error approx
47
48     # Choose upper or lower sub-interval as next bracketing interval
49     if (func(xl)*func(xr)) >= 0:      # root is in upper sub-interval
50         xl = xr
51
52     if (func(xu)*func(xr)) >= 0:      # root is in lower sub-interval
53         xu = xr
54
55     if xl == xu:          # func(xr) == 0, exactly (unlikely)
56         epsa = 0
57     else:
58         # update the root estimate
59         xr = xu - func(xu)*(xu - xl)/(func(xu) - func(xl))
60         # approximate the error
61         epsa = abs((xr-xrold)/xr)*100
62
63     # check if stopping criterion is satisfied or if maximum number of
64     # iterations has been reached
65     if (epsa<=reltol):
66         break
67     elif (iter >= maxiter):
68         print('\nMaximum # of iterations reached - exiting.\n\n')
69         break
70
71     fxr = func(xr);
72
73     return [xr, fxr, epsa, iter]
```

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for Loops

The for Loop

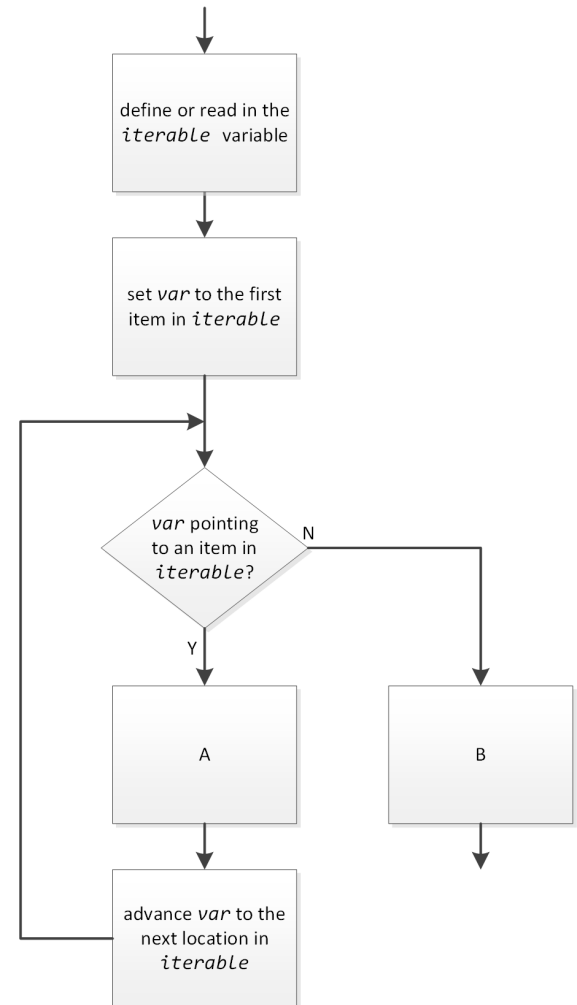
21

□ The *for loop*

- Loop executed a specified number of times

```
for var in iterable:  
    statements  
    :
```

- *iterable*: any ***iterable*** object (ndarray, list, tuple, dict, str)
 - *var*: variable that assumes each successive value in *iterable* on each iteration
 - *Statements*: code block that is executed once for each item in *iterable*
- ***Collection-based***, not counter-based
 - Iterates through each item in a collection
 - Can be counter-based, like flowchart to the right



for Loop – Example 1

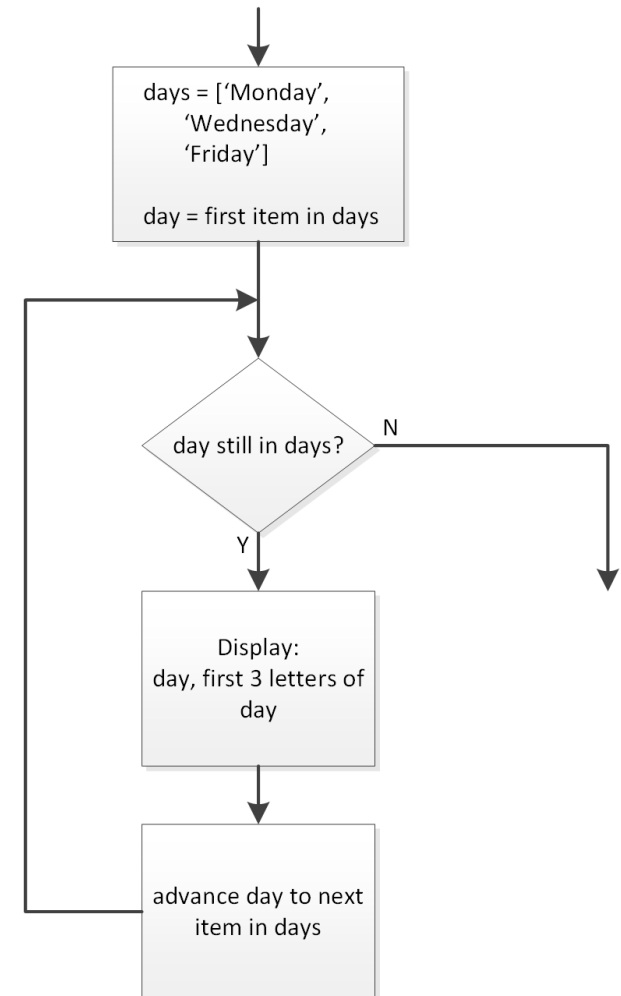
22

- A **collection-based** (or **iterator-based**) for loop
 - ▣ Iterates through each value in a list of days
 - ▣ No explicit loop counter

```
7 days = ['Monday',  
8         'Wednesday',  
9         'Friday']  
10  
11 print('\n')  
12  
13 for day in days:  
14     print(day, ', ', day[0:3])  
15
```

```
Monday , Mon  
Wednesday , Wed  
Friday , Fri
```

```
In [70]:
```



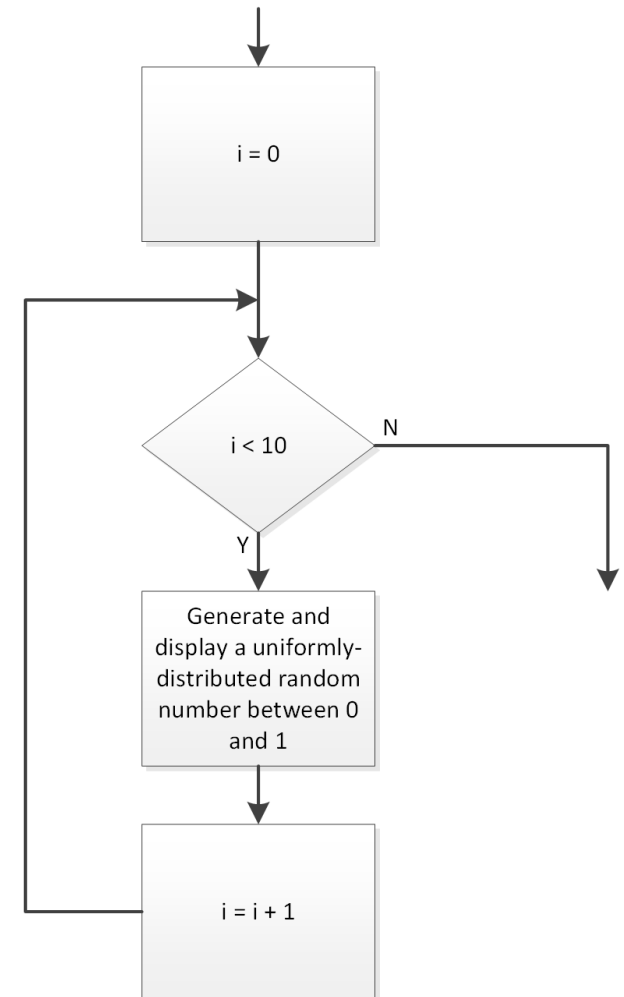
for Loop – Example 2 – range()

23

- **Counter-based** for loop
 - Use Python's range() function:
range(start, stop, step)
 - Generate a list of loop counter values to iterate through
 - Technically, still collection-based

```
19 rng = np.random.default_rng()
20
21 print('\n')
22
23 for i in range(10):
24     x = rng.uniform(low=0, high=1)
25     print('x = {:.4f}'.format(x))
26
```

```
x = 0.0735
x = 0.2565
x = 0.0224
x = 0.5613
x = 0.1624
x = 0.2274
x = 0.9905
x = 0.6892
x = 0.7598
x = 0.7589
```



for Loop – Example 3 – enumerate()

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- Sometimes we may want a combination of a collection-based and counter-based for loop
 - ▣ Iterate over both the *values* and *indices* of all items in an iterable
 - ▣ Use Python's **enumerate()** function
 - Generates an (index, value) pair for each item in the iterable
- For example, consider a list of numbers:
$$x = [2, 4, 6, 8, 10]$$
- Generate (index, value) pairs for each item in x:
$$i, val = enumerate(x)$$
- Generates the following (i, val) pairs:
$$(0, 2), (1, 4), (2, 6), (3, 8)$$
- Can iterate over these (index, value) pairs with a for loop

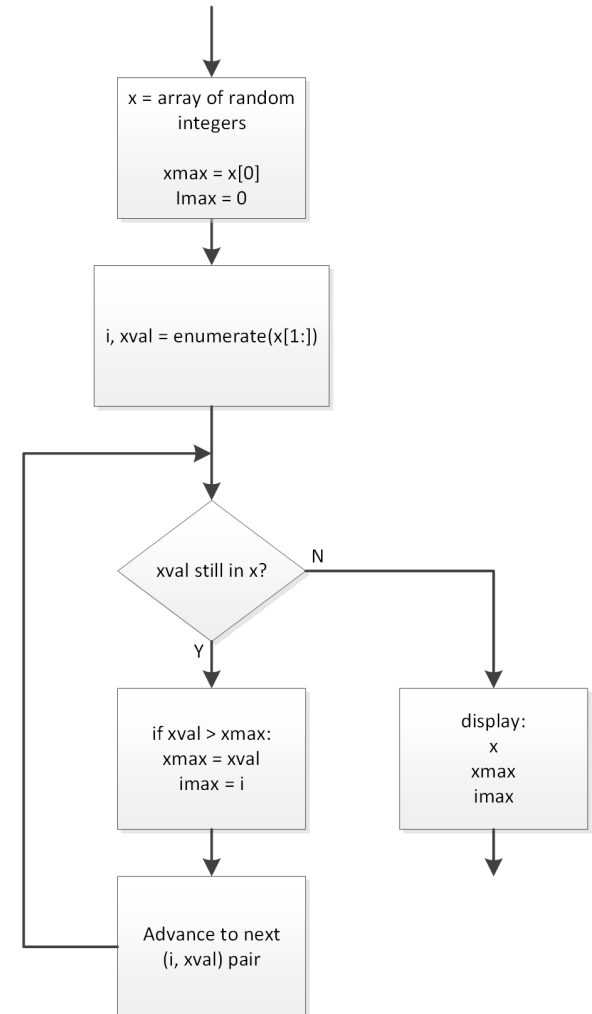
for Loop – Example 3 – enumerate()

25

- Loop through an array of numbers to find the maximum value and its index
 - Use `enumerate()` to simultaneously loop through array values and their indices

```
29
30 x = rng.integers(0, 100, 10)
31 xmax = x[0]
32 imax = 0
33
34 for i, xval in enumerate(x[1:]):
35     if xval > xmax:
36         xmax = xval
37         imax = i
38
39 print('\nx = ', x)
40 print('\nxmax: x[{:d}] = {:d}'.format(i, xmax))
41
```

```
x = [69 91 50 65 92 79 84 61 33 30]
xmax: x[8] = 92
In [131]:
```



Exercise – for Loop, enumerate()

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Exercise

- The step response of a first-order system is given by

$$y(t) = 1 - e^{-\frac{t}{\tau}}$$

- Write a script to do the following:

- ▣ Generate an array of τ values:

$$\tau = [1.0 \quad 1.5 \quad 2.0 \quad 2.5 \quad 3.0] \text{ sec}$$

- ▣ Generate a time vector with 2000 values between 0 and $5 * \max(\tau)$
- ▣ In a **for loop**, using the **enumerate** function, iterate through the values in τ and:
 - Calculate $y(t)$
 - Store the result as one column of a matrix, y
- ▣ Outside of the for loop, plot each of the columns of y on a single set of axes

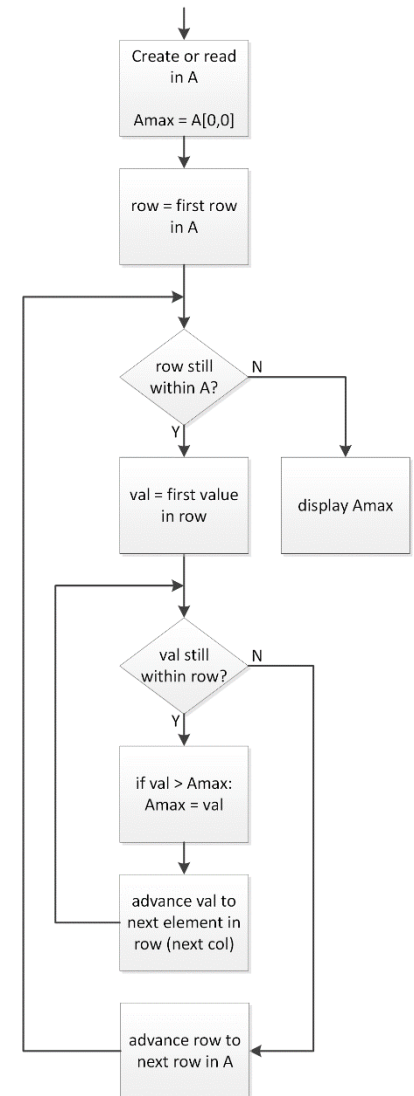
27

Nested Loops

Nested Loop – Example 1

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- Use a nested for loop to find the maximum value in a matrix or 2-D array
 - ▣ **Outer loop** steps through rows
 - ▣ **Inner loop** steps through columns
 - ▣ Store the largest value seen as the maximum value
- Consider an ($m \times n$) matrix, A
 - ▣ $A[0]$ indexes the first row, so
 - for row in A:
 - ▣ Steps through the rows in A one-by-one
 - row = $A[0]$, row = $A[1]$, up to row = $A[-1]$
 - ▣ An inner loop steps through each element in each row
 - for row in A:
 - for val in row:
 - <code to check for max>
 - val = row[0], val = row[1], and so on



Nested Loop – Example 1

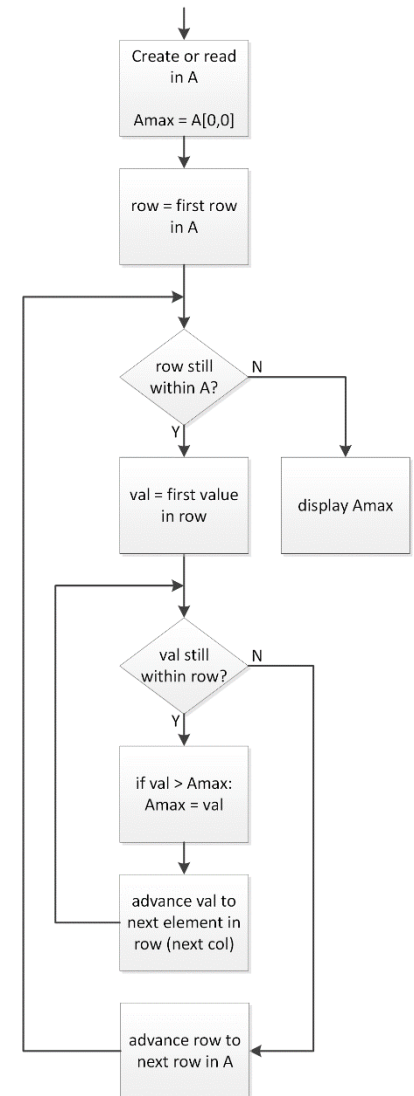
29

```
8
9 # initialize rng
10 rng = np.random.default_rng()
11
12 # create an array of random integers
13 A = rng.integers(low=0, high=100, size=(5,5))
14
15 # %% find maximum value in A
16 # initialize Amax
17 Amax = A[0,0]
18
19 # nested for loop to find max value of A
20 for row in A:
21     for val in row:
22         if val > Amax:
23             Amax = val
24
25 print('\n', A)
26 print('\nAmax = {}'.format(Amax))
27
```

```
[[47 95 54 61 66]
 [ 2 20 32 30 91]
 [35  1 60 83 73]
 [29 89 18 94 81]
 [95 53  5 67 90]]
```

Amax = 95

In [157]:



Nested for Loop – Example 2

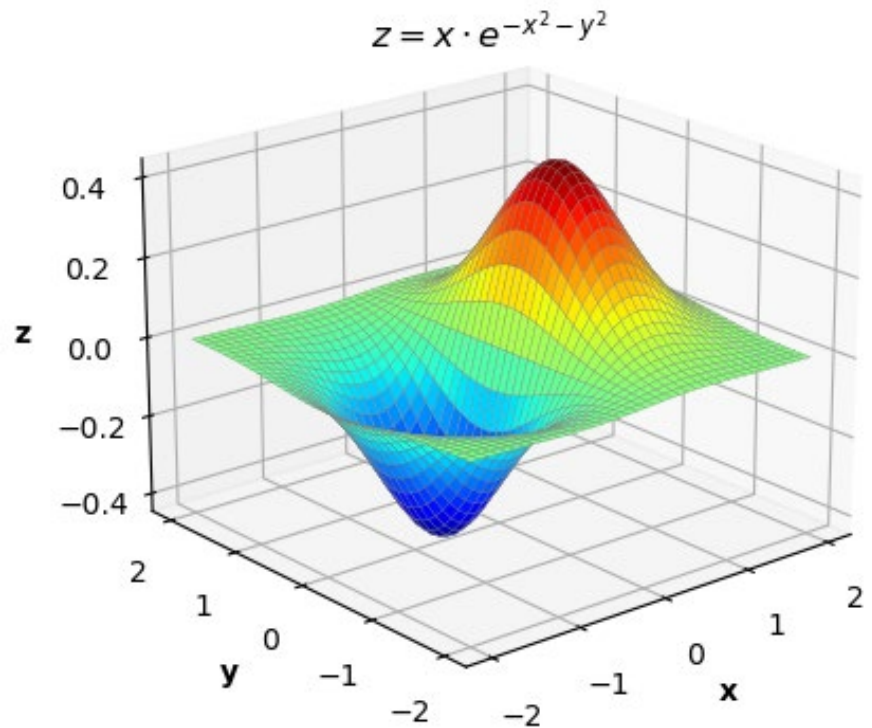
30

- Evaluate a function of two variables:

$$z = x \cdot e^{-x^2 - y^2}$$

over a range of $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$

- A surface in three-dimensional space
- Later in the course, we'll learn how to generate such a plot



Nested for Loop – Example 2

31

$$z = x \cdot e^{-x^2 - y^2}$$

- Evaluate the function over a range of x and y
- First, define x and y vectors
- Initialize the Z matrix
- Use a nested for loop to step through all points in this range of the x - y plane
 - Use `enumerate()` to iterate through indices and values

```
34
35 x = np.arange(-2, 2.1, 0.1)
36 y = np.arange(-2, 2.1, 0.1)
37
38 Z = np.empty((len(y), len(x)))
39
40 for j, xval in enumerate(x):
41     for i, yval in enumerate(y):
42         Z[i,j] = xval*np.exp(-xval**2 - yval**2)
43
44
```

Nested Loops

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- We just saw how we can use nested loops to:
 - ▣ Find the maximum value in a matrix or 2-D array
 - ▣ Evaluate a function of two variables
- A good illustration of nested loops, **BUT**
- ***There are easier, more efficient ways to do both of these things in Python***
 - ▣ Looping is slow – avoid if possible
 - ▣ Operate directly on arrays

```
70
71 # %% a better way to evaluate a 2-D
72 # function over a region of the x-y plane
73
74 x = np.arange(-2, 2.1, 0.1)
75 y = np.arange(-2, 2.1, 0.1)
76
77 X, Y = np.meshgrid(x,y)
78
79 Z = X*np.exp(-X**2 - Y**2)
80
```

```
32 # %% a better way to find the maximum
33 # value in a 2-D array
34
35 A = rng.integers(low=0, high=100, size=(5,5))
36
37 Amax = np.max(A)
38
39 print('\n', A)
40 print('\nAmax = {}'.format(Amax))
41
```


33

The Spyder Debugger

Debugging

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- You've probably already realized that it's not uncommon for your code to have errors
 - ▣ Computer code errors referred to as **bugs**

- Three main categories of errors
 - ▣ **Syntax errors** prevent your code from running and generate a Python error message
 - ▣ **Runtime errors** – not syntactically incorrect, but generate an error upon execution – e.g., indexing beyond matrix dimensions
 - ▣ **Algorithmic errors** don't prevent your code from executing, but do produce an unintended result

- Syntax and runtime errors are usually more easily fixed than algorithmic errors

- **Debugging** – the process of identifying and fixing errors is an important skill to develop
 - ▣ Spyder has a built-in **debugger** to facilitate this process

Debugging

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- Identifying and fixing errors is difficult because:
 - ▣ Programs run seemingly instantaneously
 - ▣ Incorrect output results, but can't see the intermediate steps that produced that output

- ***Basic debugging principles:***
 - ▣ ***Slow code execution down*** – allow for stepping through line-by-line
 - ▣ ***Provide visibility into the code execution*** – allow for monitoring of intermediate steps and variable values

Spyder Debugger – Breakpoints

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- **Breakpoint** – specification of a line of code at which Spyder should pause execution
- Set by clicking next to the number to the left of a line of code in a script
 - ▣ Spyder will execute the script **up to** this line, then pause

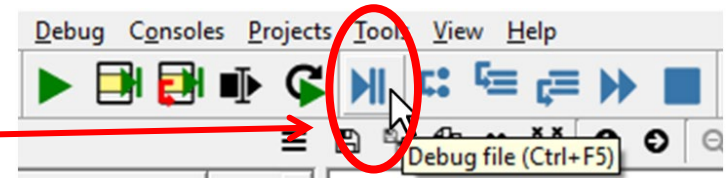
- Clicking here sets a breakpoint
 - ▣ Indicated by red circle

```
8   B = rng.integers(1, 11, (5,4))
9
10  # initialize array of zeros
11  C = np.zeros(np.shape(B), dtype=int)
12
13  m = np.shape(B)[0]      # rows in B
14  n = np.shape(B)[1]      # cols in B
15
16  for i in range(0, m-1):
17      for j in range(0, n-1):
18          C[i,j] = B[j,i]**2
19
20  print('\nB = \n{}'.format(B))
21  print('\nC = \n{}'.format(C))
```

Spyder Debugger – Breakpoints

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- Click 'Debug file' to begin execution
- Execution halts at the breakpoint
 - ▣ Before executing that line
- Console prompt changes to IPdb [n]:
 - ▣ Can now interactively enter commands



```
8 B = rng.integers(1, 11, (5,4))
9
10 # initialize array of zeros
11 C = np.zeros(np.shape(B), dtype=int)
12
13 m = np.shape(B)[0] # rows in B
14 n = np.shape(B)[1] # cols in B
15
16 for i in range(0, m-1):
17     for j in range(0, n-1):
18         C[i,j] = B[j,i]**2
19
```

A screenshot of a code editor showing Python code. A red diamond breakpoint is set on line 13. A red arrow points from the second list item to this breakpoint.

```
IPdb [1]: !continue
> c:\users\webbk\box\kwebb\classes\enr102_1
11 C = np.zeros(np.shape(B), dtype=int)
12
1--> 13 m = np.shape(B)[0] # rows in B
14 n = np.shape(B)[1] # cols in B
15

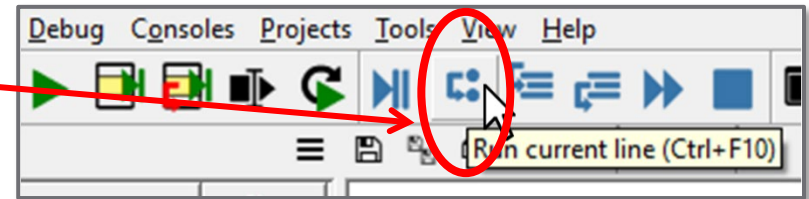
IPdb [2]:
```

A screenshot of the IPdb console. It shows the execution of the code from the previous screenshot, with a red arrow pointing from the third list item to the console prompt.

Spyder Debugger – Breakpoints

38

- Click 'Run current line' to execute the current line of code



- Arrow indicator advances to the next line

```
8 B = rng.integers(1, 11, (5,4))
9
10 # initialize array of zeros
11 C = np.zeros(np.shape(B), dtype=int)
12
13 m = np.shape(B)[0] # rows in B
14 n = np.shape(B)[1] # cols in B
15
16 for i in range(0, m-1):
17     for j in range(0, n-1):
18         C[i,j] = B[j,i]**2
```

- Variable, m, defined on previous line (line 16) now exists in the namespace
 - ▣ Available in the console

```
IPdb [2]: !next
IPdb [2]: m
5
IPdb [3]:
```

Debugger – Example

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- Recall a previous example of an algorithm to square every element in a matrix
- Let's say we run our script and get the following result:

```
5 # define a matrix of random ints
6 rng = np.random.default_rng()
7
8 B = rng.integers(1, 11, (5,4))
9
10 # initialize array of zeros
11 C = np.zeros(np.shape(B), dtype=int)
12
13 m = np.shape(B)[0] # rows in B
14 n = np.shape(B)[1] # cols in B
15
16 for i in range(0, m-1):
17     for j in range(0, n-1):
18         C[i,j] = B[j,i]**2
19
20 print('\nB = \n{}'.format(B))
21 print('\nC = \n{}'.format(C))
22
```

```
B =
[[ 1  6  1  7]
 [ 3  6  4  7]
 [ 8  6  3  2]
 [ 4  9  1 10]
 [ 4  9  7  7]]

C =
[[ 1  9 64  0]
 [36 36 36  0]
 [ 1 16  9  0]
 [49 49  4  0]
 [ 0  0  0  0]]

In [149]:
```

- Resulting matrix is ***transposed***
 - Use the ***debugger*** to figure out why

Debugger – Example

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- ❑ Set a **breakpoint** in the innermost for loop
- ❑ Click '**Debug file**'
- ❑ Code executes up to the breakpoint
- ❑ Variable Explorer shows $i=0$ and $j=0$
- ❑ Click '**Run current line**'
- ❑ Display $B[i, j]$ and $C[i, j]$ in the console
 - ▣ Both are as expected

```
12
13 m = np.shape(B)[0]      # rows in B
14 n = np.shape(B)[1]      # cols in B
15
16 for i in range(0, m-1):
17     for j in range(0, n-1):
18         C[i,j] = B[j,i]**2
19
```

Name	Type	Size	Value
B	Array of int64	(5, 4)	$\begin{bmatrix} 7 & 3 & 1 & 8 \\ 2 & 6 & 7 & 3 \end{bmatrix}$
C	Array of int32	(5, 4)	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
i	int	1	0
j	int	1	0
m	int	1	5

```
IPdb [4]: !next
```

```
IPdb [4]: B[i,j]
7
```

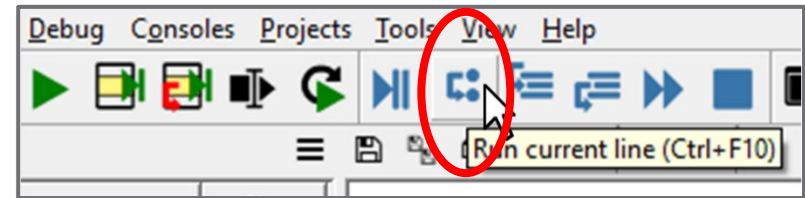
```
IPdb [5]: C[i,j]
49
```

```
IPdb [6]:
```


Debugger – Example

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- Click '***Run current line***' twice
 - ▣ Execute the next iteration of the loop
- Now, $i=0$ and $j=1$
 - ▣ First row, second column
- $B[i, j] = 10$
- But, $C[i, j] = 16$
 - ▣ Should be 100



Name /	Type	Size	Value								
B	Array of int64	(5, 4)	<table border="1"><tr><td>8</td><td>10</td><td>6</td><td>8</td></tr><tr><td>4</td><td>9</td><td>6</td><td>7</td></tr></table>	8	10	6	8	4	9	6	7
8	10	6	8								
4	9	6	7								
C	Array of int32	(5, 4)	<table border="1"><tr><td>64</td><td>16</td><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	64	16	0	0	0	0	0	0
64	16	0	0								
0	0	0	0								
i	int	1	0								
j	int	1	1								

```
IPdb [2]: !next
IPdb [2]: B[i,j]
10
IPdb [3]: C[i,j]
16
IPdb [4]: |
```

Debugger – Example

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- We see that $C[1, 2] = 16 = 4**2 = B[2, 1]**2$
- This leads us to an error on line 21 of the code
 - ▣ Should be $B[i, j]**2$, not $B[j, i]**2$

```
7
8   B = rng.integers(1, 11, (5,4))
9
10  # initialize array of zeros
11  C = np.zeros(np.shape(B), dtype=int)
12
13  m = np.shape(B)[0]      # rows in B
14  n = np.shape(B)[1]      # cols in B
15
16  for i in range(0, m-1):
17      for j in range(0, n-1):
18  ●   C[i,j] = B[j,i]**2
19
20  print('\nB = \n{}'.format(B))
21  print('\nC = \n{}'.format(C))
22
23
```

```
IPdb [4]: !continue

B =
[[ 8 10  6  8]
 [ 4  9  6  7]
 [10  3  2  5]
 [ 7 10 10  1]
 [ 6  9  5  8]]

C =
[[ 64  16 100  0]
 [100  81  9  0]
 [ 36  36  4  0]
 [ 64  49  25  0]
 [ 0  0  0  0]]

In [151]:
```

Exercise – Nested Loops, Debugger

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Exercise

- Write a script to do the following:
 - ▣ Create a 5x5 matrix of zeros, X
 - ▣ Initialize a random number generator:

```
rng = np.random.default_rng()
```
 - ▣ In a ***nested loop*** step through all elements in X
 - Outer loop steps through rows, inner loop steps through columns
 - Replace each element in X with a random integer:

```
X[i,j] = rng.integers(100)
```
- Set a ***breakpoint*** at the start of the outer loop and run the ***debugger***
- Step through code line-by-line observing the evolution of the matrix X