SECTION 7: FAULT ANALYSIS
Introduction
Faults in three-phase power systems are short circuits
- Line-to-ground
- Line-to-line

Result in the flow of excessive current
- Damage to equipment
  - Heat – burning/melting
  - Structural damage due to large magnetic forces

Bolted short circuits
- True short circuits – i.e., zero impedance

In general, fault impedance may be non-zero

Faults may be opens as well
- We’ll focus on short circuits
Types of Faults

- Type of faults from most to least common:
  - Single line-to-ground faults
  - Line-to-line faults
  - Double line-to-ground faults
  - Balanced three-phase (symmetrical) faults

- We’ll look first at the least common type of fault – the symmetrical fault – due to its simplicity
Subtransient Fault Current
Fault Current

- Faults occur nearly instantaneously
  - Lightening, tree fall, arcing over insulation, etc.
- Step change from steady-state behavior
  - Like throwing a switch to create the fault at $t = 0$
- Consider an unloaded synchronous generator
  - Equivalent circuit model:
    
    - $R$: generator resistance
    - $L$: generator inductance
    - $i(t) = 0$ for $t < 0$
    - Source phase, $\alpha$, determines voltage at $t = 0$
      - Short circuit can occur at any point in a 60 Hz cycle

$$v(t) = \sqrt{2}V_G \sin(\omega t + \alpha)$$
Fault Current

- The governing differential equation for \( t > 0 \) is

\[
\frac{di}{dt} + i(t) \frac{R}{L} = \frac{\sqrt{2}V_g}{L} \sin(\omega t + \alpha)
\]

- The solution gives the fault current

\[
i(t) = \frac{\sqrt{2}V_g}{Z} \left[ \sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta) e^{-t\frac{R}{L}} \right]
\]

where \( Z = \sqrt{R^2 + (\omega L)^2} \) and \( \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \)

- This total fault current is referred to as the *asymmetrical fault current*
  - It has a **steady-state** component

\[
i_{ac}(t) = \frac{\sqrt{2}V_g}{Z} \sin(\omega t + \alpha - \theta)
\]

  - And a **transient** component

\[
i_{dc}(t) = -\frac{\sqrt{2}V_g}{Z} \sin(\alpha - \theta) e^{-t\frac{R}{L}}
\]
Fault Current

- Magnitude of the transient fault current, $i_{dc}$, depends on $\alpha$
  - $i_{dc}(0) = 0$ for $\alpha = \theta$
  - $i_{dc}(0) = \sqrt{2}I_{ac}$ for $\alpha = \theta - 90^\circ$
  - $I_{ac} = \frac{V_G}{Z}$ is the rms value of the steady-state fault current

- Worst-case fault current occurs for $\alpha = \theta - 90^\circ$

$$i(t) = \frac{\sqrt{2}V_G}{Z} \left[ \sin \left( \omega t - \frac{\pi}{2} \right) + e^{-tL} \right]$$
Fault Current

- Important points here:
  - Total fault current has both steady-state and transient components – asymmetrical
  - Magnitude of the asymmetry (transient component) depends on the phase of the generator voltage at the time of the fault
  - In this class, we will use the steady-state current component, $I_{ac}$, as our primary fault current metric
The reactance of the generator was assumed constant in the previous example.

Physical characteristics of real generators result in a time-varying reactance following a fault.

Time-dependence modeled with three reactance values:
- $X''_d$: subtransient reactance
- $X'_d$: transient reactance
- $X_d$: synchronous reactance

Reactance increases with time, such that $X''_d < X'_d < X_d$.
Sub-Transient Fault Current

- Transition rates between reactance values are dictated by two time constants:
  - \( \tau''_d \): short-circuit subtransient time constant
  - \( \tau'_d \): short-circuit transient time constant

- Neglecting generator resistance, i.e. assuming \( \theta = 90^\circ \), the synchronous portion of the fault current is

\[
i_{ac}(t) = \sqrt{2}V_G \left[ \left( \frac{1}{X''_d} - \frac{1}{X'_d} \right) e^{-\frac{t}{\tau''_d}} + \left( \frac{1}{X'_d} - \frac{1}{X_d} \right) e^{-\frac{t}{\tau'_d}} + \frac{1}{X_d} \right] \sin \left( \omega t + \alpha - \frac{\pi}{2} \right)
\]

- At the instant of the fault, \( t = 0 \), the rms synchronous fault current is

\[
I''_F = \frac{V_G}{X''_d}
\]

  - This is the rms subtransient fault current, \( I''_F \)
  - This will be our primary metric for assessing fault current
Symmetrical Three-Phase Short Circuits
Symmetrical 3-Ø Short Circuits

- Next, we’ll calculate the subtransient fault current resulting from a balanced three-phase fault.

- We’ll make the following simplifying assumptions:
  - Transformers modeled with leakage reactance only
    - Neglect winding resistance and shunt admittances
    - Neglect Δ-Y phase shifts
  - Transmission lines modeled with series reactance only
  - Synchronous machines modeled as constant voltage sources in series with subtransient reactances
    - Generators and motors
  - Induction motors are neglected or modeled as synchronous motors
  - Non-rotating loads are neglected
Symmetrical 3-ϕ Short Circuits

☐ We’ll apply superposition to determine three-phase subtransient fault current

☐ Consider the following power system:

Assume there is a balanced three-phase short of bus 1 to ground at \( t = 0 \)
Symmetrical 3-Φ Short Circuits

- The instant of the fault can be modeled by the switch closing in the following line-to-neutral schematic.

- The short circuit (closed switch) can be represented by two back-to-back voltage sources, each equal to $V_F$. 
Symmetrical 3-ϕ Short Circuits

Applying superposition, we can represent this circuit as the sum of two separate circuits:

**Circuit 1**

**Circuit 2**
Assume that the value of the fault-location source, $V_F$, is the *pre-fault voltage* at that location.

Circuit 1, then, represents the *pre-fault circuit*, so

$$I_{F1}'' = 0$$

The $V_F$ source can therefore be removed from circuit 1.
Symmetrical 3-ϕ Short Circuits

- The current in circuit 1, $I_L$, is the pre-fault line current.
- Superposition gives the fault current:
  \[ I_F'' = I_{F1}'' + I_{F2}'' = I_{F2}'' \]
- The generator fault current is:
  \[ I_G'' = I_{G1}'' + I_{G2}'' \]
  \[ I_G'' = I_L'' + I_{G2}'' \]
- The motor fault current is:
  \[ I_M'' = I_{M1}'' + I_{M2}'' \]
  \[ I_M'' = -I_L'' + I_{M2}'' \]
Symmetrical 3-φ Fault – Example

- For the simple power system above:
  - Generator is supplying rated power
  - Generator voltage is 5% above rated voltage
  - Generator power factor is 0.95 lagging

- A bolted three-phase fault occurs at bus 1

- Determine:
  - Subtransient fault current
  - Subtransient generator current
  - Subtransient motor current
Symmetrical 3-ϕ Fault – Example

- First convert to per-unit
  - Use $S_b = 100 \text{ MVA}$
- Base voltage in the transmission line zone is
  \[ V_{b,tl} = 138 \text{ kV} \]
- Base impedance in the transmission line zone is
  \[ Z_{b,tl} = \frac{V_{b,tl}^2}{S_b} = \frac{(138 \text{ kV})^2}{100 \text{ MVA}} = 190.4 \Omega \]
- The per-unit transmission line reactance is
  \[ X_{tl} = \frac{20 \Omega}{190.4 \Omega} = 0.105 \text{ p.u.} \]
Symmetrical 3-\(\phi\) Fault – Example

- The two per-unit circuits are

- These can be simplified by combining impedances
Symmetrical 3-ϕ Fault – Example

- Using circuit 2, we can calculate the subtransient fault current

\[ I''_F = \frac{1.05\angle 0^\circ}{j0.116} = 9.079\angle -90^\circ \text{ p.u.} \]

- To convert to kA, first determine the current base in the generator zone

\[ I_{b,G} = \frac{S_b}{\sqrt{3}V_{b,G}} = \frac{100 \text{ MVA}}{\sqrt{3} \cdot 13.8 \text{ kV}} = 4.18 \text{ kA} \]

- The subtransient fault current is

\[ I''_F = (9.079\angle -90^\circ) \cdot 4.18 \text{ kA} \]

\[ I''_F = 37.98\angle -90^\circ \text{ kA} \]
Symmetrical 3-Ø Fault – Example

- The **pre-fault line current** can be calculated from the pre-fault generator voltage and power

\[
I_L = \left( \frac{S_G/3}{\sqrt{3}V_G''} \right)^* = \left( \frac{S_G}{\sqrt{3}V_G''} \right)^* = \frac{(100 \angle \cos^{-1}(0.95) \text{ MVA})^*}{(\sqrt{3} \cdot 1.05 \cdot 13.8 \angle 0^\circ \text{ kV})^*}
\]

\[
I_L = \frac{100 \angle -18.19^\circ \text{ MVA}}{\sqrt{3} \cdot 1.05 \cdot 13.8 \angle 0^\circ \text{ kV}}
\]

\[
I_L = 3.98 \angle -18.19^\circ \text{ kA}
\]

- Or, in per-unit:

\[
I_L = \frac{3.98 \angle -18.19^\circ \text{ kA}}{4.18 \text{ kA}} = 0.952 \angle -18.19^\circ \text{ p.u.}
\]

- This will be used to find the generator and motor fault currents
Symmetrical 3-ϕ Fault – Example

- The generator’s contribution to the fault current is found by applying current division

\[ I''_{G2} = I''_{F} \frac{0.505}{0.505 + 0.15} = 7.0 \angle -90^\circ \text{ p.u.} \]

- Adding the pre-fault line current, we have the \textit{subtransient generator fault current}

\[ I''_G = I_L + I''_{G2} \]
\[ I''_G = 0.952 \angle -18.19^\circ + 7.0 \angle -90^\circ \]
\[ I''_G = 7.35 \angle -82.9^\circ \text{ p.u.} \]

- Converting to kA

\[ I''_G = (7.35 \angle -82.9^\circ) \cdot 4.18 \text{ kA} \]
\[ I''_G = 30.74 \angle -82.9^\circ \text{ kA} \]
Symmetrical 3-ϕ Fault – Example

- Similarly, for the motor

\[ I''_{M2} = I''_F \cdot \frac{0.15}{0.505 + 0.15} = 2.08\angle - 90^\circ \ p.u. \]

- Subtracting the pre-fault line current gives the subtransient motor fault current

\[ I''_M = -I_L + I''_{M2} \]

\[ I''_M = -0.952\angle - 18.19^\circ + 2.08\angle - 90^\circ \]

\[ I''_M = 2.0\angle - 116.9^\circ \]

- Converting to kA

\[ I''_M = (2.0\angle - 116.9^\circ) \cdot 4.18 \ kA \]

\[ I''_M = 8.36\angle - 116.9^\circ \ kA \]
Symmetrical Components
Symmetrical Components

- In the previous section, we saw how to calculate subtransient fault current for balanced three-phase faults.

- Unsymmetrical faults are much more common.
  - Analysis is more complicated.

- We’ll now learn a tool that will simplify the analysis of unsymmetrical faults.
  - The *method of symmetrical components*
Symmetrical Components

- The *method of symmetrical components*: Represent an asymmetrical set of $N$ phasors as a sum of $N$ sets of symmetrical component phasors. These $N$ sets of phasors are called *sequence components*.

- Analogous to:
  - Decomposition of electrical signals into differential and common-mode components
  - Decomposition of forces into orthogonal components

- For a three-phase system ($N = 3$), sequence components are:
  - Zero sequence components
  - Positive sequence components
  - Negative sequence components
Sequence Components

- **Zero sequence components**
  - Three phasors with equal magnitude and equal phase
  - $V_a0, V_b0, V_c0$

- **Positive sequence components**
  - Three phasors with equal magnitude and $\pm 120^\circ$, positive-sequence phase
  - $V_a1, V_b1, V_c1$

- **Negative sequence components**
  - Three phasors with equal magnitude and $\pm 120^\circ$, negative-sequence phase
  - $V_a2, V_b2, V_c2$
Sequence Components

- Note that the absolute phase and the magnitudes of the sequence components is not specified.
  - Magnitude and phase define a unique set of sequence components.

- Any set of phasors – balanced or unbalanced – can be represented as a sum of sequence components.

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} =
\begin{bmatrix}
V_{a0} \\
V_{b0} \\
V_{c0}
\end{bmatrix} +
\begin{bmatrix}
V_{a1} \\
V_{b1} \\
V_{c1}
\end{bmatrix} +
\begin{bmatrix}
V_{a2} \\
V_{b2} \\
V_{c2}
\end{bmatrix}
\]  

(1)
The phasors of each sequence component have a fixed phase relationship
- If we know one, we know the other two
- Assume we know phase $a$ – use that as the reference

For the zero sequence components, we have
$$V_0 = V_{a0} = V_{b0} = V_{c0} \quad (2)$$

For the positive sequence components,
$$V_1 = V_{a1} = (1\angle 120^\circ) \cdot V_{b1} = (1\angle 240^\circ) \cdot V_{c1} \quad (3)$$

And, for the negative sequence components,
$$V_2 = V_{a2} = (1\angle 240^\circ) \cdot V_{b2} = (1\angle 120^\circ) \cdot V_{c2} \quad (4)$$

Note that we’re using phase $a$ as our reference, so
$$V_0 = V_{a0}, \quad V_1 = V_{a1}, \quad V_2 = V_{a2}$$
Next, we define a complex number, \( a \), that has unit magnitude and phase of 120°

\[ a = 1 \angle 120° \] (5)

- Multiplication by \( a \) results in a rotation (a phase shift) of 120°
- Multiplication by \( a^2 \) yields a rotation of 240° = −120°

Using (5) to rewrite (3) and (4)

\[ V_1 = V_{a1} = aV_{b1} = a^2V_{c1} \] (6)
\[ V_2 = V_{a2} = a^2V_{b2} = aV_{c2} \] (7)
Sequence Components

- Using (2), (6), and (7), we can rewrite (1) in a simplified form:

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix}
\begin{bmatrix}
V_0 \\
V_1 \\
V_2
\end{bmatrix}
\] (8)

- The vector on the left is the vector of phase voltages, \( V_p \).
- The vector on the right is the vector of (phase \( a \)) sequence components, \( V_s \).
- We’ll call the \( 3 \times 3 \) transformation matrix \( A \).

- We can rewrite (8) as:

\[ V_p = AV_s \] (9)
Sequence Components

We can express the sequence voltages as a function of the phase voltages by inverting the transformation matrix

\[ V_s = A^{-1}V_p \]  \hspace{1cm} (10)

where

\[ A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \]  \hspace{1cm} (11)

So

\[ \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \]  \hspace{1cm} (12)
Sequence Components

- The same relationships hold for three-phase currents
- The phase currents are
  \[ I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \]
- And, the sequence currents are
  \[ I_s = \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \]
The transformation matrix, $A$, relates the phase currents to the sequence currents

$$I_p = AI_s$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

And vice versa

$$I_s = A^{-1}I_p$$

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$
Before applying sequence components to unbalanced systems, let’s first look at the sequence components for a balanced, positive-sequence, three-phase system.

For a balanced system, we have

\[ V_b = V_a \cdot 1\angle -120^\circ = a^2 V_a \]
\[ V_c = V_a \cdot 1\angle120^\circ = aV_a \]

The sequence voltages are given by (12)

- The zero sequence voltage is

\[ V_0 = \frac{1}{3} [V_a + V_b + V_c] = \frac{1}{3} [V_a + a^2 V_a + aV_a] \]

\[ V_0 = \frac{1}{3} V_a [1 + a^2 + a] \]

Applying the identity \( 1 + a^2 + a = 0 \), we have

\[ V_0 = 0 \]
The **positive sequence component** is given by

\[ V_1 = \frac{1}{3} [V_a + aV_b + a^2V_c] \]

\[ V_1 = \frac{1}{3} [V_a + a \cdot a^2V_a + a^2 \cdot aV_a] \]

\[ V_1 = \frac{1}{3} [V_a + a^3V_a + a^3V_a] \]

Since \( a^3 = 1\angle0^\circ \), we have

\[ V_1 = \frac{1}{3} [3V_a] \]

\[ V_1 = V_a \]
The **negative sequence component** is given by

\[ V_2 = \frac{1}{3} [V_a + a^2 V_b + aV_c] \]

\[ V_2 = \frac{1}{3} [V_a + a^2 \cdot V_a + a \cdot a V_a] \]

\[ V_2 = \frac{1}{3} [V_a + a^4 V_a + a^2 V_a] \]

Again, using the identity \( 1 + a^2 + a = 0 \), along with the fact that \( a^4 = a \), we have

\[ V_2 = 0 \]
Sequence Components – Balanced System

- So, for a **positive-sequence, balanced**, three-phase system, the **sequence voltages** are
  \[ V_0 = 0, \quad V_1 = V_a, \quad V_2 = 0 \]

- Similarly, the **sequence currents** are
  \[ I_0 = 0, \quad I_1 = I_a, \quad I_2 = 0 \]

- This is as we would expect
  - No zero- or negative-sequence components for a positive-sequence balanced system
  - Zero- and negative-sequence components are only used to account for imbalance
Sequence Components

- We have just introduced the concept of *symmetric components*
  
  - Allows for decomposition of, possibly unbalanced, three-phase phasors into *sequence components*

- We’ll now apply this concept to power system networks to develop *sequence networks*
  
  - *Decoupled networks* for each of the sequence components
  
  - Sequence networks become *coupled only at the point of imbalance*
  
  - Simplifies the analysis of unbalanced systems
Sequence Networks
Sequence Networks

- **Power system components each have their own set of sequence networks**
  - Non-rotating loads
  - Transmission lines
  - Rotating machines – generators and motors
  - Transformers

- Sequence networks for overall systems are interconnections of the individual sequence network

- Sequence networks become coupled in a particular way at the fault location depending on type of fault
  - Line-to-line
  - Single line-to-ground
  - Double line-to-ground

- Fault current can be determined through simple analysis of the coupled sequence networks
Sequence Networks – Non-Rotating Loads
Consider a balanced Y-load with the neutral grounded through some non-zero impedance.

Applying KVL gives the phase-\(a\)-to-ground voltage

\[
V_{ag} = Z_y I_a + Z_n I_n
\]

\[
V_{ag} = Z_y I_a + Z_n (I_a + I_b + I_c)
\]

\[
V_{ag} = (Z_y + Z_n) I_a + Z_n I_b + Z_n I_c
\]  \hspace{1cm} (15)

For phase \(b\):

\[
V_{bg} = Z_y I_b + Z_n I_n = Z_y I_b + Z_n (I_a + I_b + I_c)
\]

\[
V_{bg} = Z_n I_a + (Z_y + Z_n) I_b + Z_n I_c
\]  \hspace{1cm} (16)

Similarly, for phase \(c\):

\[
V_{ag} = Z_n I_a + Z_n I_b + (Z_y + Z_n) I_c
\]  \hspace{1cm} (17)
Putting (15) – (17) in matrix form

\[
\begin{bmatrix}
V_{ag} \\
V_{bg} \\
V_{cg}
\end{bmatrix} =
\begin{bmatrix}
(Z_y + Z_n) & Z_n & Z_n \\
Z_n & (Z_y + Z_n) & Z_n \\
Z_n & Z_n & (Z_y + Z_n)
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

or

\[
V_p = Z_p I_p
\]  \hspace{1cm} (18)

where \(V_p\) and \(I_p\) are the phase voltages and currents, respectively, and \(Z_p\) is the \textit{phase impedance matrix}.

We can use (9) and (13) to rewrite (18) as

\[
AV_s = Z_p AI_s
\]

Solving for \(V_s\)

\[
V_s = A^{-1} Z_p AI_s
\]

or

\[
V_s = Z_s I_s
\]  \hspace{1cm} (19)
Sequence Networks – Non-Rotating Loads

\[ V_s = Z_s I_s \]  

where \( Z_s \) is the \textit{sequence impedance matrix}

\[
Z_s = A^{-1} Z_p A = \begin{bmatrix}
(Z_y + 3Z_n) & 0 & 0 \\
0 & Z_y & 0 \\
0 & 0 & Z_y
\end{bmatrix}
\]  

\[ \Box \] Equation (19) then becomes a set of three uncoupled equations

\[
V_0 = (Z_y + 3Z_n)I_0 = Z_0 I_0
\]  

\[
V_1 = Z_y I_1 = Z_1 I_1
\]  

\[
V_2 = Z_y I_2 = Z_2 I_2
\]
Equations (21) – (23) describe the uncoupled sequence networks:

- **Zero-sequence network:**

- **Positive-sequence network:**

- **Negative-sequence network:**
We can develop similar sequence networks for a balanced Δ-connected load

- \( Z_y = Z_\Delta / 3 \)
- There is no neutral point for the Δ-network, so \( Z_n = \infty \) - an open circuit

- **Zero-sequence network:**

- **Positive-sequence network:**

- **Negative-sequence network:**
Sequence Networks – 3-φ Lines
Balanced, three-phase lines can be modeled as

\[
\begin{align*}
V_{an} & = Z_{aa}I_a + V_{aa'} \\
V_{bn} & = Z_{ba}I_b + V_{bb'} \\
V_{cn} & = Z_{ca}I_c + V_{cc'}
\end{align*}
\]

The voltage drops across the lines are given by the following system of equations

\[
\begin{bmatrix}
V_{aa'} \\
V_{bb'} \\
V_{cc'}
\end{bmatrix} = \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix} \begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix} = \begin{bmatrix}
V_{an} - V_{a'n} \\
V_{bn} - V_{b'n} \\
V_{cn} - V_{c'n}
\end{bmatrix}
\] (24)
Sequence Networks – 3-ϕ Lines

- Writing (24) in compact form

\[ V_p - V_{p'} = Z_p I_p \]  \hspace{1cm} (25)

- \( Z_p \) is the \textit{phase impedance matrix}
  - Self impedances along the diagonal
  - Mutual impedances elsewhere
  - Symmetric
  - Diagonal, if we neglect mutual impedances
We can rewrite (25) in terms of sequence components

\[ AV_s - AV_{s'} = Z_p AI_s \]

\[ V_s - V_{s'} = A^{-1} Z_p AI_s \]

\[ V_s - V_{s'} = Z_s I_s \]  \hspace{1cm} (26)

where \( Z_s \) is the sequence impedance matrix

\[ Z_s = A^{-1} Z_p A \]  \hspace{1cm} (27)

\( Z_s \) is diagonal so long as the system impedances are balanced, i.e.

- Self impedances are equal: \( Z_{aa} = Z_{bb} = Z_{cc} \)
- Mutual impedances are equal: \( Z_{ab} = Z_{ac} = Z_{bc} \)
Sequence Networks – 3-ϕ Lines

- For balanced lines, $Z_s$ is diagonal

$$Z_s = \begin{bmatrix} Z_{aa} + 2Z_{ab} & 0 & 0 \\ 0 & Z_{aa} - Z_{ab} & 0 \\ 0 & 0 & Z_{aa} - Z_{ab} \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

- Because $Z_s$ is diagonal, (26) represents three uncoupled equations

$$V_0 - V_0' = Z_0 I_0 \quad (28)$$
$$V_1 - V_1' = Z_1 I_1 \quad (29)$$
$$V_2 - V_2' = Z_2 I_2 \quad (30)$$
Equations (28) – (30) describe the voltage drop across three uncoupled sequence networks:

- **Zero-sequence network:**

- **Positive-sequence network:**

- **Negative-sequence network:**
Sequence Networks – Rotating Machines
Consider the following model for a *synchronous generator*

Similar to the Y-connected load
- Generator includes *voltage sources* on each phase

Voltage sources are *positive sequence*
- Sources will appear *only in the positive-sequence network*
Sequence Networks – Synchronous Generator

- Sequence networks for Y-connected synchronous generator
  - Zero-sequence network:
  
  ![Zero-sequence network diagram]

  - Positive-sequence network:
    
    ![Positive-sequence network diagram]

  - Negative-sequence network:
    
    ![Negative-sequence network diagram]
Sequence Networks – Motors

- **Synchronous motors**
  - Sequence networks identical to those for synchronous generators
  - Reference current directions are reversed

- **Induction motors**
  - Similar sequence networks to synchronous motors, except source in the positive sequence network set to zero
Sequence Networks – Transformers
Per-unit sequence networks for transformers
- Simplify by neglecting transformer shunt admittances
- Consider a Y-Y transformer

Similar to the Y-connected load, the voltage drops across the neutral impedances are $3I_0Z_N$ and $3I_0Z_n$
- $3Z_N$ and $3Z_n$ each appear in the zero-sequence network
- Can be combined in the per-unit circuit as long as shunt impedances are neglected
Impedance accounting for leakage flux and winding resistance for each winding can be referred to the primary
- Add together into a single impedance, $Z_s$, in the per-unit model

Y-Y transformer sequence networks

- **Zero-sequence network:**

- **Positive-sequence network:**

- **Negative-sequence network:**
Sequence Networks – Y-Δ Transformers

- Y-Δ transformers differ in a couple of ways
  - Must account for phase shift from primary to secondary
    - For positive-sequence network, Y-side voltage and current lead Δ-side voltage and current
    - For negative-sequence network, Y-side voltage and current lag Δ-side voltage and current
  - No neutral connection on the Δ side
    - Zero-sequence current cannot enter or leave the Δ winding
Sequence Networks – Y-Δ Transformers

- Sequence networks for Y-Δ Transformers
  - **Zero-sequence network:**
    
    ![Zero-sequence network diagram]
    
    \[ I_{H0} \rightarrow 3Z_N \rightarrow Z_s \rightarrow I_{X0} = 0 \]
    
    \[ V_{H0} \rightarrow V_{X0} \]
  
  - **Positive-sequence network:**
    
    ![Positive-sequence network diagram]
    
    \[ I_{H1} \rightarrow Z_s \rightarrow I_{X1} \]
    
    \[ V_{H1} \rightarrow e^{j30^\circ} : 1 \rightarrow V_{X1} \]
  
  - **Negative-sequence network:**
    
    ![Negative-sequence network diagram]
    
    \[ I_{H2} \rightarrow Z_s \rightarrow I_{X2} \]
    
    \[ V_{H2} \rightarrow e^{-j30^\circ} : 1 \rightarrow V_{X2} \]
Sequence Networks – Δ-Δ Transformers

- Δ-Δ transformers
  - Like Y-Y transformers, no phase shift
  - No neutral connections
    - Zero-sequence current cannot flow into or out of either winding
Sequence Networks – $\Delta$-$\Delta$Transformers

- Sequence networks for $\Delta$-$\Delta$Transformers

  - **Zero-sequence network:**

  ![Zero-sequence network diagram](image)

  - **Positive-sequence network:**

  ![Positive-sequence network diagram](image)

  - **Negative-sequence network:**

  ![Negative-sequence network diagram](image)
Power in Sequence Networks
Power in Sequence Networks

- We can relate the power delivered to a system’s sequence networks to the three-phase power delivered to that system.

- We know that the complex power delivered to a three-phase system is the sum of the power at each phase:

\[
S_p = V_{an}I_a^* + V_{bn}I_b^* + V_{cn}I_c^*
\]

- In matrix form, this looks like

\[
S_p = \begin{bmatrix}
V_{an} & V_{bn} & V_{cn}
\end{bmatrix}
\begin{bmatrix}
I_a^* \\
I_b^* \\
I_c^*
\end{bmatrix}
\]

\[
S_p = V_p^T I_p^*
\]  \hspace{1cm} (28)
Recall the following relationships

\[ V_p = AV_s \]  \hspace{1cm} (9)

\[ I_p = AI_s \]  \hspace{1cm} (13)

Using (9) and (13) in (28), we have

\[ S_p = (AV_s)^T (AI_s)^* \]

\[ S_p = V^T_s A^T A^* I^*_s \]  \hspace{1cm} (29)

Computing the product in the middle of the right-hand side of (29), we find

\[ A^T A^* = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3I_3 \]

where \( I_3 \) is the 3×3 identity matrix
Power in Sequence Networks

- Equation (29) then becomes
  \[ S_p = V_s^T 3I_3 I_s^* \]
  \[ S_p = 3V_s^T I_s^* \]
  \[ S_p = 3 [V_0 \ V_1 \ V_2] \begin{bmatrix} I_0^* \\ I_1^* \\ I_2^* \end{bmatrix} \]
  \[ S_p = 3 (V_0 I_0^* + V_1 I_1^* + V_2 I_2^*) \]

- The total power delivered to a three-phase network is three times the sum of the power delivered to the three sequence networks
  - The three sequence networks represent only one of the three phases – recall, we chose to consider only phase \( a \)
Example Problems
A bolted, symmetric, three-phase fault occurs 60% of the way from bus 1 to bus 2. Determine the subtransient fault current in per-unit and in amperes. The load is consuming rated power at rated voltage and unity power factor.
Determine the sequence components for the following unbalanced set of three-phase voltage phasors:

\[ V_a = 1\angle0^\circ \text{ p.u.} \]
\[ V_b = 0.5\angle-60^\circ \text{ p.u.} \]
\[ V_c = 2\angle200^\circ \text{ p.u.} \]
Determine the phase components for the following set of sequence components:

\[ V_0 = 1 \angle 60^\circ \text{ p.u.} \]
\[ V_1 = 1 \angle 0^\circ \text{ p.u.} \]
\[ V_2 = 0 \text{ p.u.} \]
Unsymmetrical Faults
Unsymmetrical Faults

- The majority of faults that occur in three-phase power systems are unsymmetrical
  - Not balanced
  - Fault current and voltage differ for each phase
- The method of symmetrical components and sequence networks provide us with a tool to analyze these unsymmetrical faults
- We’ll examine three types of unsymmetrical faults
  - Single line-to-ground (SLG) faults
  - Line-to-line (LL) faults
  - Double line-to-ground (DLG) faults
Basic procedure for fault analysis:

1. Generate sequence networks for the system

2. Interconnect sequence networks appropriately at the fault location

3. Perform circuit analysis on the interconnected sequence networks
To simplify our analysis, we’ll make the following assumptions

1. System is balanced before the instant of the fault
2. Neglect pre-fault load current
   - All pre-fault machine terminal voltages and bus voltages are equal to $V_F$
3. Transmission lines are modeled as series reactances only
4. Transformers are modeled with leakage reactances only
5. Non-rotating loads are neglected
6. Induction motors are either neglected or modeled as synchronous motors
Unsymmetrical Fault Analysis

- Each sequence network includes all interconnected power-system components
  - Generators, motors, lines, and transformers

- Analysis will be simplified if we represent each sequence network as its Thévenin equivalent
  - From the perspective of the fault location

- For example, consider the following power system:

```
150 MVA
13.8 kV
X_{G}'' = 0.1
X_{0} = 0.06
X_{2} = 0.12

T_{1}

X_{\text{line}} = 0.12

T_{2}

150 MVA
13.8 kV
X_{M}'' = 0.25
X_{0} = 0.15
X_{2} = 0.27
X_{n} = 0.1

150 MVA
13.8 kV Δ / 230 kV Y
X_{T1} = 0.15 \text{ p.u.}

150 MVA
230 kV Y / 13.8 kV Δ
X_{T2} = 0.15 \text{ p.u.}
```
Unsymmetrical Fault Analysis – Sequence Networks

- The sequence networks for the system are generated by interconnecting the sequence networks for each of the components.

- The zero-sequence network:

- The positive-sequence network:
  - Assuming the generator is operating at the rated voltage at the time of the fault.
Unsymmetrical Fault Analysis – Sequence Networks

- The negative sequence network:

- Now, let’s assume there is some sort of fault at bus 1
  - Determine the Thévenin equivalent for each sequence network from the perspective of bus 1
  - Simplifying the zero-sequence network to its Thévenin equivalent
The positive-sequence network simplifies to the following circuit with the following Thévenin equivalent:

Similarly, for the negative-sequence network, we have:

Next, we’ll see how to interconnect these networks to analyze different types of faults.
Single-Line-to-Ground Fault
Unsymmetrical Fault Analysis – SLG Fault

- The following represents a generic three-phase network with terminals at the fault location:

- If we have a single-line-to-ground fault, where phase $a$ is shorted through $Z_f$ to ground, the model becomes:
The phase-domain fault conditions:

\[ I_a = \frac{V_{ag}}{Z_f} \]  

\[ I_b = I_c = 0 \]  

Transforming these phase-domain currents to the sequence domain:

\[
\begin{bmatrix}
I_0 \\
I_1 \\
I_2 
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a 
\end{bmatrix} \begin{bmatrix}
\frac{V_{ag}}{Z_f} \\
0 \\
0 
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
\frac{V_{ag}}{Z_f} \\
\frac{V_{ag}}{Z_f} \\
\frac{V_{ag}}{Z_f} 
\end{bmatrix}
\]

This gives one of our sequence-domain fault conditions:

\[ I_0 = I_1 = I_2 \]
We know that

\[ I_a = \frac{V_{ag}}{Z_f} = I_0 + I_1 + I_2 \] (5)

and

\[ V_{ag} = V_0 + V_1 + V_2 \] (6)

Using (5) and (6) in (1), we get

\[ I_0 + I_1 + I_2 = \frac{1}{Z_f} (V_0 + V_1 + V_2) \]

Using (4), this gives our second **sequence-domain fault condition**

\[ I_0 = I_1 = I_2 = \frac{1}{3Z_f} (V_0 + V_1 + V_2) \] (7)
The sequence-domain fault conditions are satisfied by connecting the sequence networks in series along with three times the fault impedance.

We want to find the phase domain fault current, $I_F$

\[
I_F = I_a = I_0 + I_1 + I_2 = 3I_1
\]

\[
I_1 = \frac{V_F}{Z_0 + Z_1 + Z_2 + 3Z_F}
\]

\[
I_F = \frac{3V_F}{Z_0 + Z_1 + Z_2 + 3Z_F}
\]
Returning to our example power system

The interconnected sequence networks for a *bolted fault* at bus 1:
SLG Fault - Example

- The fault current is

\[ I_F = \frac{3V_F}{Z_0 + Z_1 + Z_2 + 3Z_F} \]

\[ I_F = \frac{3.0 \angle 30^\circ}{j0.473} = 6.34 \angle -60^\circ \text{ p. u.} \]

- The current base at bus 1 is

\[ I_b = \frac{S_b}{\sqrt{3}V_{b1}} = \frac{150 \text{ MVA}}{\sqrt{3} \text{ 230 kV}} = 376.5 \text{ A} \]

- So the fault current in kA is

\[ I_F = (6.34 \angle -60^\circ)(376.5 \text{ A}) \]

\[ I_F = 2.39 \angle -60^\circ \text{ kA} \]
Line-to-Line Fault
Now consider a **line-to-line fault** between phase $b$ and phase $c$ through impedance $Z_F$.

Phase-domain fault conditions:

\[ I_a = 0 \]  
\[ I_b = -I_c = \frac{V_{bg} - V_{cg}}{Z_F} \]  

Transforming to the sequence domain

\[
\begin{bmatrix}
I_0 \\
I_1 \\
I_2
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix} \begin{bmatrix}
0 \\
I_b \\
-I_b
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
0 \\
(a - a^2)I_b \\
(a^2 - a)I_b
\end{bmatrix}
\]  

So, the first two sequence-domain fault conditions are

\[ I_0 = 0 \]  
\[ I_2 = -I_1 \]
To derive the remaining sequence-domain fault condition, rearrange (10) and transform to the sequence domain

\[ V_{bg} - V_{cg} = I_bZ_F \]

\[
(V_0 + a^2V_1 + aV_2) - (V_0 + aV_1 + a^2V_2) = (I_0 + a^2I_1 + aI_2)Z_F
\]

\[ a^2V_1 + aV_2 - aV_1 - a^2V_2 = (a^2 - a)I_1Z_F \]

\[ (a^2 - a)V_1 - (a^2 - a)V_2 = (a^2 - a)I_1Z_F \]

The last sequence-domain fault condition is

\[ V_1 - V_2 = I_1Z_F \]  \hspace{1cm} (14)
Unsymmetrical Fault Analysis – LL Fault

- Sequence-domain fault conditions
  \[
  I_0 = 0 \\
  I_2 = -I_1 \\
  V_1 - V_2 = I_1 Z_F
  \]

- These can be satisfied by:
  - Leaving the zero-sequence network open
  - Connecting the terminals of the positive- and negative-sequence networks together through \( Z_F \)
The fault current is the phase $b$ current, which is given by

$$ I_F = I_b = I_0 + a^2 I_1 + a I_2 $$

$$ I_F = a^2 I_1 - a I_1 $$

$$ I_F = -j\sqrt{3} I_1 = \frac{-j\sqrt{3} V_F}{Z_1 + Z_2 + Z_F} $$

$$ I_F = \frac{\sqrt{3} V_F \angle -90°}{Z_1 + Z_2 + Z_F} \quad (15) $$
Now consider the same system with a bolted line-to-line fault at bus 1.

The sequence network:
LL Fault - Example

\[ I_1 = \frac{1\angle30^\circ}{j0.349} = 2.87\angle - 60^\circ \]

- The subtransient fault current is given by (15) as

\[ I_F = (\sqrt{3}\angle - 90^\circ)(2.87\angle - 60^\circ) \]

\[ I_F = 4.96\angle - 150^\circ \text{ p.u.} \]

- Using the previously-determined current base, we can convert the fault current to kA

\[ I_F = I_{b1} \cdot 4.96\angle - 150^\circ \]

\[ I_F = (4.96\angle - 150^\circ)(376.5A) \]

\[ I_F = 1.87\angle - 150^\circ \text{ kA} \]
Double-Line-to-Ground Fault
Now consider a **double line-to-ground fault**

- Assume phases $b$ and $c$ are shorted to ground through $Z_F$

Phase-domain fault conditions:

\[
I_a = 0 \quad \text{(16)}
\]

\[
I_b + I_c = \frac{V_{bg}}{Z_F} = \frac{V_{cg}}{Z_F} \quad \text{(17)}
\]

It can be shown that (16) and (17) transform to the following sequence-domain fault conditions (analysis skipped here)

\[
I_0 + I_1 + I_2 = 0 \quad \text{(18)}
\]

\[
V_1 = V_2 \quad \text{(19)}
\]

\[
I_0 = \frac{1}{3Z_F} (V_0 - V_1) \quad \text{(20)}
\]
Unsymmetrical Fault Analysis – DLG Fault

- **Sequence-domain fault conditions**
  
  \[ I_0 + I_1 + I_2 = 0 \]  \hspace{1cm} (18)
  
  \[ V_1 = V_2 \]  \hspace{1cm} (19)
  
  \[ I_0 = \frac{1}{3Z_F} (V_0 - V_1) \]  \hspace{1cm} (20)

- **To satisfy these fault conditions**
  - Connect the positive- and negative-sequence networks together directly
  - Connect the zero- and positive-sequence networks together through \(3Z_F\)
Unsymmetrical Fault Analysis – DLG Fault

The fault current is the sum of the phase $b$ and phase $c$ currents, as given by (17)

- In the sequence domain the fault current is

$$I_F = I_b + I_c = 3I_0$$

$$I_F = 3I_0$$ (21)

$I_0$ can be determined by a simple analysis (e.g. nodal) of the interconnected sequence networks.
Now determine the subtransient fault current for a bolted double line-to-ground fault at bus 1.

Here, because $Z_F = 0$, $V_0 = V_1 = V_2$.
To find $I_F$, we must determine $I_0$

We can first find $V_0$ by applying voltage division

$$V_0 = V_F \frac{Z_2 || Z_0}{Z_1 + Z_2 || Z_0}$$

$$V_0 = 1.0 \angle 30^\circ \frac{j0.18 || j0.124}{j0.169 + j0.18 || j0.124}$$

$$V_0 = 0.303 \angle 30^\circ$$
Next, calculate $I_0$

$$I_0 = \frac{-V_0}{Z_0} = \frac{-0.303\angle30^\circ}{j0.124} = 2.44\angle120^\circ \text{ p.u.}$$

The per-unit fault current is

$$I_F = 3I_0 = 7.33\angle120^\circ \text{ p.u.}$$

Using the current base to convert to kA, gives the subtransient DLG fault current

$$I_F = (7.33\angle120^\circ)(376.5 \text{ A})$$

$$I_F = 2.76\angle120^\circ \text{ kA}$$
Example Problems
Draw the sequence networks for the following power system. Assume the generator is operating at rated voltage.

- Generator (Gen) 100 MVA, 13.8 kV, $X_g'' = 0.15$ p.u., $X_n = 0.05$ p.u.
- Transformer $T_1$: 100 MVA, 13.8/138 kV, $X_{T1} = 0.07$ p.u.
- Line: $X_{line} = 0.1$ p.u.
- Transformer $T_2$: 100 MVA, 138/13.8 kV, $X_{T2} = 0.07$ p.u.
- Load (M) 100 MVA, 13.8 kV, $X_{M''} = 0.2$ p.u.
Reduce the sequence networks to their Thévenin equivalents for a fault occurring half of the way along the transmission line.
Determine the subtransient fault current resulting from a DLG fault, half way along the transmission line, through an impedance of j0.2 p.u.