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ROCKING RESPONSE OF RIGID BLOCKS TO EARTHQUAKES

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SUMMARY

This investigation deals with the rocking response of rigid blocks subjected to earthquake ground motion. A numerical procedure and computer program are developed to solve the non-linear equations of motion governing the rocking motion of rigid blocks on a rigid base subjected to horizontal and vertical ground motion.

The response results presented show that the response of the block is very sensitive to small changes in its size and slenderness ratio and to the details of ground motion. Systematic trends are not apparent: The stability of a block subjected to a particular ground motion does not necessarily increase monotonically with increasing size or decreasing slenderness ratio. Overturning of a block by a ground motion of particular intensity does not imply that the block will necessarily overturn under the action of more intense ground motion.

In contrast, systematic trends are observed when the problem is studied from a probabilistic point of view with the ground motion modelled as a random process. The probability of a block exceeding any response level, as well as the probability that a block overturns, increases with increase in ground motion intensity, increase in slenderness ratio of the block and decrease in its size.

It is concluded that probabilistic estimates of the intensity of ground shaking may be obtained from its observed effects on monuments, minarets, tombstones and other similar objects provided suitable data in sufficient quantity is available, and the estimates are based on probabilistic analyses of the rocking response of rigid blocks, considering their non-linear dynamic behaviour.

INTRODUCTION

Toward the latter part of the 19th century and in the early part of this century, before instruments had been developed to record strong ground motions, procedures were proposed to estimate the intensity of ground shaking from its observed effects on tombstones and monumental columns, whether they overturned or remained standing.[1–4] In these procedures, the free standing tombstone or column was idealized as a rigid block resting on a rigid horizontal base, and the ground motion was either idealized as an instantaneous impulse or its effects were represented by a static horizontal force acting on the block. Formulas were derived to calculate the ground acceleration necessary to overturn a block of given dimensions.[1–4] The great interest in this approach led to a proposal for its use in seismological observatories to determine the maximum ground acceleration during earthquakes. The idea was to erect a family of columns, all with rectangular sections but varying in slenderness ratio.[1–4] When subjected to ground motion some of these columns might overturn; others would remain standing. From such information, using the above-mentioned formulas, the maximum ground acceleration could be estimated.

Modern strong-motion accelerographs have now been available for a few decades and have been deployed in seismic areas. Several hundred recordings have been obtained, most of them in the last ten years. However, because of the rarity of strong earthquakes, their seeming proclivity to occur in uninstrumented areas, and the localized extent of the really strong shaking, there are wide gaps in the present day collection of strong ground motion accelerograms. For example, the shaking in the vicinity of the causative fault has been recorded only a few times during Richter Magnitude 7 earthquakes and never in a truly great (Magnitude 8) earthquake. Thus, estimates of ground acceleration based on the approach mentioned earlier could provide useful supplements to the modern recorded data, which would be useful in specifying design earthquakes of very long return periods, suitable for nuclear power plants and major dams near population centres. Available for such

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calculations is a wealth of historical information from seismic areas of the world which were the centres of the ancient Roman, Greek, Chinese and Indian civilizations. Some of the historical monuments in these areas were destroyed by earthquakes, others withstood many destructive earthquakes. Such an approach was recently employed to estimate the accelerations in the epicentral region of the 1975, Ohita earthquake in Japan.[5]

In almost every destructive earthquake in the Eastern Mediterranean and Middle Eastern regions, free-standing columns of Greek and Roman monuments and minarets have survived undamaged in earthquakes that have caused spectacular destruction around them. In some cities in India, free-standing stone columns that supported heavy statues remained standing although at the end of the earthquake they were surrounded by heaps of debris that had been seemingly more stable structures. Tall, slender stone pillars in graveyards have survived strong ground motion whereas box-like electric power transformers have rocked and overturned. During the Chilean earthquakes of 1960, several golf-ball-on-a-tee type of elevated water tanks survived the ground shaking whereas much more stable-appearing reinforced concrete elevated water tanks were severely damaged. In order to explain such anomalous behaviour, the above-mentioned analyses of overturning of rigid blocks, proposed several decades ago, are inadequate and it is necessary to consider the dynamics of the system.

Motivated by the observations of damage to water tanks in the 1960 Chilean earthquakes, Housner was the first to investigate systematically the dynamics of a rigid block on a rigid horizontal base undergoing horizontal motion.[6] Representing the ground acceleration as a rectangular pulse and as a half-cycle sine-wave pulse, equations were derived for the minimum accelerations (which depend on the duration of the pulse) required to overturn a block. The approach employed in obtaining these equations is valid only for single pulse excitations which, if large enough, would overturn the block without rocking and the associated impacts. Smaller accelerations than specified by these equations may set the block to rocking but will not overturn it. It is, however, possible to overturn the block with smaller accelerations if a number of pulses act successively, a characteristic of ground accelerations during earthquakes. Using an energy approach and idealizing the ground motion as white noise, Housner presented an approximate analysis of the dynamics of rigid blocks subjected to such excitations. From these results he showed that there is a scale effect which makes the larger of two geometrically similar blocks more stable than the smaller block. Moreover, the stability of a tall, slender block subjected to earthquake motion is much greater than would be inferred from its stability against a static horizontal force, employed to represent earthquake effects in the very simple analysis mentioned above. Thus, Housner's work led to important results.

Motivated by the need to design concrete blocks to provide radiation shielding in particle accelerator laboratories, rigid blocks were tested on the Berkeley shaking table.[7] These experiments demonstrated that the rocking response of a rigid block is very sensitive to the boundary conditions, the impact coefficient of restitution and the ground motion details.

In the first part of this report, the dynamics of rocking rigid blocks is studied with the aim of understanding the sensitivity of response to system and ground motion properties. The problem is next examined from a probabilistic point of view to identify, if possible, any predictable behavioural trends in spite of the response sensitivity. Finally, the implications of the results for estimating the intensity of ground shaking from its observed effects on tombstones, monumental columns and other similar objects are presented.

GOVERNING EQUATIONS

A rigid block subjected to horizontal and vertical ground accelerations of the rigid base is shown in rotated position in Figure 1. The coefficient of friction is assumed to be sufficiently large so that there will be no sliding between the block and the base. Depending on the ground acceleration, the block may move rigidly with the ground or be set into rocking; in the latter case, it will oscillate about the centres of rotation 0 and 0′. It is assumed that the block and base surfaces in contact are perfectly smooth so that the block will rock around the edges 0 and 0′ and no intermediate location.

When subjected to base accelerations a_g^x in the horizontal direction and a_g^y in the vertical direction, the block will be set into rocking when the overturning moment of the horizontal inertia force about one edge exceeds the restoring moment due to the weight of the block and vertical inertia force:

$$\frac{W}{g}a_g^x\frac{H}{2}>\left(W+\frac{W}{g}a_g^y\right)\frac{B}{2} \tag{1}$$

$$a_g^x>\frac{B}{H}g\left(1+\frac{a_g^y}{g}\right)$$

where W is the weight of the block, g = acceleration of gravity, and it is assumed that the geometric and gravity centrès for the block coincide.

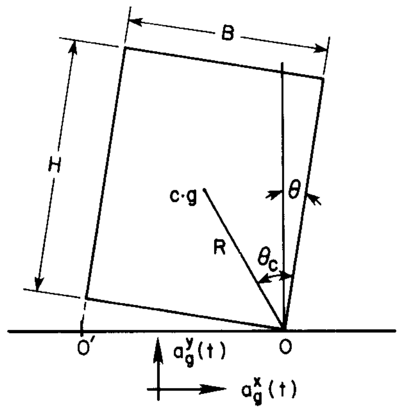

Figure 1. Rocking block

The rigid block will oscillate about the centres of rotation 0 and 0′ when it is set to rocking. The equations of motion of the block, governing the angle θ from the vertical (Figure 1), subjected to horizontal and vertical ground accelerations $a_g^x(t)$ and $a_g^y(t)$ are derived by considering the equilibrium of moments about the centres of rotation. These equations may be expressed as

$$I_0\ddot{\theta}+W\left(1+\frac{a_g^y(t)}{g}\right)R\sin(\theta_c-\theta)=-\frac{W}{g}R\cos(\theta_c-\theta)\,a_g^x(t) \tag{2}$$

when the block is rocking about 0, and

$$I_0\ddot{\theta}+W\left(1+\frac{a_g^y(t)}{g}\right)R\sin(\theta_c+\theta)=-\frac{W}{g}R\cos(\theta_c+\theta)\,a_g^x(t) \tag{3}$$

when it is rocking about 0′. In addition to the quantities defined earlier and in Figure 1, I_0 = mass moment of inertia of the block about 0 or 0′ and $\theta_c=\cot^{-1}(H/B)$. Because of the trigonometric functions of θ, each of these equations is non-linear. Another source of non-linearity is the switching of equations back and forth between equations (2) and (3) as the block rocks alternately about 0 and 0′.

For slender blocks, θ_c is small and it is reasonable to linearize the two equations of motion, individually, as follows:

$$I_0\ddot{\theta}+WR\left(1+\frac{a_g^y(t)}{g}\right)(\theta_c-\theta)=-\frac{W}{g}Ra_g^x(t) \tag{4}$$

for rocking about 0, and

$$I_0\ddot{\theta}-WR\left(1+\frac{a_g^y(t)}{g}\right)(\theta_c+\theta)=-\frac{W}{g}Ra_g^x(t) \tag{5}$$

for rocking about 0′.

The relation between a static moment applied about 0 and 0′ and the resulting angle of rotation θ, identified from these linearized equations, is shown in Figure 2. Interpreting this relationship in terms of the usual concepts of structural stiffness, the system has infinite stiffness until the magnitude of the applied moment reaches $WR\theta_c$; thereafter the stiffness is negative. When θ exceeds θ_c, the critical angle, the block will overturn under static moment, but, as will be seen later, not necessarily under dynamic conditions. Obviously, the properties of the rocking block are much different to those of a linear single-degree-of-freedom system where the stiffness is positive and constant. The properties are reminiscent of a rigid-plastic system except that, for the rocking block, the second slope is negative and behaviour is not hysteretic.

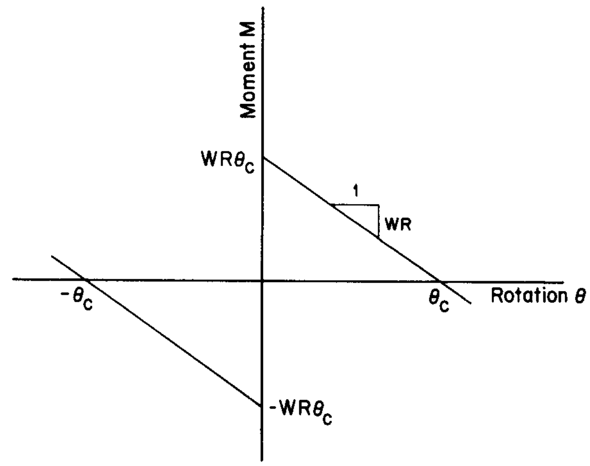

Figure 2. Moment–rotation relation for slender blocks ($H/B \geqslant 3$)

When the block is rotated through an angle θ and released from rest with initial displacement, it will rotate about the point 0 as it falls back into the vertical position. If the impact is assumed to be such that there is no bouncing of the block, the rotation continues smoothly about the point 0′ and the momentum about 0′ is conserved. Thus,

$$I_0\dot{\theta}_1 - \frac{W}{g}RB\dot{\theta}_1 \sin\theta_c = I_0\dot{\theta}_2 \tag{6}$$

where $\dot{\theta}_1$ and $\dot{\theta}_2$ are the angular velocities before and after impact. The ratio of kinetic energy quantities after and before impact is

$$r = \tfrac{1}{2}I_0\dot{\theta}_2^2/\tfrac{1}{2}I_0\dot{\theta}_1^2 = (\dot{\theta}_2/\dot{\theta}_1)^2 \tag{7}$$

Using equation (6), r can be expressed as

$$r = \left[1 - \frac{W}{g}\frac{R^2}{I_0}(1-\cos 2\theta_c)\right]^2 \tag{8}$$

Just before and just after impact, the angle of rotation θ is zero and the potential energy stored in the system is zero. At these two instants, the total energy in the system is therefore all in the form of kinetic energy. Thus, the energy loss due to impact $= 1-r$.

Following the concepts of classical analytical dynamics, a coefficient of restitution is defined as

$$e = \dot{\theta}_2/\dot{\theta}_1 \tag{9a}$$

which from equation (8), after noting that $I_0 = \tfrac{4}{3}(W/g)R^2$, can be expressed as

$$e = 1 - \tfrac{3}{2}\sin^2\theta_c \tag{9b}$$

Thus, the energy loss due to impact $= 1-e^2$. The higher the coefficient of restitution, the smaller the energy loss due to impact.

As shown in Figure 3, the coefficient of restitution, e, varies from one for very slender blocks ($\theta_c = 0$, $H/B = \infty$) to zero for blocks with $H/B = 1/\sqrt{2}$. For smaller values of H/B, e is negative indicating that the angular velocity changes sign after impact, resulting in bouncing of the block about the point of rotation prior to impact.

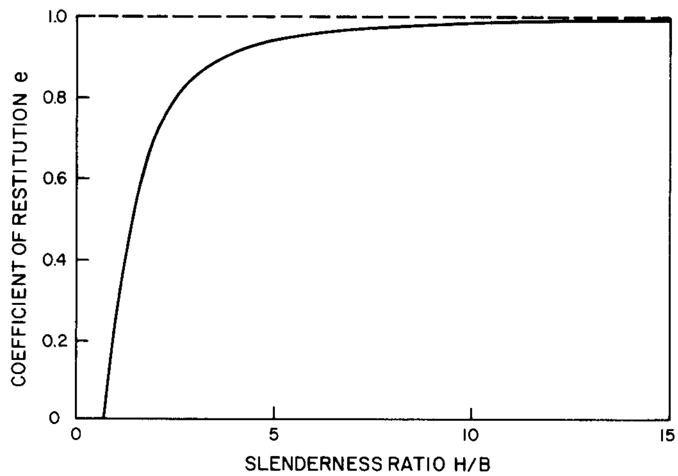

Figure 3. Variation of coefficient of restitution, based on conservation of angular momentum, with slenderness ratio

The coefficient of restitution determined from conservation of angular momentum [equation (9b)] depends only on the slenderness ratio H/B or θ_c; it is independent of both the angular velocity before impact and size of the block. This result is exactly valid for the idealized conditions of rigid block and rigid base but only approximately valid if the idealized conditions do not exist, since the kinetic energy loss depends on the materials of the block and the base. The influence of such variations in the coefficient of restitution on the rocking response of the block is examined later.

APPROXIMATE ANALYSIS OF OVERTURNING

Free vibrations

When the block is rotated about 0 through an angle θ and released, the resulting free vibration of slender blocks is governed by equation (4) with the base accelerations set to zero:

$$I_0\ddot{\theta} + WR(\theta_c - \theta) = 0$$

or

$$\ddot{\theta} - p^2\theta = -p^2\theta_c \tag{10}$$

where

$$p^2 = \frac{WR}{I_0}$$

For a homogeneous rectangular block

$$p^2 = \frac{WR}{\frac{4}{3}(W/g)R^2} = \frac{3g}{4R}$$

and equation (10) is then independent of the weight W of the block.

This equation, subject to the initial conditions $\theta = \theta_0$ and $\dot{\theta} = \dot{\theta}_0$ at $t = 0$, has the solution

$$\theta = \theta_c - (\theta_c - \theta_0)\cosh pt + \frac{\dot{\theta}_0}{p}\sinh pt \tag{11}$$

Equation (11) with $\dot{\theta}_0 = 0$ describes the rotation of the block about the point 0 as it falls back into the vertical position after it is released from rest with initial displacement θ_0. The block will then rotate about 0′ and if there is no energy loss during impact, the block will rotate through an angle $\theta = -\theta_0$. The block will then fall back again to the vertical position and will rise about point 0 until θ is again equal to θ_0. At this instant, one complete cycle of vibration will have been completed. The time T required to complete this cycle is the period of free vibration. It will be equal to the time required for the block to fall from $\theta = \theta_0$ to $\theta = 0$, determined from equation (12), multiplied by four:

$$T = \frac{4}{p}\cosh^{-1}\left(\frac{1}{1-\theta_0/\theta_c}\right) \tag{12}$$

The period of free vibration is strongly and non-linearly dependent on the amplitude ratio θ_0/θ_c (Figure 4) and increases from zero to infinity as the amplitude ratio increases from zero to one.[6]

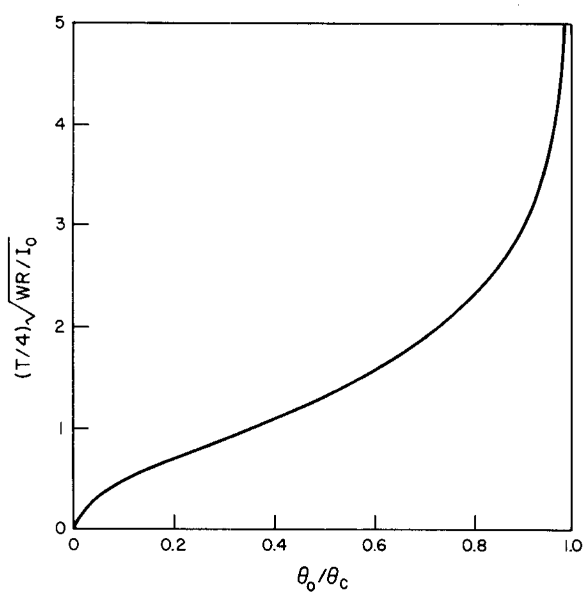

Figure 4. Free vibration period T of block rocking with amplitude θ_0 (Reference 6)

Overturning by single-pulse excitations

Consider a rigid block resting on a base which is subjected to a single pulse of horizontal acceleration. This may be a rectangular pulse with constant acceleration a_{g0} lasting for a time t_1 or acceleration varying as a half-cycle sine-wave pulse of amplitude a_{g0} and duration t_1. Motion of the block is initiated by base acceleration a_g^x if [from equation (2)] $a_{g0}/g > B/H$, or for slender blocks, $a_{g0}/g > \theta_c$. These inequalities specify the acceleration as a fraction of g required to begin tilting the block.

Even if motion is initiated, the block may or may not overturn depending on the magnitude of a_{g0} and the duration t_1. The duration t_1 of a rectangular pulse with acceleration a_{g0} required to overturn the block was determined[6] and given by the following equation:

$$\cosh t_1\sqrt{\left(\frac{WR}{I_0}\right)} = 1 + \left[1\bigg/2\frac{a_{g0}}{g\theta_c}\left(\frac{a_{g0}}{g\theta_c}-1\right)\right] \tag{13}$$

Similarly, the conditions under which a half-cycle sine-pulse of duration t_1 and maximum acceleration a_{g0} will overturn the block are governed by

$$\frac{a_{g0}}{g\theta_c} = \sqrt{\left[1 + \frac{I_0}{WR}\left(\frac{\pi}{t_1}\right)^2\right]} \tag{14}$$

These results were derived starting with the linearized equation (4). Thus, they are valid only for slender blocks. Furthermore, the approach employed[6] in obtaining these results is valid only for single pulse excitations which, if large enough, would overturn the block without rocking back and forth and the associated impacts. Smaller accelerations than are specified by these equations may set the block to rocking but will not overturn it.

Graphs of equations (13) and (14) are presented in Figure 5. In each case, the graph represents the boundary between overturning and stable regions. The acceleration a_{g0} required for overturning is plotted against pt_1 for selected values of the slenderness ratio H/B, and against the critical angle $\theta_c = \cot^{-1}(H/B)$ for selected values

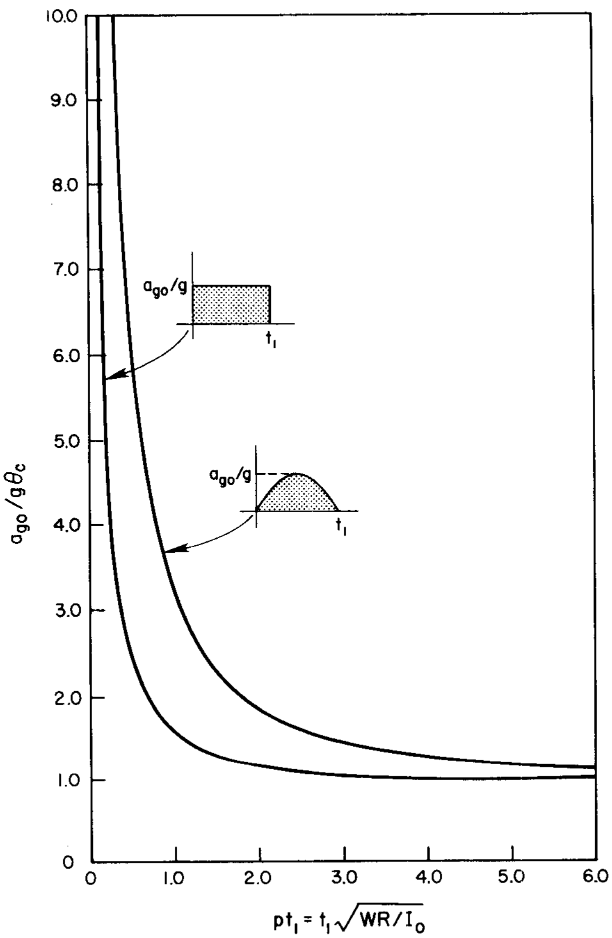

Figure 5. Maximum acceleration a_{g0} of single-pulse required for overturning a block

of pt_1 in Figure 6. For a fixed duration t_1 and size parameter R, a_{g0} decreases as H/B increases. Similarly, for a fixed t_1 and H/B, a_{g0} increases with R. Thus, larger accelerations are required to overturn larger blocks and smaller accelerations are required to overturn relatively slender blocks. The acceleration required to overturn a block depends on the shape of the acceleration pulse; smaller acceleration is required for a rectangular pulse than for a half-cycle sine-pulse of the same duration t_1.

Overturning by earthquake motion

The minimum accelerations required to overturn a block by single acceleration pulses of two types, rectangular and half-sine, are given by equations (13) and (14). Smaller accelerations than specified by these equations may set the block to rocking but will not overturn it. It is, however, possible that a block will

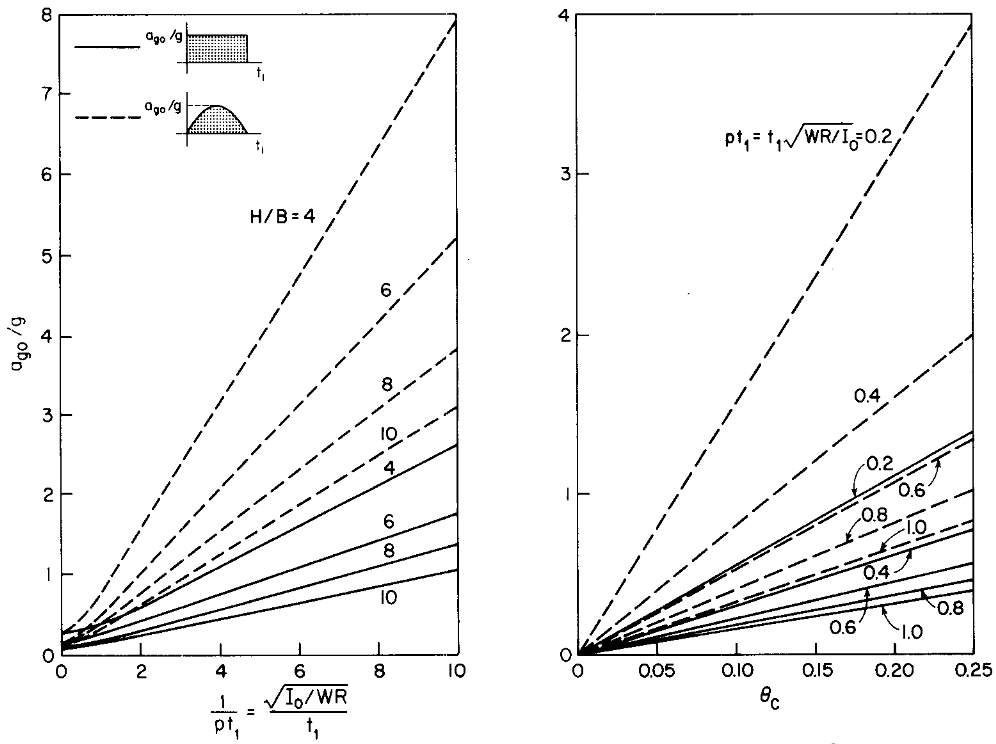

Figure 6. Maximum acceleration a_{g0} of single-pulse required for overturning of blocks

overturn under the action of a sequence of pulses, characteristic of earthquake ground motion, each with accelerations smaller than specified by these equations.

Housner presented an ingenious, approximate analysis of overturning of blocks by earthquake ground motion. The ground motion was idealized as white noise and linearized equations of motion were used. The approximate analysis, based on energy concepts, is too subtle to be summarized here and the interested reader should refer to the original source.[6] One of the principal results was the following: For a given pseudo-velocity spectrum value S_v, a block having a critical angle θ_c given by

$$\theta_c = \frac{S_v}{\sqrt{(gR)}}\sqrt{\frac{WR^2}{gI_0}} \tag{15a}$$

$$= 0{\cdot}866\frac{S_v}{\sqrt{(gR)}} \quad \text{for rectangular blocks} \tag{15b}$$

will have approximately a 50 per cent probability of being overturned. The dimensions of the structure enter only through the length R and the slenderness ratio H/B through the angle θ_c. The earthquake intensity necessary to overturn a block increases with size for a fixed slenderness ratio; increases with θ_c, or decreases with increasing slenderness ratio, for a fixed size R. For a block of fixed size and slenderness ratio, equation (15b) suggests that the probability of overturning increases with intensity of ground motion. Thus, these conclusions, derived earlier for single-pulse excitations, are also valid for idealized earthquake motions but only in a probabilistic sense. Housner[6] also concluded that 'the behaviour of a rocking block could be quite variable in that relatively small ground motion may fortuitously build up the amplitude at the beginning of the ground motion and lead to overturning the block'.

RESPONSE BEHAVIOUR

Analysis procedure

The rocking response of a rigid block to prescribed ground acceleration is determined by numerically solving equations (2) and (3) with the condition for initiation of rocking defined by equation (1) and the impact condition by either the conservation of angular momentum [equation (6)] or a specified coefficient of restitution [equation (9a)]. Typical values for the time-step used in the numerical integration of the governing equations were $\Delta t = 1/400$ s. The sign of a_g^x at the time-instant when equation (1) is satisfied; i.e. motion is initiated, determines whether equation (2) or (3) is to be used for the next time-step. Equation (3) is used for positive a_g^x and equation (2) for negative a_g^x. The same equation is used for subsequent time steps until the sign of rotation θ changes. When such a change in sign occurs, say during time step t_j to t_{j+1}, this time step is divided into two parts. Using the equation of motion valid at t_j, the exact time $\bar{t}$, $t_j < \bar{t} < t_{j+1}$, at which θ becomes zero is determined by an iterative procedure. Just before the impact $\theta(\bar{t}-) = 0$ and velocity is $\dot{\theta}(\bar{t}-)$. Immediately after impact $\theta(\bar{t}+) = 0$ and $\dot{\theta}(\bar{t}+) = e\dot{\theta}(\bar{t}-)$ where e is the coefficient of restitution which either has a specified value [equation (9a)] or is determined by conservation of angular momentum [equation (9b)]. With these initial conditions, the equation of motion, other than the one valid at $t = t_j$, is solved over the time interval $\bar{t}$ to t_{j+1} and subsequent time intervals until the rotation angle θ again changes sign. The above-mentioned process is then repeated.

Using a fourth-order Runge–Kutta integration scheme, the numerical solution procedure summarized above was implemented in a computer program. The accuracy of the computer program has been checked by comparing its results with analytical results for single-pulse excitations[6] and with experimental results from shaking table experiments using earthquake-type excitations.[7] It was observed that in order to obtain accurate results it is necessary to use a high-order integration scheme, such as the one employed here, along with a very short time-step; the typical value used = 1/400 s.

Simulated ground motions

Simulated motions were developed to model the properties of the following ground motions recorded on firm ground in the region of strong shaking during earthquakes of magnitude 6·5 to 7·5:

El Centro, California; 30 December 1934
El Centro, California; 18 May 1940
Olympia, Washington; 13 April 1949
Taft, California; 21 July 1952

These recorded accelerograms are presented in Figure 7.

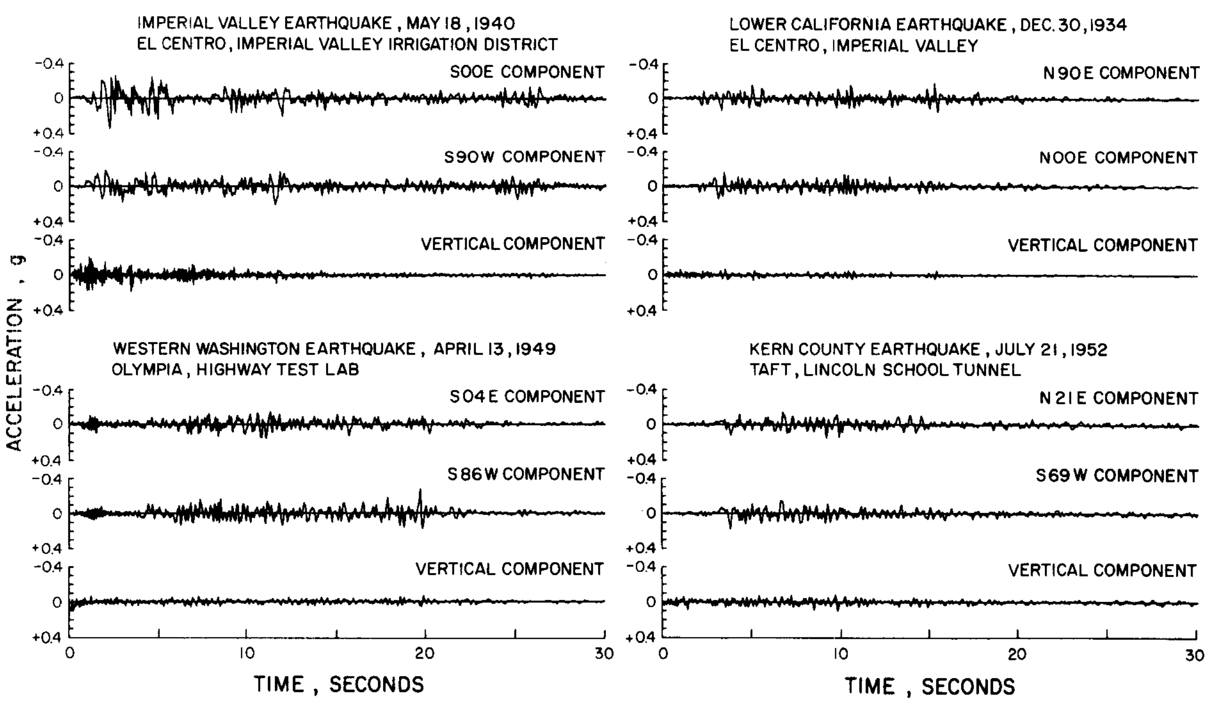

Figure 7. Ground accelerations recorded during four earthquakes

The simulation procedure adopted herein is essentially identical to earlier studies.[8,9] It consisted of generating samples of stationary Gaussian white noise; multiplying the white noise by an intensity function of time (Figure 8) to represent a segment of strong shaking at constant intensity preceded by a quadratic build-up of intensity and followed by an exponential decay in intensity; passing the resulting function through a second order linear filter to impart the desired frequency content, as indicated by the spectral density (Figure 8), and finally performing a baseline correction on the filtered function.

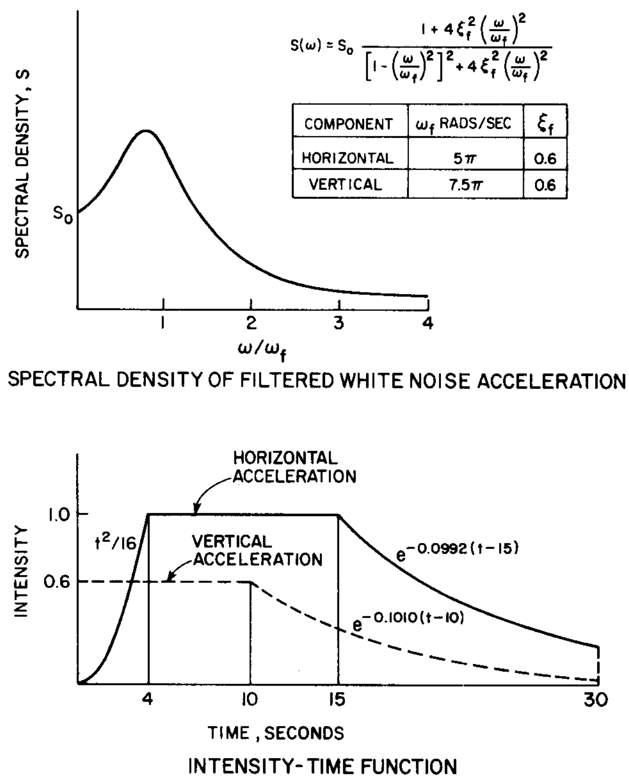

Figure 8. Spectral density and intensity–time functions for simulated (horizontal and vertical) ground accelerations

In an earlier study,[8] the parameters for the intensity function and the filter (frequency = 2·5 cps and damping ratio = 60 per cent) were estimated from the properties of the horizontal components of the recorded motions. Similar parameter values (Figure 8) were employed in simulating the horizontal ground motions for this investigation. Parameter values for simulating vertical ground motions have apparently not been reported in the literature. They were therefore estimated from the properties of the vertical component of recorded motions using the procedures of Reference 8. Figure 8 shows the parameters for the intensity function and for the filter (frequency = 3·75 cps and damping ratio = 60 per cent).

Using the above-mentioned random process model and parameter values, two sets of twenty simulated ground motions were generated to model the properties of the horizontal and vertical components of the recorded motions, respectively. The simulated horizontal motions were all scaled by the same factor such that the ensemble average of the peak acceleration would be 1 g. Similarly, the simulated vertical motions were all scaled such that the ensemble average of the peak acceleration would be 0·6 g. The resulting motions are presented in Figures 9 and 10.

Influence of system parameters

Using the computer program mentioned earlier, the response of several rigid blocks to the same simulated ground motion was determined. The time–variation of the rotation of the block is presented in Figure 11. In those cases where the rotation θ continues to increase significantly beyond θ_c, the block overturns. It is seen

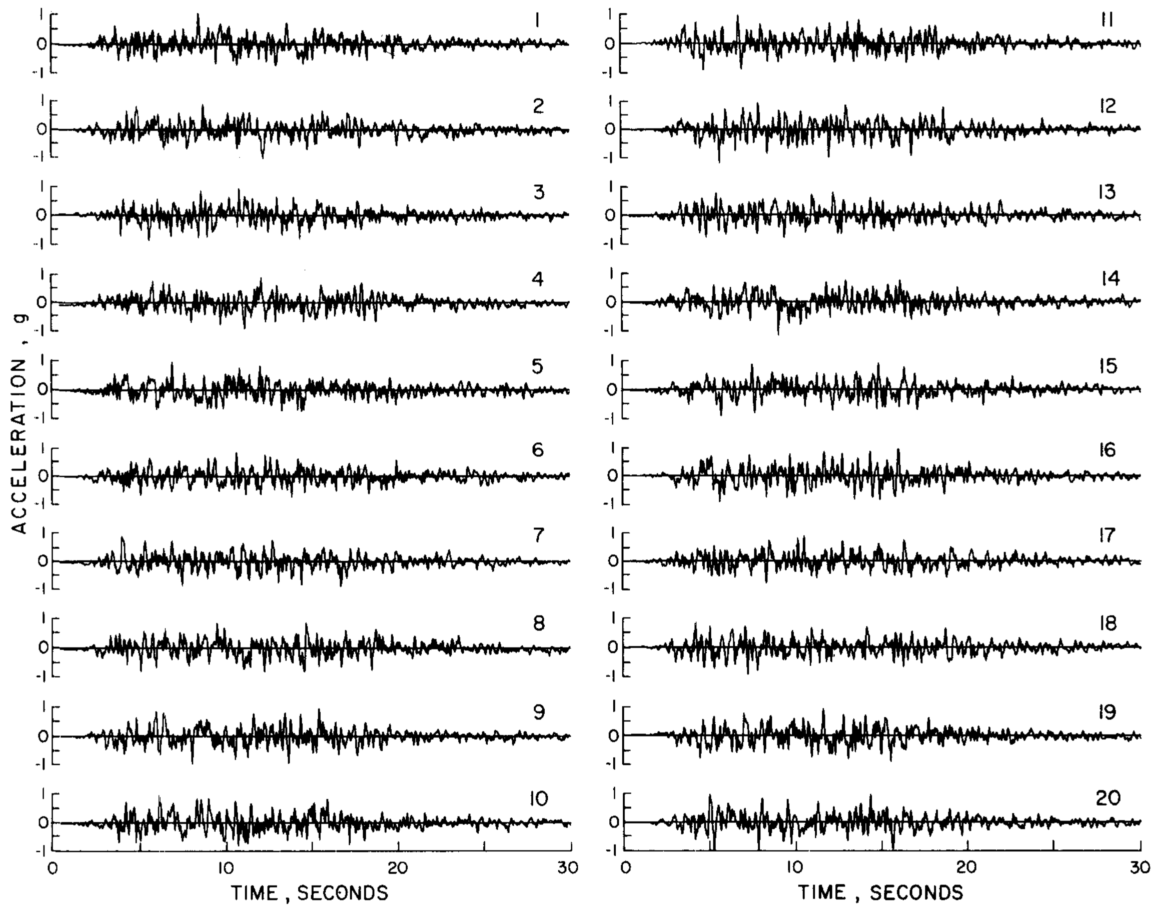

Figure 9. Simulated horizontal ground accelerations with ensemble average of peak acceleration = 1 g

from this figure that the rocking response of the block can be very sensitive to small changes in the system parameters. Small variations in the slenderness ratio H/B and size parameter R lead to large changes in the response. In contrast to the conclusions from single-pulse excitations, stability of a block does not necessarily increase monotonically with increasing size or decreasing slenderness ratio. Similarly, contrary to what intuition would suggest, decrease in the value of the coefficient of restitution—which implies increase in energy dissipation—does not necessarily lead to smaller response of the block.

Influence of ground motion properties

The ground motion assumed in Housner's approximate analysis consisting of a single-pulse ground acceleration, followed by a constant velocity of the ground, is unrealistic for earthquakes. The ground motion shown in Figure 12, consisting of three acceleration pulses, although much simpler than earthquake ground motion, is useful for studies of structural response.[10] The response of several blocks, all having the same size parameter R but varying slenderness ratio H/B, to this simpler ground motion is determined by the computer program. From the results presented in Figure 12, it is seen that response of two blocks with slightly different slenderness ratios can be considerably different. However, the response and overturning tendency of the block increase as the slenderness ratio increases. Thus a consistent trend in the change of response with the change of slenderness ratio is noticed.

The horizontal ground motions of Figure 9 are all members of a random process, defined in the average sense by a power spectral density and an intensity–time function. The variations from one simulated motion to the next are intended to represent the probabilistic variations expected in ground motions recorded under

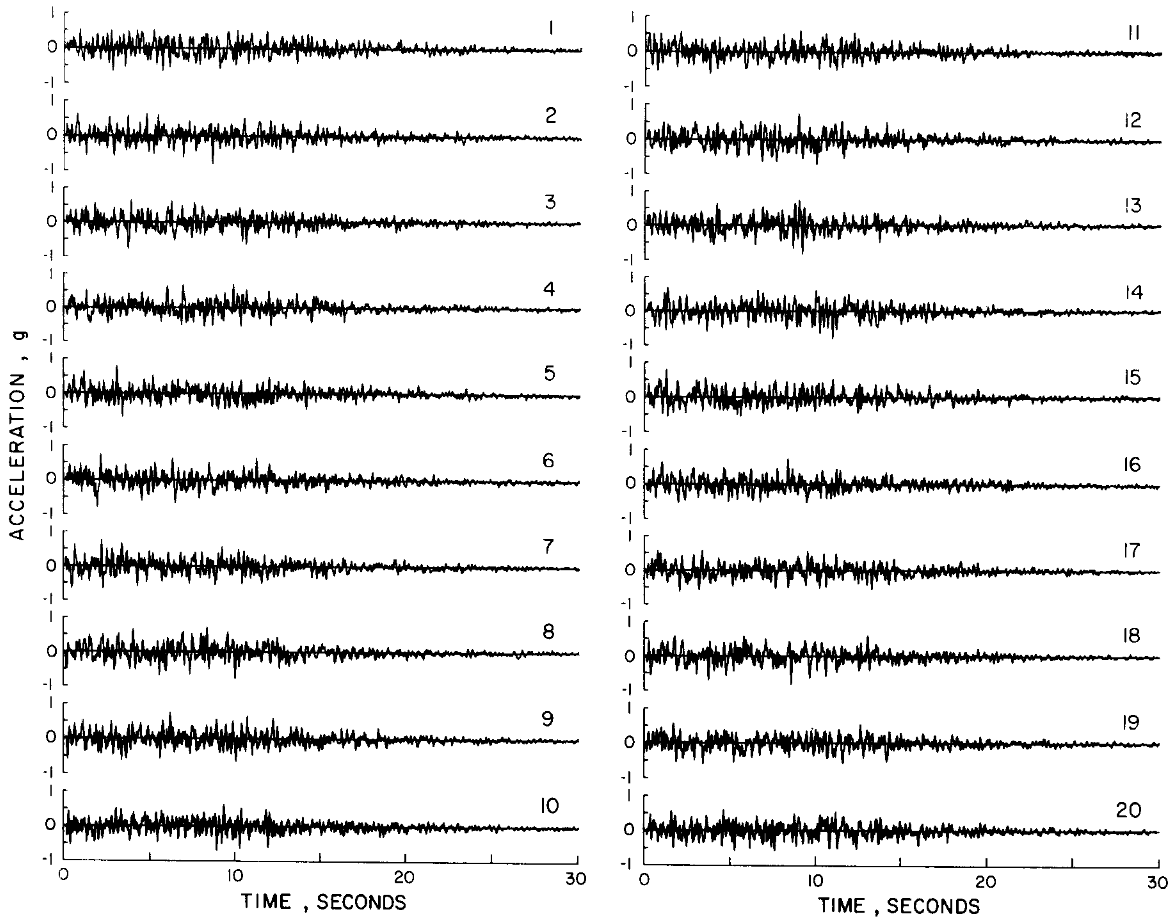

Figure 10. Simulated vertical ground accelerations with ensemble average of peak acceleration = 0·6 g

seemingly identical conditions. Using the computer program, the response of a single rigid block to several of the ground motions of Figure 9, multiplied by 0·4 to represent an ensemble with average maximum acceleration = 0·4 g, was determined.

The rotation of the same block due to each earthquake is presented in Figure 13(a) and the maximum rotation due to earthquake j is $\theta_{j,\max}$. It is seen that $\theta_{j,\max}$ varies with j over the entire range of possible values, from a small fraction of the critical angle to values in excess of the critical angle indicating overturning of the block. Thus, the rocking response of the block is extremely sensitive to the detailed characteristics of the ground motion. Such sensitivity was observed in experiments on the Berkeley shaking table where repeated measurements of the response of a block subjected to a prescribed table acceleration differed significantly from one another. The differences were apparently because, even with the same prescribed horizontal accelerations, the table undergoes slightly different pitching motions.[7]

Consider the simulated ground motion No. 17 (Figure 9) but scaled by several factors 0·35, 0·40, 0·45, 0·50, 0·55 and 0·6. The result is a family of ground motions, with all properties identical, except the intensity. Using the computer program the response of a single rigid block to this family of ground motions was determined. The results presented in Figure 13(b) indicate that overturning of a block by ground motion of a particular intensity does not imply the block will necessarily overturn under more intense ground motion. In contrast, if a single-pulse excitation with some maximum acceleration a_{g0} and duration t_1 is necessary to overturn a block of a particular size and slenderness ratio, the block will also overturn under a similar pulse but with larger acceleration.

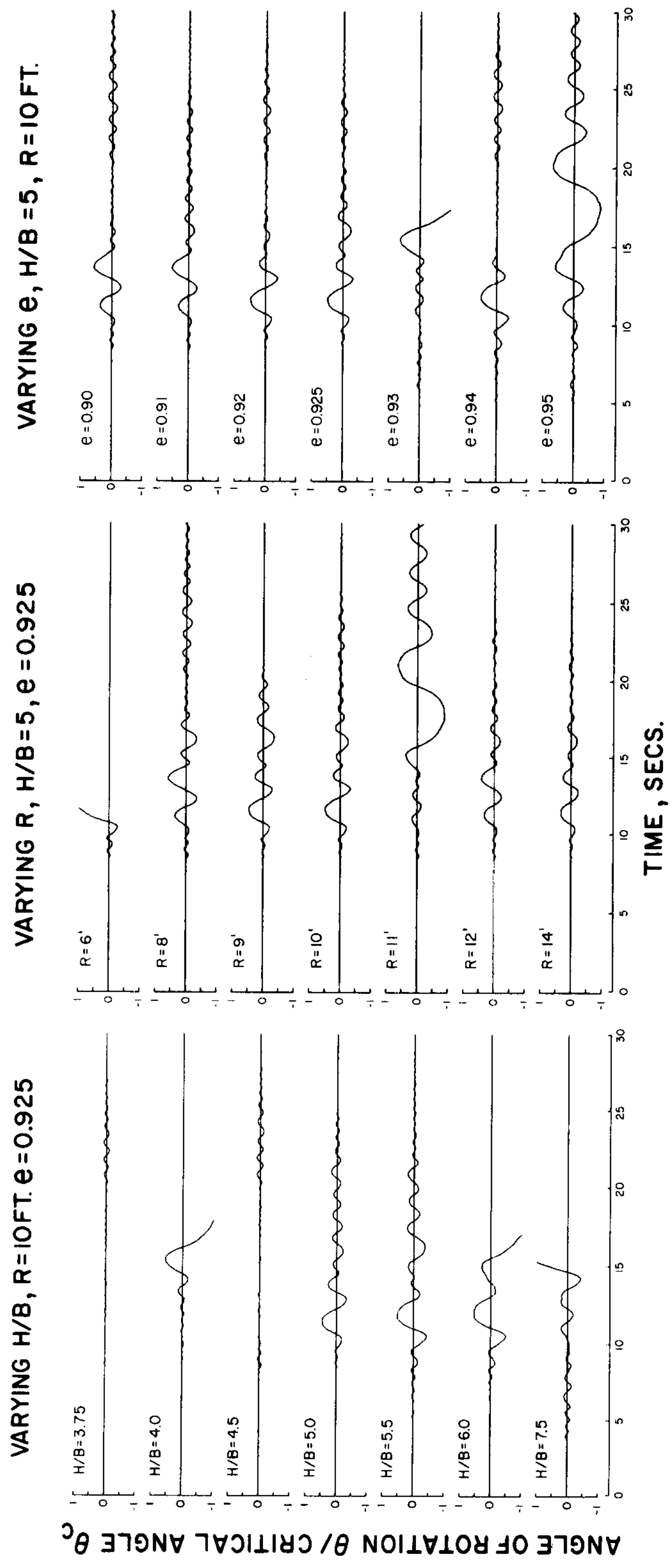

Figure 11. Response of several blocks to the same simulated motion. Parameters: slenderness ratio H/B, size R and coefficient of restitution are varied one at a time

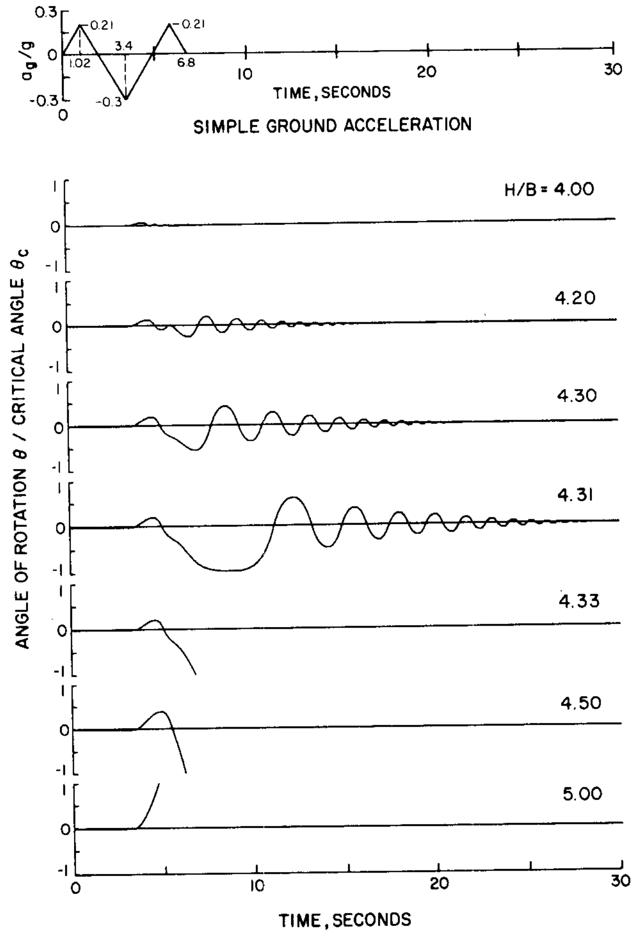

Figure 12. Response of blocks of varying slenderness ratio H/B to the simple ground acceleration shown. Size parameter $R = 10$ ft and coefficient of restitution $e = 0{\cdot}925$ for all blocks

Figure 14 shows the response of a single rigid block to four simulated ground motions in Figure 9, all scaled by 0·4 to represent an ensemble with an average maximum acceleration = 0·4 g, acting in the horizontal direction. Figure 14 also shows the block response when in each case the excitation includes a simulated ground motion from Figure 10, scaled by 0·4 to represent a member of an ensemble with average maximum acceleration = 0·24 g, acting in the vertical direction. The vertical ground motion may overturn a block which is stable under the action of horizontal ground motion alone (case 1), stabilize a block which overturns due to horizontal ground motion alone (case 2), delay the time of overturning of a block (case 3) or greatly reduce the rocking of a block (case 4). Thus, the influence of vertical ground motion is apparently not systematic.

Response sensitivity

Experiments on the Berkeley shaking table indicated that the rocking response of a rigid block can be extremely sensitive to ground motion details. Repeated measurements of the response of a block subjected to prescribed table accelerations were not identical.[7] These differences were apparently due to the fact that the table undergoes slightly different pitching motions, even with the same prescribed horizontal accelerations. Thus, the sensitivity of the response to small changes in the system parameters or ground motions, revealed by the analytical results presented earlier, may be surprising but it is real. It is relevant to note that lightly damped rigid-plastic systems also display considerable response sensitivity[11] and the properties of a rocking block (Figure 2) are reminiscent of these systems except that the behaviour of a rock block is not hysteretic and its

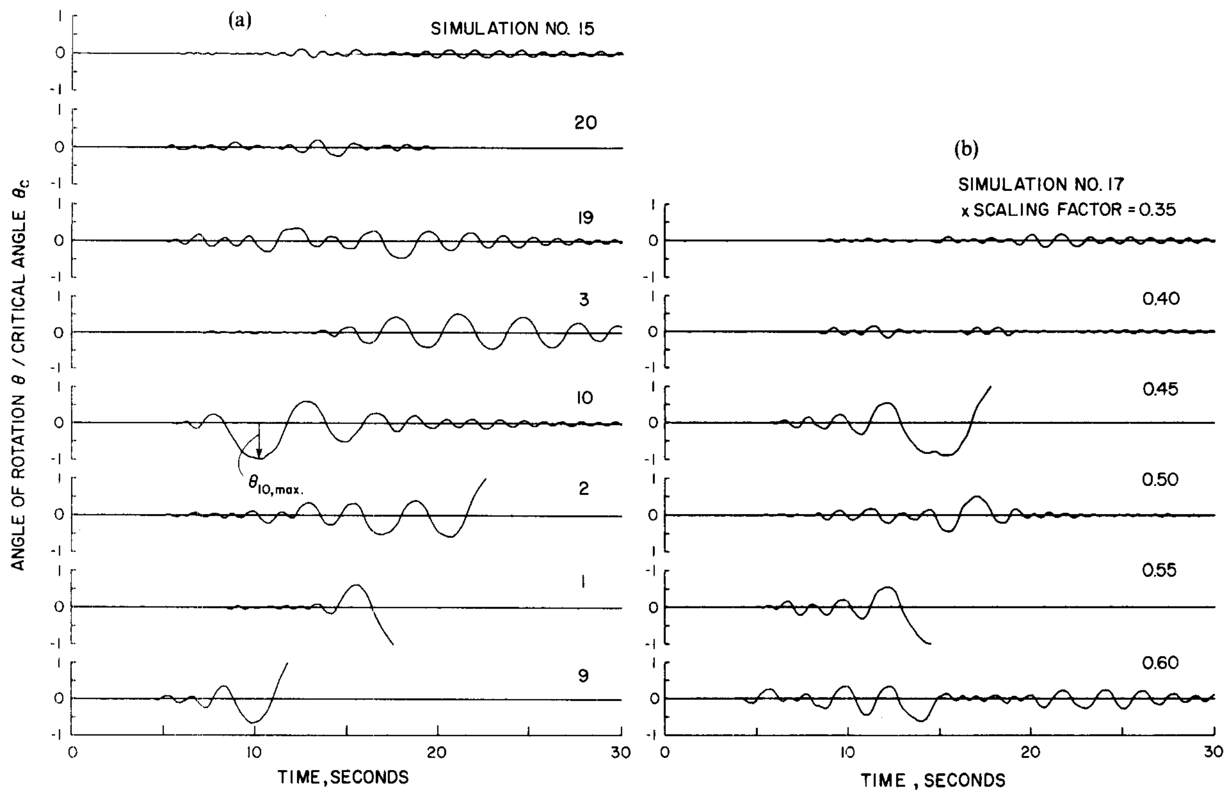

Figure 13(a). Response of a block, $R = 12$ ft and $H/B = 5$, to several simulated motions from an ensemble with average peak acceleration = 0·4 g

Figure 13(b). Response of a block, $R = 10$ ft and $H/B = 5$, to simulated motion No. 17 (Figure 9) scaled by factors as shown

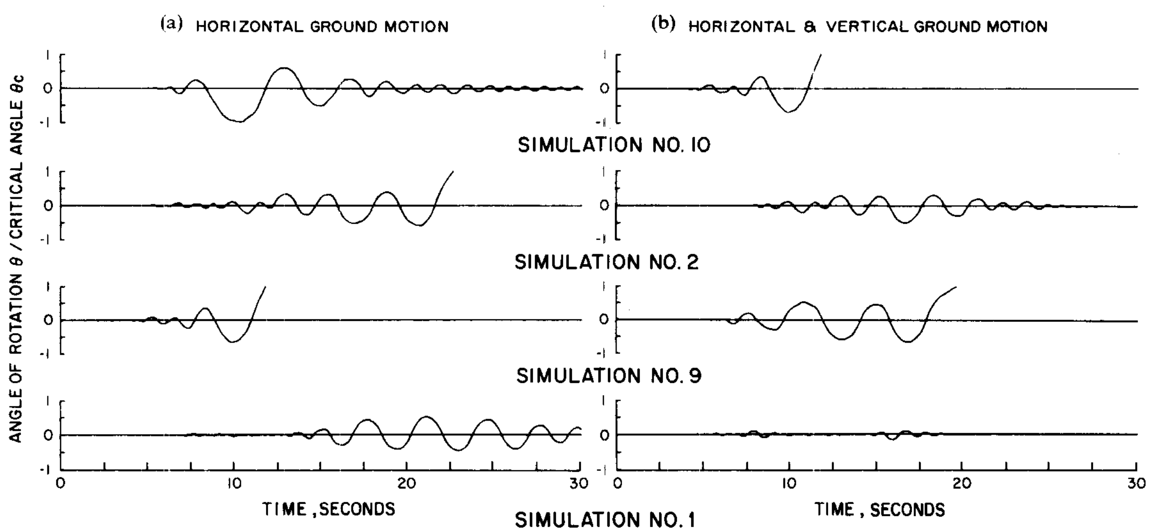

Figure 14. Response of a block with $R = 12$ ft and $H/B = 5$ to simulated ground motions. The response to (a) horizontal ground motion and (b) horizontal and vertical ground motions is presented for each situation

second slope is negative. As the amplitude of rocking approaches incipient overturning the block is effectively undamped.[6] The effective damping for a block rocking at large amplitudes is therefore small.

Although sensitivity of the rocking response of a rigid block is most obvious when it is subjected to irregular earthquake ground motion, it is present even in the case of single-pulse excitations. In the range of small values for the parameter pt_1, small changes in the pulse duration t_1 or size parameter R could shift a block from stable to unstable region or vice versa (Figure 5). Similarly, in the range of large pt_1, small changes in the acceleration a_{g0} or slenderness parameter θ_c could affect the stability of a block (Figure 5). Given especially favourable circumstances, a small change in the initial conditions could produce a large change in the response of the block. As seen in Figure 15, by changing the initial value of normalized rotation from 0 to 0·01, the maximum normalized rotation response increases from approximately 0·5 to more than 1, resulting in overturning of the block. Small changes in the displacement and velocity at a particular time can grow exponentially [equation (11)] provided the block continues to tip in one direction for an extended duration (see Figure 15).

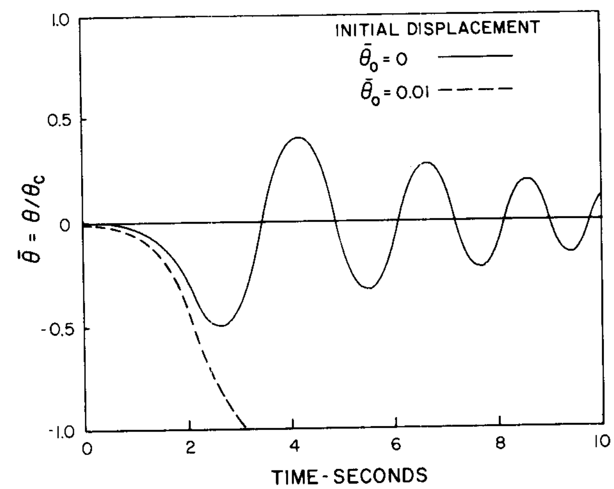

Figure 15. Response of a block (R = 10 ft, H/B = 4) to a rectangular pulse of acceleration = 0·26 g and duration = 2·1 s

The extreme sensitivity of the response of a rocking block to various factors can best be explained by examining the change in potential energy of a block as it rotates from an angle θ to incipient overturning, $\theta = \theta_c$:

$$V(\theta) = WR[1-\cos(\theta_c - \theta)] \tag{16}$$

The plot of equation (16) in Figure 16 shows how the additional energy required to overturn a block varies with initial rotation θ. It is apparent that very little additional energy is needed to overturn a block beyond the larger values of θ. Small changes in the system parameters—size parameter R, slenderness ratio H/B, coefficient of restitution e—may influence the energy input to the system slightly but sufficiently to overturn a block once it is rotated beyond a certain angle. Similarly, once significant rocking of the block has developed during earthquake motion, seemingly small differences in the details of subsequent ground motion could greatly affect the response of the block. If the subsequent motion inputs additional energy, although small, at the right time it could be sufficient to overturn a block.

RESPONSE EXCEEDANCE AND OVERTURNING PROBABILITIES

The influence of system parameters and ground motion properties in the rocking response of rigid blocks is studied next from a probabilistic point of view. Using the computer program mentioned earlier, the response of several rigid blocks to earthquake ground motion was analysed. Determined for each block was the time–history of rotation $\theta(t)$ and the maximum normalized rotation $\bar{\theta}_{max} = \theta_{max}/\theta_c$ due to each of the twenty

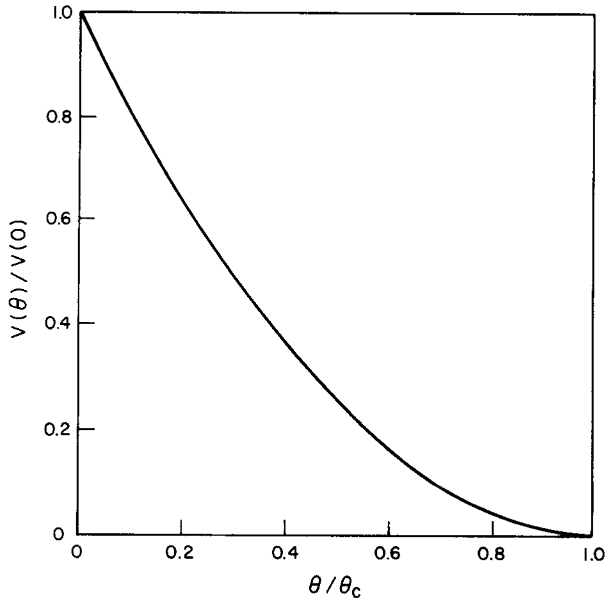

Figure 16. Additional energy $V(\theta)$ required to overturn (under static conditions) a block with initial rotation θ. The graph depends on θ_c but only slightly

horizontal ground motions (Figure 9), scaled by an appropriate factor α. Thus, for each block, twenty values of $\bar{\theta}_{max}$ were obtained corresponding to the twenty excitations with average value of peak acceleration $= \alpha g$. The twenty values of $\bar{\theta}_{max}$ were arranged in ascending order and plotted in the form of a cumulative probability distribution function (CDF).

Such plots for a block with size parameter $R = 10$ ft and slenderness ratio $H/B = 5$ are presented in Figure 17 for various values of the coefficient of restitution e. The influence of e on the response appears only at each impact; hence the effect is small. Thus the CDF for various values of e are clustered together with considerable overlapping of curves. From a probabilistic point of view, the coefficient of restitution influences the response in no obviously systematic way and, as will be seen later, to a much lesser degree than the other system parameters. Thus, the coefficient of restitution is not varied in the subsequent results. Its value is defined by equation (9b), based on conservation of angular momentum, depending only on the slenderness ratio.

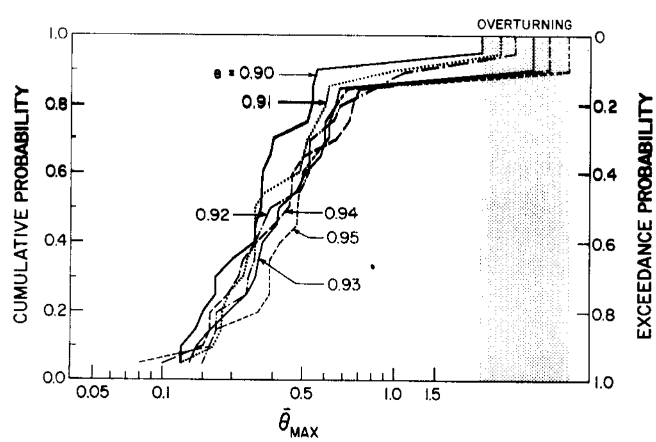

Figure 17. Cumulative probability distribution functions for maximum rotation of a block, $R = 10$ ft and $H/B = 5$, for various values of the coefficient of restitution e. Ensemble average of peak horizontal ground acceleration $= 0{\cdot}4\,g$

The influence of intensity of ground motion on the cumulative distribution functions for the response is examined in Figure 18. Four ensembles of ground motions with average peak acceleration = 0·3 g, 0·4 g, 0·5 g and 0·6 g are derived by appropriately scaling the ensemble of Figure 9. Presented in Figure 18 are the CDF for the response of a block of fixed size and slenderness ratio corresponding to the four ground motion ensembles. It is apparent that the probability of the block exceeding any response level increases with ground motion intensity.

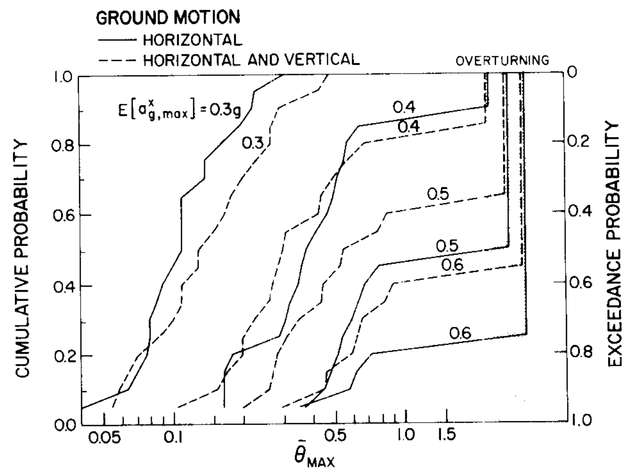

Figure 18. Cumulative probability distribution functions for maximum rotation of a block, $R = 10$ ft and $H/B = 5$. Ensemble average of peak accleration is varied: 0·3 g, 0·4 g, 0·5 g and 0·6 g for horizontal ground motion; 0·18 g, 0·24 g, 0·30 g and 0·36 g for vertical ground motion

Presented in Figure 19 are the cumulative probability distribution functions for several blocks subjected to the ensemble of horizontal ground motions with average peak acceleration = 0·4 g. In one case, all blocks have the same size parameter but varying slenderness ratio. In the other, the slenderness ratio is fixed but the size is varied. It is seen that the rotational response of a rigid block is extremely sensitive to the details of the ground motion as noted earlier from Figure 13(a). For example, $\bar{\theta}_{max}$ for a block with $R = 10$ ft and $H/B = 4{\cdot}0$, when subjected to the twenty simulated motions, varies from as little as 0·01 to more than 1, indicating overturning. It is seen that the CDF is influenced by the two parameters in a systematic manner, with the exceedance probability for any level of response increasing with increase in slenderness ratio and decrease in size.

It is seen from Figures 18 and 19 that a block may overturn when subjected to some members of an ensemble of ground motions but remain stable under the action of other members of the same ensemble. The probability that a block will overturn when subjected to ground motion of specified intensity is roughly estimated as the fractional number of ground motions that overturn the block. The influence of the system parameters and ground motion intensity on the overturning probability is presented in Figures 20 and 21. Each point on these plots is the result of analysing the responses of a block to an ensemble of twenty ground motions. It is seen that the probability that a block overturns increases with increasing slenderness ratio for a fixed size parameter and ground motion intensity (Figures 20 and 21), decreases with increase in size for a fixed slenderness ratio and ground motion intensity (Figures 20 and 21) and increases with ground motion intensity for a fixed size parameter and slenderness ratio (Figure 20).

The numerical results generally follow these trends but not without exception. Note, for example, the following cases which violate the general trends: The overturning probability is the same for blocks with $R = 20$ ft as H/B increases from 9 to 10, for blocks with $R = 30$ ft as H/B increases from 7 to 8, for blocks with $R = 40$ ft as H/B increases from 8 to 9, for blocks with $R = 6$ ft as ground motion intensity increases from 0·45 g to 0·50 g, for blocks with $R = 20$ ft as ground motion intensity increases from 0·40 g to 0·45 g, for blocks with $H/B = 4$ as ground motion increases from 0·40 g to 0·45 g. In one case, the overturning probability for a particular block decreases as the intensity increases (Figure 20). Some of these apparent inconsistencies may be the consequence of the small sample size of 20 and are expected to disappear when a larger sample is used.

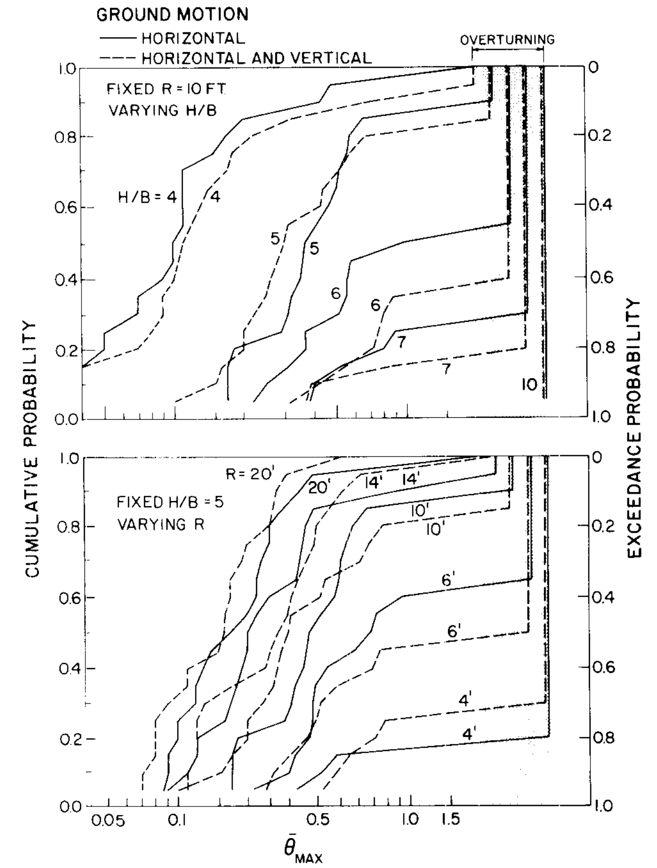

Figure 19. Cumulative probability distribution functions for maximum rotation of blocks subjected to ensembles of horizontal and vertical ground motions, with average values of peak acceleration = 0·4 g and 0·24 g, respectively

Thus, usually, the larger of two blocks having the same slenderness ratio is more stable in the sense that it is less likely to overturn; the more slender of two blocks having the same size parameters is less stable and a block is more likely to overturn if it is subjected to more intense ground motion.

Results of analyses of several blocks subjected to ensembles of horizontal ground motion with selected intensity were presented and interpreted above. These analyses were repeated with simultaneous application of horizontal and vertical ground motions, with similarly numbered members of the two ensembles paired together (Figures 9 and 10). Both ensembles were scaled by the same factor α. Results of these analyses are superimposed on Figures 17–21, summarizing the earlier results.

It is seen that the CDF for the maximum rotation of a block is influenced by vertical ground motion but not in a systematic manner; the exceedance probability is decreased at some response levels but increased at other levels. Similarly, the probability that a block will overturn may increase or decrease by including vertical ground motions. The trends noted earlier regarding the influence of size and slenderness ratio of a block and of ground motion intensity on the stability of a block are still apparent when vertical ground motion is included. However, there are more exceptions to the trends and some of them are larger in magnitude when vertical ground motion is included.

ESTIMATION OF GROUND MOTION INTENSITY

As mentioned in the beginning of this paper, by idealizing the ground motion as an instantaneous impulse or representing its effects by a static horizontal force, formulas were derived to calculate the ground acceleration

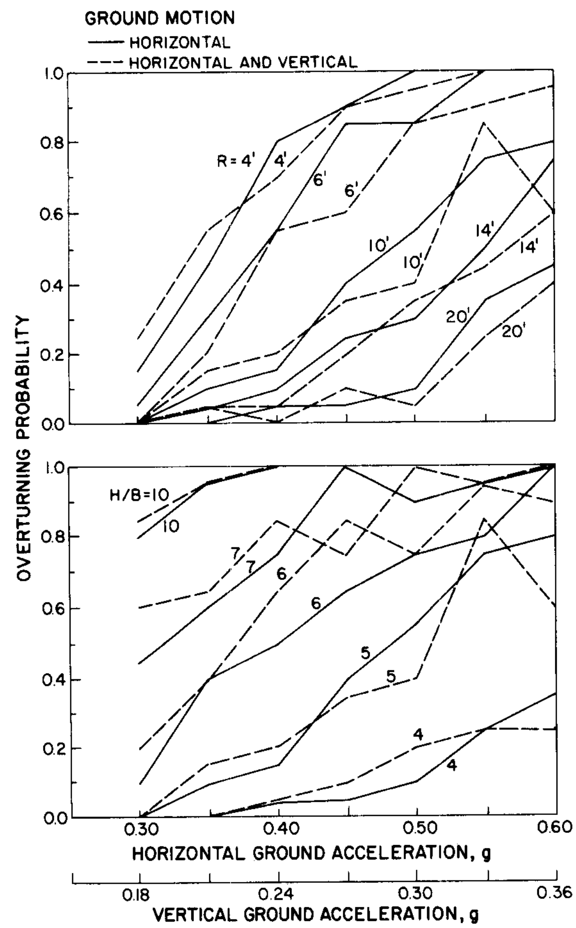

Figure 20. Overturning probability for blocks subjected to ensembles of horizontal and vertical ground motions, with average values of peak acceleration varied as shown

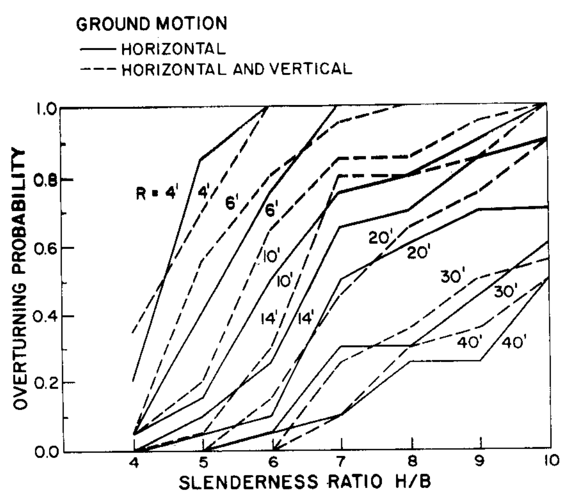

Figure 21. Overturning probability for blocks subjected to ensembles of horizontal and vertical ground motions, with average values of peak acceleration = 0·4 g and 0·24 g, respectively

necessary to overturn a rigid block of given dimensions.[1-4] These formulas were employed to estimate the accelerations in the epicentral region of the 1975 Ohita earthquake in Japan.[5] However, they are unreliable because they do not consider rocking of the block and the associated impacts that would occur during an earthquake. More recently, response procedures which consider the rocking block to be a linear system have been proposed to estimate the intensity of ground shaking from its observed effects on rigid objects.[12] This procedure is also not promising because the behaviour of a rocking block (Figure 2) has no resemblance to that of a linear system. The remaining question is: Do the non-linear analyses of system dynamics presented in this paper provide results that are useful in estimating ground motion intensity?

Consider first an idealized situation: A large number of tombstones, benches, monumental columns and similar objects, which may be idealized as rigid blocks, are concentrated in a small area and it is assumed that they are subjected to identical ground motion during an earthquake. Several of these blocks have the same size parameter R and slenderness ratio H/B and the entire collection covers a wide range of R and H/B. Dynamic analysis of a rigid block—with specific R and H/B, on a rigid base, subjected to a specified ground motion—would predict that the block is either stable or that it overturns. This conclusion would apply to all blocks with the same R and H/B. Thus, dynamic analyses of an idealized block of a particular R and H/B provide the results for all blocks with the same parameters. However, the rocking response of the block can be very sensitive to small changes in the system parameters and stability of the block does not necessarily increase monotonically with increasing size or decreasing slenderness ratio (Figure 11). Furthermore, overturning of a block by ground motion of a particular intensity does not imply that the block will necessarily overturn under more intense ground motion [Figure 13(b)]. Thus, reliable predictions of intensity of ground shaking would not be possible from analytical predictions of response of blocks to a ground motion specified deterministically. As a corollary, reliable estimates of the intensity of ground shaking would not be possible from its observed effects on the instrument proposed many years ago for seismological observatories, consisting of a family of columns, all with rectangular section but varying in slenderness ratio.

Contrary to the analytical prediction that blocks with the same R and H/B either are all stable or all overturn when subjected to a specified ground motion, in reality some may overturn but others may remain standing during an earthquake. Identical sized blocks would perform differently because their response is sensitive to the contact conditions between the base of the block and the ground.[7] An overturning probability for a particular R and H/B could be calculated as the fractional number of blocks with the same R and H/B that overturned during the earthquake. However, this is not the same as the overturning probability, presented in the preceding section, which reflects probabilistic variations in ground motion of a specified intensity.

Consider a second situation: A large collection of objects, which may be idealized as rigid blocks, are distributed over an area small enough that the intensity of ground motion is essentially the same but large enough that the ground motions experienced by the various blocks are at most weakly correlated. The collection includes several objects having the same dimensions (R and H/B). A wide range of values for the parameters R and H/B are represented in the collection. The ensemble of ground motions experienced by the various objects may be interpreted as members of a random process. From a survey of the area after an earthquake, the fractional number of blocks, all with the same R and H/B, that overturned could be readily calculated. The ground motion intensity may be estimated from such information along with the results presented in Figure 20, provided the random process model selected in this paper is appropriate for the particular earthquake. The estimates obtained from the above analysis would differ depending on the particular R and H/B considered. The average of the several estimates thus obtained may be considered as the best estimate for the intensity of the ground motion. Precise estimates should not be expected from this approach because: the random process model selected in this paper may not be appropriate for the ground motion during the particular earthquake; the ground motions experienced by various blocks may not be statistically independent and the behaviour of the block is very sensitive to the conditions of the contact between its base and the ground. However, probabilistic descriptions of the ground motion intensity, e.g. $P[I_1 < \text{intensity} < I_2] = p$, should be possible.

Finally, consider the situation outlined in the beginning of this paper: monuments, minarets, tombstones and other similar objects have experienced many earthquakes in seismic areas of the world which were the centres of the ancient Roman, Greek, Chinese and Indian civilizations. Whether a particular object remained standing or overturned during a given earthquake would not provide sufficient information to make reliable estimates of the intensity of ground shaking during that earthquake, because the response of a rigid block to several ground motions, all of the same intensity but differing in details, could vary from essentially no rotation to large rotations, indicating overturning [Figure 13(a) and 19]. If an historical earthquake has affected several objects with the same geometry and dimensions, and it can be assumed that the various objects experienced ground motions of essentially the same intensity but differing in details, then the situation would be similar to the second situation considered above. The ground motion intensity could then be described on a probabilistic basis as outlined above. Special care is necessary in estimating an upper bound for the intensity of ground motion from the information that an ancient monument has survived several historical earthquakes, because the survival probability depends not only upon the most intense ground motion at the site during a single earthquake, but also on the unknown intensities of all earthquakes experienced by the monument.

Given suitable data in sufficient quantities regarding the effects of an earthquake on monuments, minarets, tombstones and other similar objects in an area, whether they overturned or remained standing, probabilistic estimates of the ground motion intensity in the area should be possible. Such estimates must be based on results of a probabilistic study of the type presented in this paper, which considers the non-linear dynamic behaviour of the rocking block system. Because the response of a rocking block is extremely sensitive to variations in system parameters and ground motion details, a high degree of precision or confidence would not be possible in these estimates. (A similar conclusion was obtained from analytical study of the response of flexible structures.[13]) For the same reasons, estimates of the intensity of ground shaking by deterministic methods, which have been used in the past and which consider observed effects on a single object, would be totally unreliable.

CONCLUSION

The principal conclusions of this study concerned with dynamics of rigid blocks rocking on a rigid base may be summarized as follows.

The rocking response of a block is very sensitive to small changes in its size and slenderness ratio and to the details of the ground motion. The stability of a block subjected to a particular ground motion does not necessarily increase monotonically with increasing size or decreasing slenderness ratio. Overturning of a block by a ground motion of particular intensity does not imply that the block will necessarily overturn under the action of more intense ground motion. Vertical ground motion significantly affects the rocking response of a rigid block, although in no apparently systematic way.

In contrast, systematic trends are observed when the rocking response of rigid blocks is studied from a probabilistic point of view with the ground motion modelled as a random process. The probability of a block exceeding any response level, as well as the probability that a block overturns, increases with increase in ground motion intensity, with increase in slenderness ratio of the block and with decrease in its size.

Probabilistic estimates of the intensity of ground shaking may be obtained from its observed effects on monuments, minarets, tombstones and other similar objects provided suitable data in sufficient quantity is available and the estimates are based on probabilistic analyses of the rocking response of rigid blocks, considering their non-linear dynamic behaviour. These estimates will not be precise because the response of a rocking block is extremely sensitive to variations in system parameters, contact conditions between the base of the block and the ground and ground motion details. For the same reasons, deterministic estimates of intensity of ground motion from its observed effects on a single object would be totally unreliable.

APPENDIX

Notation

The following symbols have been used in this paper:

a_{g0} maximum acceleration of a single-pulse excitation

a_g^x ground acceleration in the horizontal (x) direction

a_g^y ground acceleration in the vertical (y) direction

B width of the block
e coefficient of restitution
g acceleration of gravity
H height of the block
I_0 mass moment of inertia of the block about centres of rotation
$p = \sqrt{(WR/I_0)}$
R size parameter $= \sqrt{[(B^2+H^2)/4]}$
r ratio of kinetic energy quantities after and before impact
S_v constant ordinate of the undamped pseudo-velocity response spectrum for white noise
T period of free vibration of the block
t_1 duration of a single-pulse excitation
θ angle of rotation measured from the vertical
θ_c critical angle $= \cot^{-1}(H/B)$
θ_0 initial angle of rotation
$\dot{\theta}_0$ initial rotational velocity
$\dot{\theta}_1, \dot{\theta}_2$ angular velocities before and after impact
$\theta_{max} = \max_t |\theta(t)|$
$\bar{\theta} = \theta/\theta_c$
$\bar{\theta}_{max} = \theta_{max}/\theta_c$
$V(\theta)$ change in potential energy of the block as it rotates from an angle θ to incipient overturning $\theta = \theta_c$
W weight of the block

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