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EARTHQUAKE RESPONSE OF STRUCTURES WITH PARTIAL UPLIFT ON WINKLER FOUNDATION

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SUMMARY

The effects of transient foundation uplift on the earthquake response of flexible structures are investigated. The structural idealization chosen in this study is relatively simple but it incorporates the most important features of foundation uplift. In its fixed base condition the structure itself is idealized as a single-degree-of-freedom system attached to a rigid foundation mat which is flexibly supported. The flexibility and damping of the supporting soil are represented by a Winkler foundation with spring-damper elements distributed over the entire width of the foundation. Based on the response spectra presented for several sets of system parameters, the effects of foundation-mat uplift on the maximum response of structures are identified. The influence of earthquake intensity, structural slenderness ratio, ratio of foundation mass to structural mass, foundation flexibility and p–δ effects on the response of uplifting structures is also investigated.

INTRODUCTION

The earthquake induced lateral forces on a structure, computed by dynamic analysis under the assumption that the foundation and soil are firmly bonded, will often produce a base overturning moment that exceeds the available overturning resistance due to gravity loads. The computed overload implies that a portion of the foundation mat or some of the individual column footings, as the case may be, would intermittently uplift for small time durations during an earthquake. Such uplift has been observed in several earthquakes. Several examples of towers and oil tanks uplifting from the underlying soil during Arvin Tehachapi (1952), Alaska (1964) and Imperial Valley (1979) earthquakes are cited in a recent work.[1] Uplift of multistorey building foundations has rarely been observed because the uplift is expected to be small and the foundation–soil interface is often inaccessible for observation.

Housner recognized that foundation uplift may be responsible for the good performance of seemingly unstable structures during earthquakes.[2] During the Chilean earthquakes of 1960, several golf-ball-on-a-tee type of elevated water tanks survived the ground shaking whereas much more stable appearing reinforced concrete elevated water tanks were severely damaged. Motivated by this anomalous behaviour, Housner systematically investigated the dynamics of a rigid block rocking on a rigid horizontal base. He demonstrated that there is a scale effect which makes the larger of two geometrically similar blocks more stable than the smaller block. Moreover, the stability of a tall slender block subjected to earthquake motion is much greater than would be inferred from its stability against a static horizontal force. In order to take advantage of the beneficial effects of base uplift, it was proposed[3] to design the tall piers of a bridge to rock from side-to-side with vertical separation of parts of the pier from the supporting foundations. An earlier experimental study[4] suggested that tall structures allowed to rock on their foundations would be surprisingly stable during earthquakes.

More recently, the effects of foundation uplift on the earthquake response of flexible structures have been investigated analytically[1, 5, 6] and experimentally.[7, 8] Whereas the flexibility and damping of the supporting soil were not incorporated into most of these studies,[5–8] they were modelled by two spring-damper elements, one at each edge of the foundation mat, in the most recent work.[1] Because the Winkler foundation model

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leads to considerable complication in the analysis, an equivalent two-element supporting system was developed based on the dynamics of rigid blocks.[1]

Without resorting to this approximation in modelling the foundation, this paper aims to develop a better understanding of the effects of transient foundation uplift on the response of flexible structures, considering a range of the important system parameters.

SYSTEM CONSIDERED

The system considered, shown in Figure 1, consists of a linear structure of mass m, lateral stiffness k and lateral damping c, which is supported through the foundation mat of mass m_0 resting on a Winkler foundation, with spring-damper elements distributed over the entire width of the foundation mat, connected to the base which is assumed to be rigid. The column(s) is (are) assumed to be massless and axially inextensible, the foundation mat is idealized as a rigid rectangular plate of negligible thickness with uniformly distributed mass, and it is presumed that horizontal slippage between the mat and supporting elements is not possible. The stiffness and damping coefficients of the foundation model are assumed constant, independent of displacement amplitude or excitation frequency. Thus the frequency dependence of these coefficients, as for a viscoelastic half space,[9] is not recognized; nor is the strain dependence of these coefficients for soils[10] considered in this study.

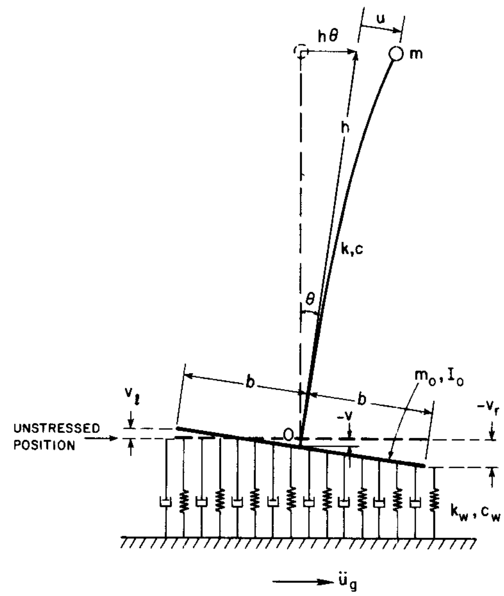

Figure 1. Flexible structure on Winkler foundation

In previous studies the impact between the foundation mat of a flexible structure,[5] or the base of a rocking rigid block,[2] and the rigid supporting soil was idealized as a perfectly inelastic collision with the vertical velocity and associated kinetic energy of the structural mass dissipated instantaneously. However, in the system idealization selected here, the energy is gradually dissipated through the viscous damping mechanism of the foundation. But if the foundation springs are relatively stiff, the typical case for buildings and soils, the energy dissipation occurs within a very short duration. Thus the two types of energy dissipating mechanisms would be expected to affect similarly the structural response. Psycharis also concluded that the impact effect can be modelled well by the simpler viscous damping mechanism.[1]

Prior to the dynamic excitation, the foundation mat rests on the Winkler spring-damper elements only through gravity and is not bonded to these supporting elements. Thus the supporting elements can provide an upward force to the foundation mat but not a downward pull. During vibration of the system this upward resultant reaction force will vary with time. At any instant of time when one edge of the foundation mat reaches the natural unstressed level of the spring elements, that edge is in the condition of incipient uplift from the supporting elements. As the upward displacement of that edge continues, an increasing portion of the foundation mat uplifts from the supporting elements.

In particular, if the dampers are absent, the relation between the displacement and reaction force per unit width of the foundation mat is shown in Figure 2(a). The upward reaction force is related linearly to the downward displacement through k_w, the spring stiffness per unit width, but no reaction force is developed if the displacement is upward; the displacements are measured from the unstressed position of the springs. If the foundation mat were bonded to the supporting elements the force–displacement relation will be linear, as shown in Figure 2(a), valid for downward as well as upward displacements.

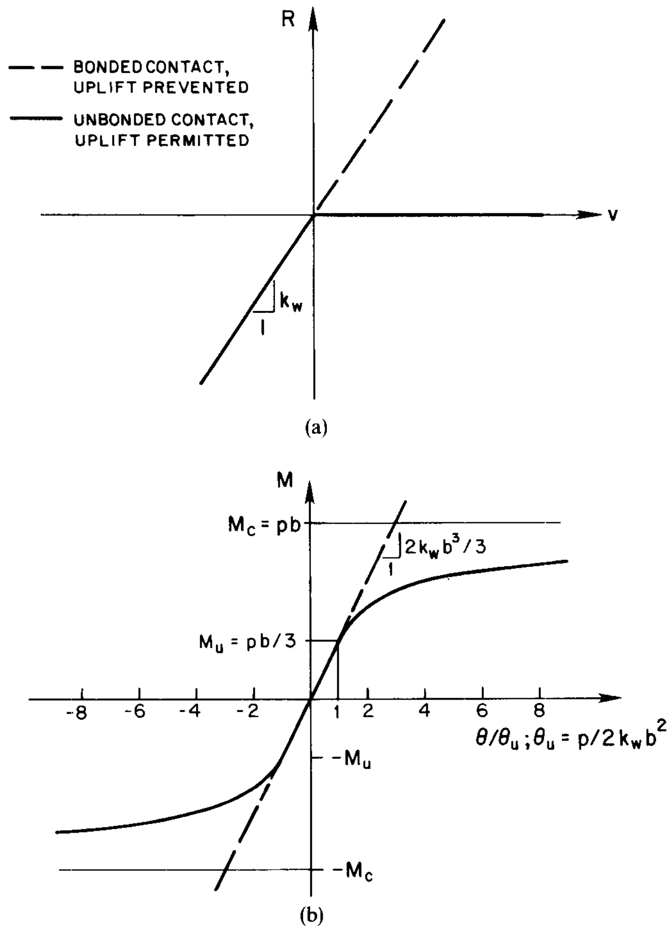

Figure 2. Properties of Winkler foundation: (a) reaction force–displacement relation for unit width of the supporting elements; (b) moment–rotation relation for foundation mat

Consider the foundation mat and its supporting elements without the superstructure with a static force p acting in the downward direction at its centre of gravity (c.g.). The relation between the static moment M applied at the c.g. and the resulting foundation-mat rotation θ, limited to angles much smaller than the slenderness ratio b/h, is shown in Figure 2(b) for unbonded as well as bonded conditions. If the mat is not bonded to the supporting elements the M–θ relation is linear, implying constant rotational stiffness, until one edge of the foundation mat uplifts from the supporting elements; thereafter the rotational stiffness decreases monotonically with increasing θ, which implies an expanding foundation-mat width over which uplift occurs. Uplift is initiated when the rotation reaches $\theta_u = p/2k_w b^2$ with the corresponding moment $M_u = pb/3$. The M–θ curve asymptotically approaches the critical moment, $M_c = pb$, corresponding to the physically unrealizable condition of uplift of the entire foundation mat from the supporting elements, except for one edge. The downward force is $p = (m+m_0)\,g$, the combined weight of the superstructure and foundation mat, prior to any dynamic excitation, but would vary with time during vibration.

Next consider the entire structural system with a gradually increasing force f_s applied at the lumped mass m in the lateral direction. If the foundation mat is bonded to the supporting elements, which along with the structure have linear properties, the lateral force can increase without limit if the overturning effects of gravity forces are neglected. However, if the mat is not bonded to the supporting elements, one edge of the foundation mat is at incipient uplift when the lateral force reaches $f_{su} = (m+m_0)\,g(b/3h)$. Thus the corresponding base shear at incipient uplift under the action of static force is

$$\mathbf{V}_{\mathbf{u}} = (m+m_0)\,g\frac{b}{3h} \tag{1a}$$

The structural deformation associated with this base shear is

$$u_{\mathrm{u}} = \frac{(m+m_0)\,g}{k}\frac{b}{3h} \tag{1b}$$

and the corresponding foundation-mat rotation

$$\theta_{\mathrm{u}} = \frac{(m+m_0)\,g}{2k_{\mathrm{w}}b^2} \tag{1c}$$

which is consistent with the preceding paragraph and Figure 2(b).

As the lateral force continues to increase beyond f_{su}, the lateral force corresponding to incipient uplift, the foundation mat separates over increasing width from its supporting elements. The lateral force $f_{sc} = (m+m_0)\,g(b/h)$ corresponds to the physically unrealizable condition that separation extends to the entire width and contact is reduced to one edge. Thus the limiting value for the base shear under the action of static forces is

$$\mathbf{V}_{\mathrm{c}} = (m+m_0)\,g\frac{b}{h} \tag{1d}$$

The structural deformation due to this base shear is

$$u_{\mathrm{c}} = \frac{(m+m_0)\,g}{k}\frac{b}{h} \tag{1e}$$

The base excitation is specified by the horizontal ground motion with displacement $u_g(t)$ and acceleration $\ddot{u}_g(t)$. The vertical component of ground motion is not considered in this study. Under the influence of this excitation the foundation mat would rotate through an angle $\theta(t)$ and undergo a vertical displacement $v(t)$, defined at its c.g. relative to the unstressed position. Prior to the dynamic excitation, the vertical displacement is $v_s = (m+m_0)\,g/2k_w\,b$, the static displacement due to the total weight of the superstructure and foundation mat. During the dynamic excitation, the vertical displacement v will remain constant at the initial static value if the foundation mat is bonded to the supporting elements, but it will vary with time in the unbonded case. The displaced configuration of the structure at any instant of time can be defined by the deformation $u(t)$, foundation-mat rotation $\theta(t)$ and vertical displacement $v(t)$ at the centre of gravity of the foundation mat.

EQUATIONS OF MOTION

The differential equations governing the small-amplitude motion of the system of Figure 1 can be derived by considering the lateral equilibrium of forces acting on the structural mass m, and moment and vertical equilibrium of forces acting on the entire system (see Appendix B, Reference 11). Assuming that the structure–foundation system and the excitation are such that the amplitudes of the resulting displacement and rotation responses are small so that $\sin\theta$ and $\cos\theta$ can be approximated by θ and 1, respectively, these equations may be expressed as

$$m\ddot{u}+m(h\ddot{\theta})+c\dot{u}+ku = -m\ddot{u}_g(t)+\frac{m(u+h\theta)}{h}(\ddot{v}+g) \tag{2a}$$

$$\frac{m_0 b^2}{3h^2}(h\ddot{\theta})-c\dot{u}+(1+\varepsilon_1^3)c_w\frac{b^3}{3h^2}(h\dot{\theta})+(1-\varepsilon_1^2)\varepsilon_2 c_w\frac{b^2}{2h}\dot{v}$$
$$-ku+(1+\varepsilon_1^3)k_w\frac{b^3}{3h^2}(h\theta)+(1-\varepsilon_1^2)\varepsilon_2 k_w\frac{b^2}{2h}v = 0 \tag{2b}$$

$$(m+m_0)\ddot{v}+(1+\varepsilon_1)c_w b\dot{v}+(1-\varepsilon_1^2)\varepsilon_2 c_w\frac{b^2}{2h}(h\dot{\theta})$$
$$+(1+\varepsilon_1)k_w bv+(1-\varepsilon_1^2)\varepsilon_2 k_w\frac{b^2}{2h}(h\theta) = -(m+m_0)g \tag{2c}$$

where contact coefficient ε_1 is equal to unity during full contact but depends continuously on foundation-mat rotation θ and vertical displacement v during partial uplift as follows:

$$\varepsilon_1 = \begin{cases} 1 & \textit{contact at both edges} \\ \varepsilon_2 v/b\theta & \textit{left or right edge uplifted} \end{cases} \tag{3a}$$

and contact coefficient ε_2 depends on whether one or both edges of the foundation mat are in contact with the supporting elements:

$$\varepsilon_2 = \begin{cases} -1 & \textit{left edge uplifted} \\ 0 & \textit{contact at both edges} \\ 1 & \textit{right edge uplifted} \end{cases} \tag{3b}$$

The vertical displacements at the edges of the foundation mat, measured from the initial unstressed position, are

$$v_i = v \pm b\theta(t), \quad i = \mathrm{l,r} \tag{4a}$$

Because the Winkler foundation cannot extend above its initial unstressed position an edge of the foundation mat would uplift at the time instant when

$$v_i(t) > 0, \quad i = \mathrm{l,r} \tag{4b}$$

The foundations of most buildings are expected to undergo only small rotations and uplift displacements, so that complete separation of the foundation mat from its supporting elements is unrealistic, i.e. the vertical displacement $v_i(t)$ will be always less than zero at one of the edges of the foundation mat.

The earthquake response of the system depends on the following dimensionless parameters:

$\omega = \sqrt{(k/m)}$, the natural frequency of the rigidly supported structure

$\xi = c/2m\omega$, the damping ratio of the rigidly supported structure

$\beta = \omega_v/\omega$, where $\omega_v = \sqrt{(2k_w b/(m+m_0))}$ is the vertical vibration frequency of the system with its foundation mat bonded to the supporting elements

$\xi_v = 2c_w b/2(m+m_0)\,\omega_v$, the damping ratio in vertical vibration of the system with its foundation mat bonded to the supporting elements

$\alpha = h/b$, a slenderness-ratio parameter

$\gamma = m_0/m$, the ratio of foundation mass to superstructure mass

The foundations of most buildings are expected to undergo only small rotations and uplift displacements, far short of overturning. At these small amplitude motions, and if p–δ effects represented by the second term in the right side of equation (2a) are neglected, the deformation response of the system to a given excitation depends on the slenderness-ratio parameter but not separately on the size. This can be shown by expressing the equations of motion and uplift criterion in terms of the above-mentioned dimensionless parameters, the deformation response is seen to depend on the h/b ratio but not separately on h. If we were to consider the large amplitude motion of the structure including the possibility of its overturning, the size parameter h would play an important role. Similarly the size parameter would influence the small-amplitude response if p–δ effects are considered, but, as will be seen later, these effects appear to be insignificant for most buildings.

The equations of motion for the system of Figure 1 are non-linear, as indicated by the dependence of the coefficients ε_1 and ε_2 on whether the foundation mat is in full or partial contact with the supporting system; and on the contact width.

The equations of motion can be specialized for the undamped system with massless foundation mat by substituting $m_0 = 0$, and $c = c_f = 0$. In particular, the inertia and damping terms in equation (2b) are zero and, following the usual approach to static condensation, θ can be expressed in terms of u and v from

$$-ku+(1+\varepsilon_1^3)\,k_w\frac{b^3}{3h^2}(h\theta)+(1-\varepsilon_1^2)\,\varepsilon_2\frac{b^2}{2h}v = 0 \tag{5}$$

and substituted into equations (2a) and (2c). The reduced system consists of the resulting two differential equations in the two unknown 'dynamic' degrees-of-freedom. As discussed later, this reduced system of equations provides a basis for approximate analysis of systems with damping and foundation-mat mass.

EQUIVALENT TWO-ELEMENT FOUNDATION SYSTEM

The solution of the non-linear equation of motion is complicated by the fact that after lift off these equations depend continuously on the varying degree of contact between the mat and its supporting elements. In contrast, the equations of motion are relatively simple for a system with foundation mat resting on two spring-damper elements, one at each edge of the foundation mat. In the latter case, the non-linear equations of motion depend on three discrete contact conditions—both edges of foundation mat are in contact with supporting elements, the left edge is uplifted, or the right edge is uplifted—but for each contact condition the governing equations are linear. Because of the relative simplicity of the two-element supporting system, it is of interest to define its properties in such a way that it can model the more complicated Winkler foundation.

The equations of motion for the idealized one-storey structure supported through a foundation mat resting on a Winkler foundation were presented in the preceding section and those for a two-element foundation in Chapter 2 of Reference 11. If the foundation mat is bonded to the supporting elements the equations of motion for the structure supported on a Winkler foundation are identical to those for the same structure on a two-element foundation with the following properties: $k_f = bk_w$, $c_f = bc_w$ and half base-width $= b/\sqrt{3}$. This two-element foundation is exactly equivalent to the Winkler foundation if uplift is not permitted.

Consider the foundation mat and its supporting elements without the superstructure with a static force p acting in the downward direction at the centre of gravity (c.g.). If the mat is not bonded to the supporting

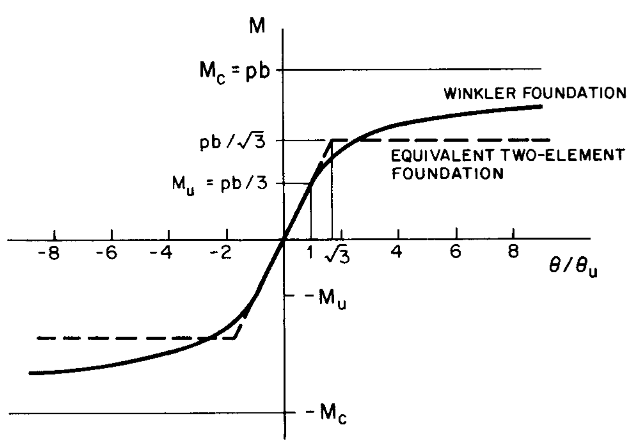

Figure 3. Moment–rotation relations for foundation mat on Winkler foundation and on equivalent two-element foundation

elements, the relation between the static moment M applied at the c.g. and the resulting foundation-mat rotation is shown in Figure 3 for the two systems. The M–θ relation is linear with the same rotational stiffness for the two systems until θ reaches $\theta_u = p/2k_w b^2$ when one edge of the foundation mat on a Winkler foundation is at incipient uplift. For large rotation angles the M–θ relation for the two systems is different. The constant rotational stiffness implied by the linear M–θ relation continues to apply for the equivalent two-element supporting system until θ reaches $\theta_u \sqrt{3}$ when one edge of the foundation mat uplifts from one of the supporting elements; thereafter no additional moment can be developed. On the other hand, the M–θ relation for the Winkler supporting system is non-linear for $\theta > \theta_u$ with the rotational stiffness decreasing monotonically with increasing θ. The M–θ curve asymptotically approaches the critical moment $M_c = pb$. Whereas the equivalent two-element supporting system is an exact representation of the Winkler foundation system for rotation angles $\theta < \theta_u$, it is only an approximation if the ground motion is intense enough to cause significant uplift. Figure 3 suggests that the approximation is likely to deteriorate for the larger angles.

The parameters of a two-element foundation system, which is equivalent to the Winkler system after uplift, can also be determined. Because the width of the foundation mat in contact with a Winkler foundation varies with time, the parameters of such an equivalent two-element system will be time-dependent. Constant parameters of a two-element foundation were determined so that this model is equivalent to the Winkler foundation with its width determined by an averaging procedure. The average contact width over a cycle of free vibration was determined from numerical results for the undamped free vibration of a rigid block rocking on a Winkler foundation, with uplift occurring during most of the time.[1] This two-element model was intended to be valid when uplift episodes dominate the response. Thus, it is possible to establish relations between the parameters of the two-element and Winkler models so that the two models are equivalent for the two cases of full contact and uplift over most of the time. However, if the structure vibrates in both states for significant portions of the response, neither of these parameter sets is expected to provide a good agreement between the responses of the two models. The two parameter sets have been combined using excitation dependent weighting functions to obtain general expressions applicable to both regimes of response.[1] These excitation dependent expressions are based on free vibration rocking response of rigid blocks and are not directly extendable to a damped, flexible structure subjected to arbitrary earthquake excitation. Consequently these expressions for parameters of the equivalent two-element system are not utilized in this investigation. Instead, the usefulness of the parameters defined earlier for the case of foundation mat bonded to the supporting elements is explored.

ANALYSIS PROCEDURE

The response of the system of Figure 1 to specified ground motion can be analysed by numerical solution of the equations of motion [equation (2)]. The equations are non-linear, as indicated by the dependence of the stiffness and damping coefficients on whether the foundation mat is in full contact with the supporting elements or it has partially uplifted; in the latter case, the coefficients depend continuously on ε_1, which is a measure of the contact width [equation (3)]. However, a linear system corresponding to any instantaneous contact condition of the foundation mat can be defined. This linear system has three degrees-of-freedom and the natural frequency of the third mode is at least an order of magnitude higher than the second mode frequency and it tends to infinity as the foundation-mat mass approaches zero (see Appendix C, Reference 11). Because the contribution of the third mode to the instantaneous response of this linear system is negligible (see Appendix C, Reference 11), it can be eliminated with the advantage that the integration time-step would not be controlled by the very short vibration period of the third mode.

The obvious procedure to eliminate the high frequency effects is to express the structural displacements as a linear combination of the first two vibration modes of the system for the instantaneous contact condition:

$$\begin{Bmatrix} u \\ h\theta \\ v \end{Bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \\ \psi_{31} & \psi_{32} \end{bmatrix} \begin{Bmatrix} Z_1 \\ Z_2 \end{Bmatrix} \tag{6a}$$

or

$$\mathbf{r} = \mathbf{\Psi Z} \tag{6b}$$

where the eigenvectors ψ_1 and ψ_2 depend continuously on the contact coefficient ε_1. Substituting this transformation into equation (2) and premultiplying both sides by the transpose of the transformation matrix Ψ leads to a reduced system of differential equations in the generalized co-ordinates Z_1 and Z_2. The stiffness and damping coefficients in the reduced equation system are also continuous functions of the contact coefficient ε_1.

The reduced system of equations is integrated numerically using an implicit method with linear variation of acceleration in each time-step with dynamic equilibrium satisfied by iteration at the end of the time-step (see Appendix D, Reference 11). Even though the high frequency, third vibration mode was eliminated, the time-step used has to be short enough to ensure convergence within a few iteration cycles. The integration time-step used is $\Delta t = 0{\cdot}001$ s, ten times shorter than that employed for the system with two-element foundation.[11]

EARTHQUAKE RESPONSES

The response of a structural system to the north–south component of the El Centro, 1940 ground motion is presented in Figure 4. Responses are shown for two conditions of contact between the foundation mat and

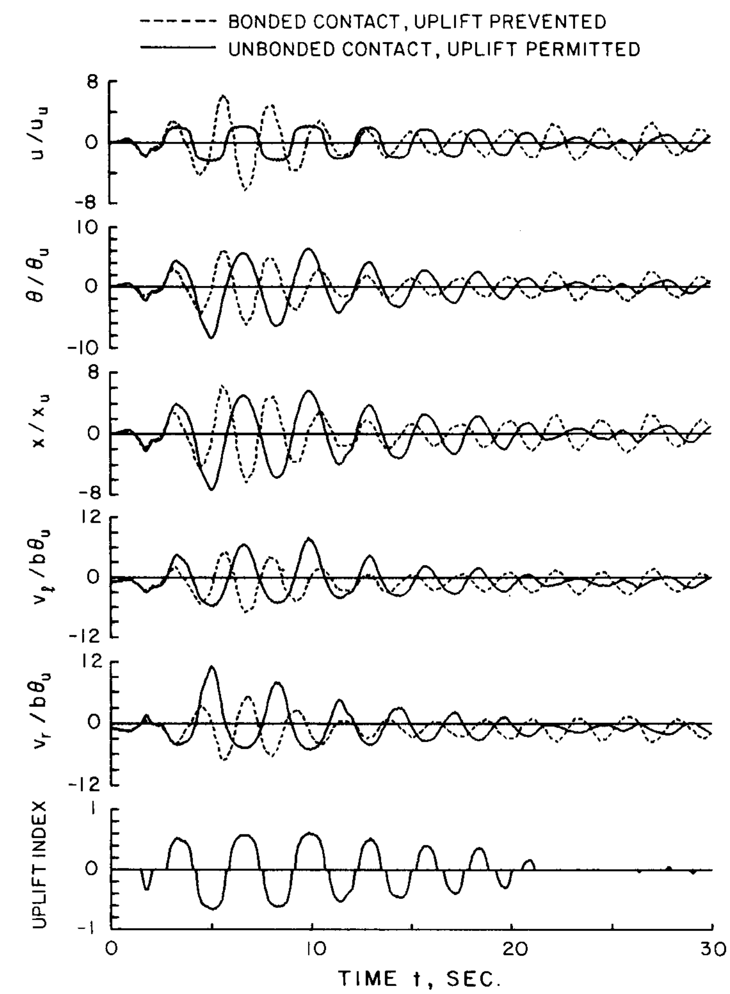

Figure 4. Response of a structure ($\alpha = 10$, $\beta = 8$, $\gamma = 0$, $T = 1$ s, $\xi = 0{\cdot}05$, $\xi_v = 0{\cdot}4$) to El Centro ground motion for two conditions of contact between the foundation mat and the supporting elements: (a) bonded contact preventing uplift; and (b) unbonded contact only through gravity with uplift permitted

the supporting spring-damper elements: (a) bonded contact preventing uplift and (b) unbonded contact only through gravity, with uplift permitted. In the first case, the structural response is entirely due to the first natural vibration mode of the system with the full width of the foundation mat in contact with the supporting elements; this mode consists of lateral deformation of the structure and rotation of the foundation mat without any vertical displacement at its c.g. Thus the response behaviour is similar to a single-degree-of-freedom system. When uplift of the foundation mat is permitted, the reponse behaviour is much more complicated. During the initial phase of the ground shaking, the foundation mat remains in contact with the supporting elements over its entire width. As the ground motion intensity builds up, the two edges of the foundation mat alternately uplift in a vibration cycle, inducing partial separation of the mat from the supporting elements. In this example the foundation mat uplifts over a significant portion of its width in every vibration cycle during the strong phase of ground shaking, with the duration of uplift depending on the amplitudes of foundation-mat rotation. As the intensity of ground motion decays toward the later phase of the earthquake, the foundation-mat uplift becomes negligible and full contact is maintained for long durations.

The effects of foundation-mat uplift on the maximum response of the selected structure due to earthquake ground motion are similar to those observed during free vibration.[11] When foundation-mat uplift is

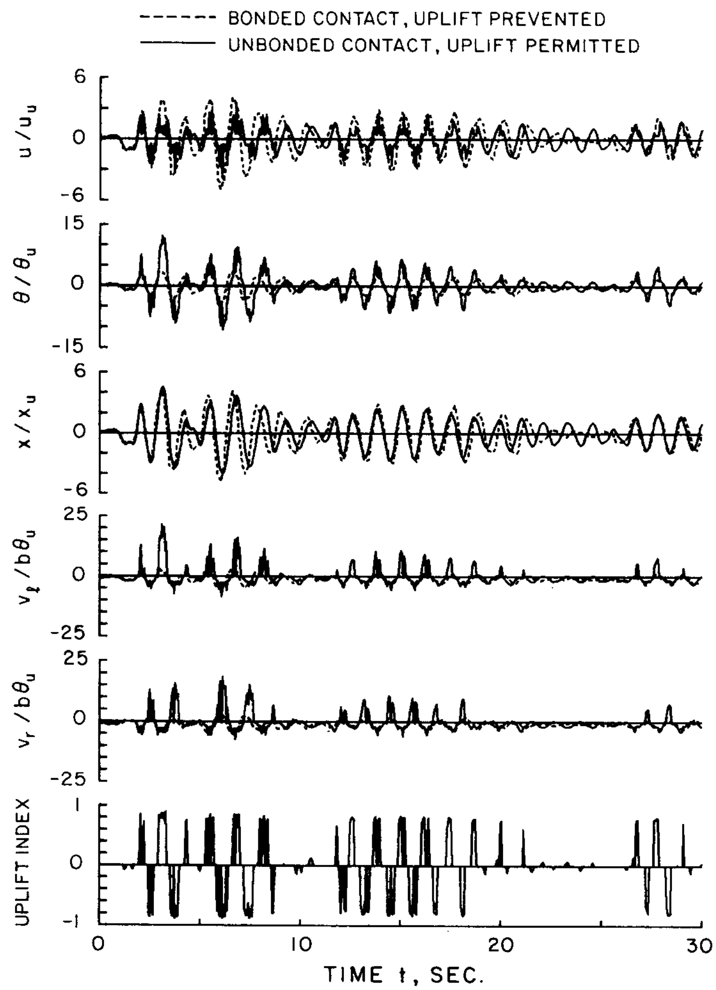

Figure 5. Response of a structure ($\alpha = 5$, $\beta = 12$, $\gamma = 0$, $T = 1$ s, $\xi = 0{\cdot}05$, $\xi_v = 0{\cdot}4$) to El Centro ground motion for two conditions of contact between the foundation mat and the supporting elements: (a) bonded contact preventing uplift; and (b) unbonded contact only through gravity with uplift permitted

permitted, the maximum deformation of the structure is about twice the deformation u_u at incipient-uplift of the foundation mat, which is a significant reduction compared to the response when foundation-mat uplift is prevented. The rotation of the foundation mat and vertical displacements of the two edges of the foundation mat are significantly increased due to the rigid body uplift motion of the system. This uplift motion provides the dominant contribution to these responses during uplift but does not affect the maximum response of the structural deformation.

The response of the relatively squatty system (smaller α) with higher stiffness in the vertical direction (larger β) to the El Centro ground motion is presented in Figure 5. While some features of the response are similar to those observed above from the previous case, important differences can be noted. In particular, foundation-mat uplift causes much larger rotation θ, and vertical edge displacements v_l and v_r of the foundation mat. The high frequency component in the deformation response is now much more pronounced, to the extent that foundation-mat uplift causes hardly any reduction in response and it leads to a slight increase in the maximum downward displacement of the foundation mat. In the previous case, the small-amplitude oscillations at a high frequency damped out and were not present in the earthquake response. Unlike in Figure 4, where the foundation-mat uplift was gradual, in this case the uplifted width of the foundation mat fluctuates very rapidly; once uplift is initiated, the uplifted width increases from zero to maximum value in almost no time; however, after a few cycles the tendency to uplift decreases. Thus, if uplift occurs in a cycle, the width is either very small or very near the maximum, but rarely an intermediate value.

In order to study the effects of foundation-mat uplift on the maximum response of structures, response spectra are presented. The base shear coefficient

$$V_{max} = \frac{\mathbf{V}_{max}}{w} = \frac{ku_{max}}{mg} = \left(\frac{2\pi}{T}\right)^2 \frac{u_{max}}{g} \tag{7}$$

where $\mathbf{V}_{max}$ is the maximum base shear, and w is the weight of the superstructure, is plotted as a function of the natural vibration period of the corresponding rigidly supported structure. For each set of system parameters α, β, γ, ξ and ξ_v, such a response spectrum plot is presented for two conditions of contact between the foundation mat and the supporting spring-damper elements: (a) bonded contact preventing uplift, and (b) unbonded contact with uplift permitted. Also presented are the results for the corresponding rigidly supported structure, which is simply the standard pseudo-acceleration response spectrum, normalized with respect to gravitational acceleration. Included in the response spectra plots is V_u, the base shear coefficient associated with the value of base shear, $\mathbf{V}_u$ at which uplift of an edge of the foundation mat is initiated [equation (1a)]:

$$V_u = \frac{\mathbf{V}_u}{w} = \frac{1+\gamma}{3\alpha} \tag{8}$$

Also included is V_c, the critical base shear coefficient associated with the static asymptotic base shear, $\mathbf{V}_c$ [equation (1d)] which corresponds to the uplift of the foundation mat from its supporting springs over the entire width, i.e. the foundation mat is standing on its edge:

$$V_c = \frac{\mathbf{V}_c}{w} = \frac{1+\gamma}{\alpha} \tag{9}$$

These base shear coefficients depend on the mass ratio γ and slenderness-ratio parameter α, but are independent of the vibration period T.

The response spectra presented in Figure 6 are for systems with massless foundations ($\gamma = 0$) and a fixed set of system parameters α, β, ξ and ξ_v subjected to El Centro ground motion. The differences between the response spectra for the two linear systems, the structure with foundation mat bonded to the supporting elements and the corresponding rigidly supported structure, are due to the change in period and damping resulting from support flexibility.[11] The base shear developed in structures with relatively long vibration periods is below the static value at incipient uplift and throughout the earthquake the foundation mat remains in contact over its entire width with the supporting elements. If foundation mat uplift is prevented,

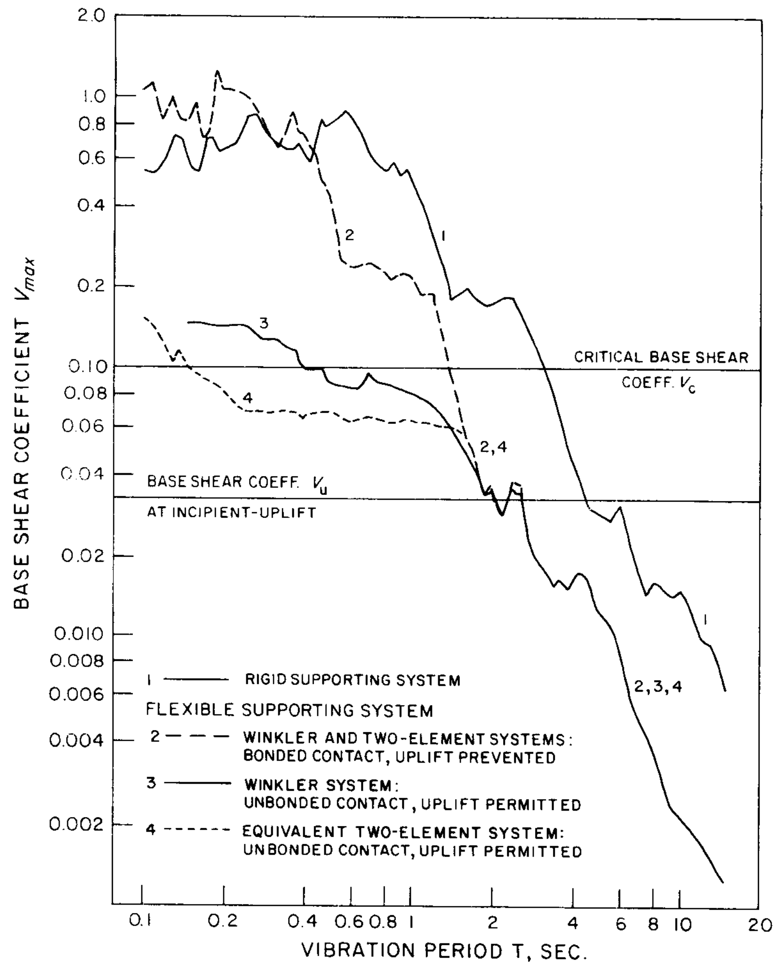

Figure 6. Response spectra for structures ($\alpha = 10, \beta = 8, \gamma = 0, \xi = 0{\cdot}05, \xi_v = 0{\cdot}4$) subjected to El Centro ground motion for four support conditions

the maximum base shear at some vibration periods may exceed the incipient-uplift value. For the selected system parameters and ground motion, Figure 6 indicates that this occurs for all vibration periods shorter than the period where the linear spectrum first attains the incipient-uplift value. If the foundation mat of such a structure rests on the Winkler spring-damper elements only through gravity and is not bonded to these elements, partial separation occurs and this has the effect of reducing the base shear. However, the base shear exceeds the value at incipient-uplift because even under static forces the base moment, and hence base shear, continue to increase considerably beyond this value [Figure 2(b)]. Furthermore the base shear is not reduced to as low as the critical value based on static consideration. Although this asymptotic value can never be exceeded under static forces, depending on the state—displacement, velocity and acceleration—of the system, the deformation and base shear may exceed the critical values during dynamic response, as seen in Figure 6. Because the base shear developed in linear structures (foundation-mat uplift prevented) tends to exceed the incipient-uplift value by increasing margins as the vibration period decreases, the foundation mat of a shorter-period structure has a greater tendency to uplift over a greater portion of its width, which in turn results in the incipient-uplift base shear being exceeded by a greater margin although it remains well below the linear response.

Also shown in Figure 6 is the response spectrum for the equivalent two-element system defined earlier. This response spectrum is identical to that for the Winkler system for the relatively long periods because the base shear developed is below the incipient-uplift value and the foundation mat does not uplift from its supporting elements, and for this condition the two-element supporting system is exactly equivalent to the Winkler supporting system. Uplift occurs for any structure if the corresponding ordinate of the linear response spectrum exceeds the static base shear coefficient at incipient uplift, which is $1/3\alpha$ for a Winkler system and $1/\sqrt{3}(\alpha)$ for the equivalent two-element system. Because the base shear developed in linear structures tends to increase as the vibration period decreases, uplift in a Winkler system is initiated at a longer period compared to the two-element system. However, because the uplift is limited in extent and duration in the range of periods bounded on the low side by the period at which uplift is initiated in a two-element system and on the high side by the period at which uplift is initiated in a Winkler system, the difference between the response spectra for the two systems is small in this period range. For shorter vibration periods outside this range the equivalent two-element system consistently underestimates the maximum response because, as shown in Figure 3, the equivalent two-element system does not adequately represent the moment–rotation relation for larger rotation angles. Overall, the equivalent two-element system is successful in displaying the main effects of foundation uplift on maximum structural response, and it provides results that are reasonably close to those for a Winkler system over a wide range of vibration periods, excluding the very short periods.

Presented in Figure 7 is the response spectrum for the downward displacement at either edge of the foundation mat for two conditions of contact between the foundation mat at its supporting spring-damper elements: (a) bonded contact preventing uplift and (b) unbonded contact with uplift permitted. The edge displacement is normalized with respect to the initial static displacement due to gravity. Just like the maximum deformation (Figure 6) the foundation mat edge displacement tends to be larger for shorter vibration period structures. Although this response quantity is affected by uplift of the foundation mat, these effects are not very significant, and over a wide range of periods a conservative estimate is provided by linear analyses preventing uplift.

The effects of ground motion intensity on the dynamic response of structures are displayed in Figure 8, wherein the response spectra for El Centro ground motion amplified by a factor of 2 are compared with the corresponding plots for the unscaled ground motion. If the foundation mat is bonded to the supporting elements and it cannot uplift, the structural system is linear and the response spectrum is amplified by the

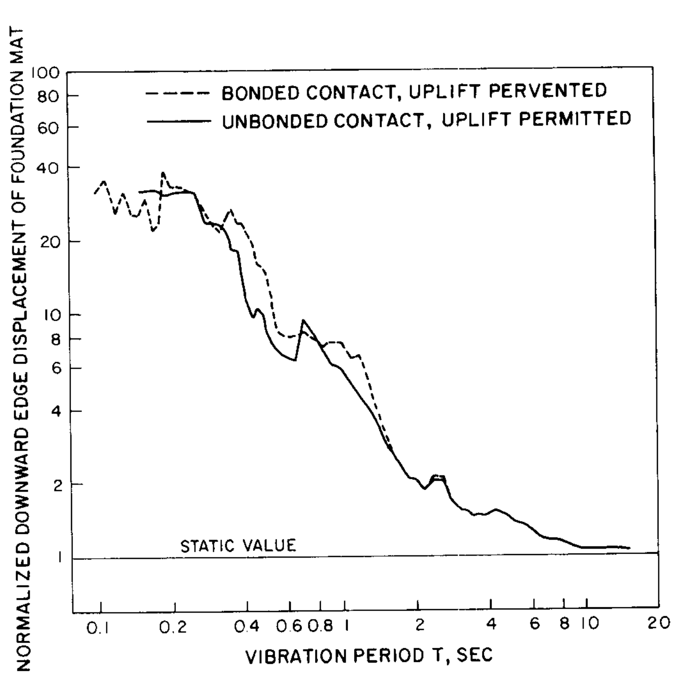

Figure 7. Response spectra for structures ($\alpha = 10$, $\beta = 8$, $\gamma = 0$, $\xi = 0{\cdot}05$, $\xi_v = 0{\cdot}4$) subjected to El Centro ground motion for two support conditions

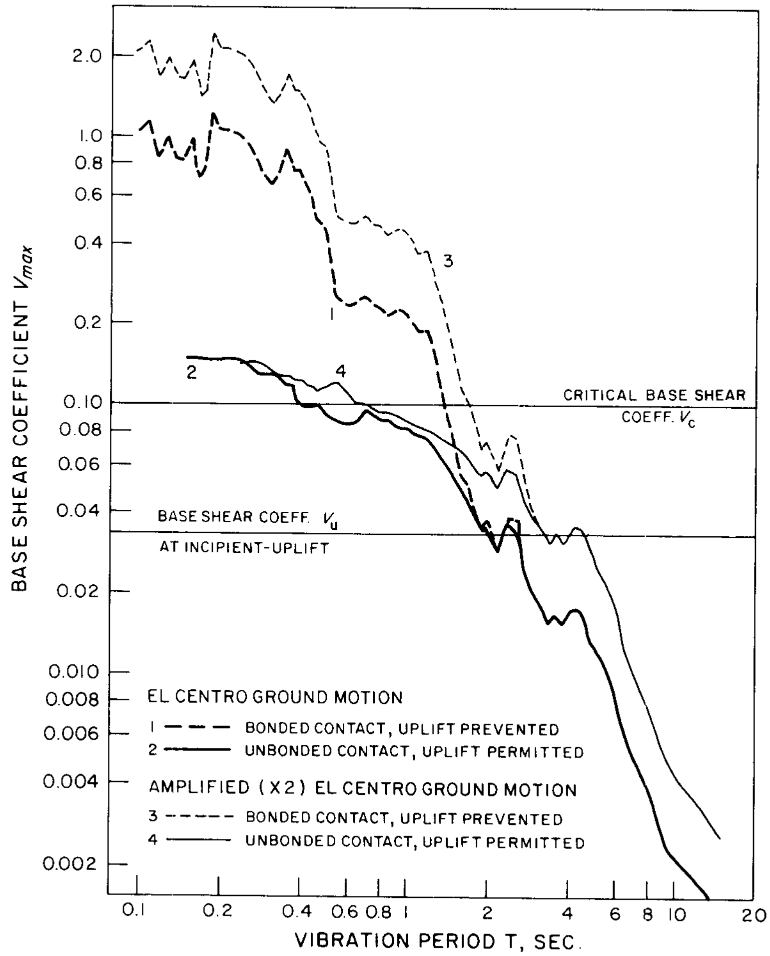

Figure 8. Effects of ground motion intensity on response spectra for structures ($\alpha = 10$, $\beta = 8$, $\gamma = 0$, $\xi = 0{\cdot}05$, $\xi_v = 0{\cdot}4$)

same factor of 2. When uplift is permitted but the uplift is limited in extent, the response is only slightly reduced from the corresponding linear values with greater reduction for the more intense earthquake, and the response spectrum is amplified by a factor somewhat less than 2. This behaviour differs from the two-element system, where uplift of the foundation mat abruptly changes the state from full contact to no contact, and the response permitting foundation-mat uplift is essentially independent of the earthquake intensity.[11] However, for structures with shorter periods the foundation mat uplifts over greater width and for more time, and the base shear is controlled by the critical value V_c which is a property of the system, independent of ground motion. Thus like the two-element system, the base shear for short-period systems is increased only slightly although the earthquake intensity is doubled. Because the base shear attains its incipient-uplift value at a slightly longer period when earthquake intensity is increased, uplift of the foundation mat is initiated at a slightly longer period.

The response spectra presented in Figure 9 are for two values of the frequency ratio β with all other system parameters kept constant. As mentioned earlier, the differences in the response spectra for the two linear systems, the structure with foundation mat bonded to the supporting elements and the corresponding rigidly supported structure, are due to the change in period and damping resulting from support flexibility.[11] The period change appears as a shift in the response spectrum to the left, with larger shift for smaller values of β, i.e. for the more flexible supporting elements. The non-linear response spectrum for the structure with foundation mat permitted to uplift shifts similarly to the left, with uplift initiated at longer periods as β

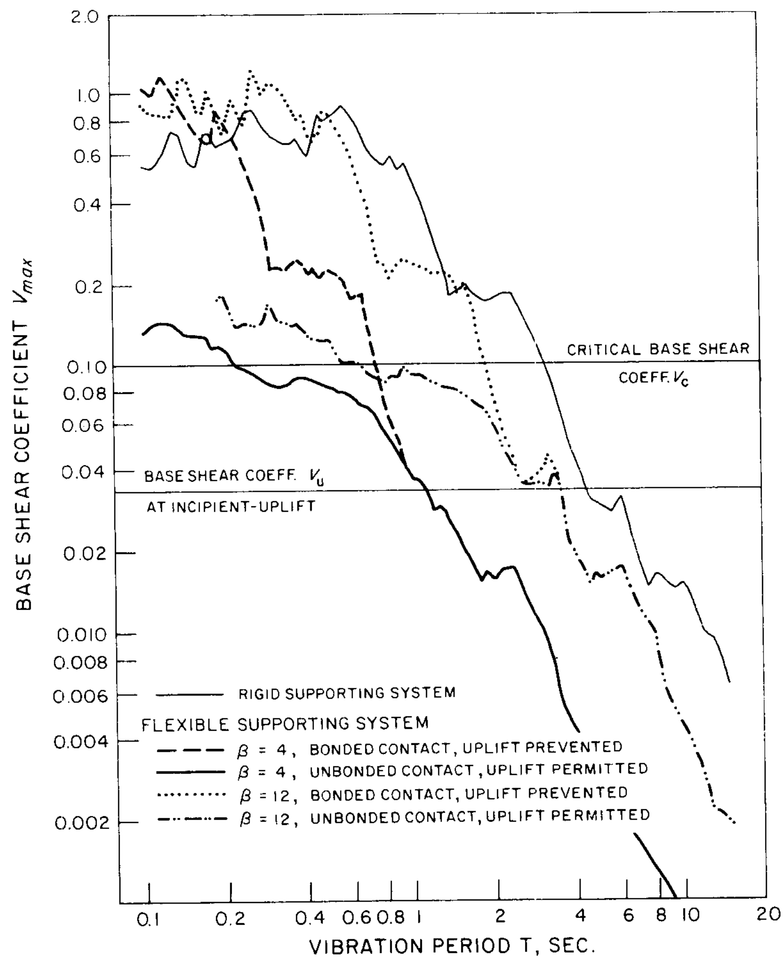

Figure 9. Response spectra for structures ($\alpha = 10$, $\gamma = 0$, $\xi = 0{\cdot}05$, $\xi_v = 0{\cdot}4$) subjected to El Centro ground motion for two values of frequency ratio $\beta = 4$ and 12

increases. But for the period shift, the shapes of the linear, as well as non-linear, response spectra are affected little by the frequency ratio β.

As presented in equations (9) and (10), the incipient-uplift and critical base shear coefficients are inversely proportional to the slenderness-ratio parameter α. This would suggest that maximum base shear will be smaller for relatively slender structures, which is confirmed by response spectra presented in Figure 10. The foundation mat of a slender structure has a greater tendency to uplift, resulting in greater reductions in the base shear. Uplift of the foundation mat occurs at all vibration periods shorter than the period where the linear response spectrum attains the incipient-uplift value. This period, in general, depends on the slenderness ratio α in a complicated manner, because the incipient-uplift base shear coefficient as well as the period shift of the linear response spectrum (relative to the standard pseudo-acceleration spectrum) both depend on α; however, in this example, this period is essentially independent of α.

The incipient-uplift and critical base shear coefficients increase with mass ratio γ as indicated by equations (9) and (10): for systems with massless foundation mat ($\gamma = 0$) these coefficients are $1/3\alpha$ and $1/\alpha$ respectively, and they increase to $2/3\alpha$ and $2/\alpha$ for systems with equal foundation-mat and structural masses ($\gamma = 1$). The response spectra for these two mass ratios are presented in Figure 11 which indicate that the effects of foundation-mat mass are to reduce the short-period range over which the foundation mat uplifts; to approximately double the maximum base shear over this period range; and to introduce a period shift in the linear response spectrum, with little influence on the shape of the spectrum.

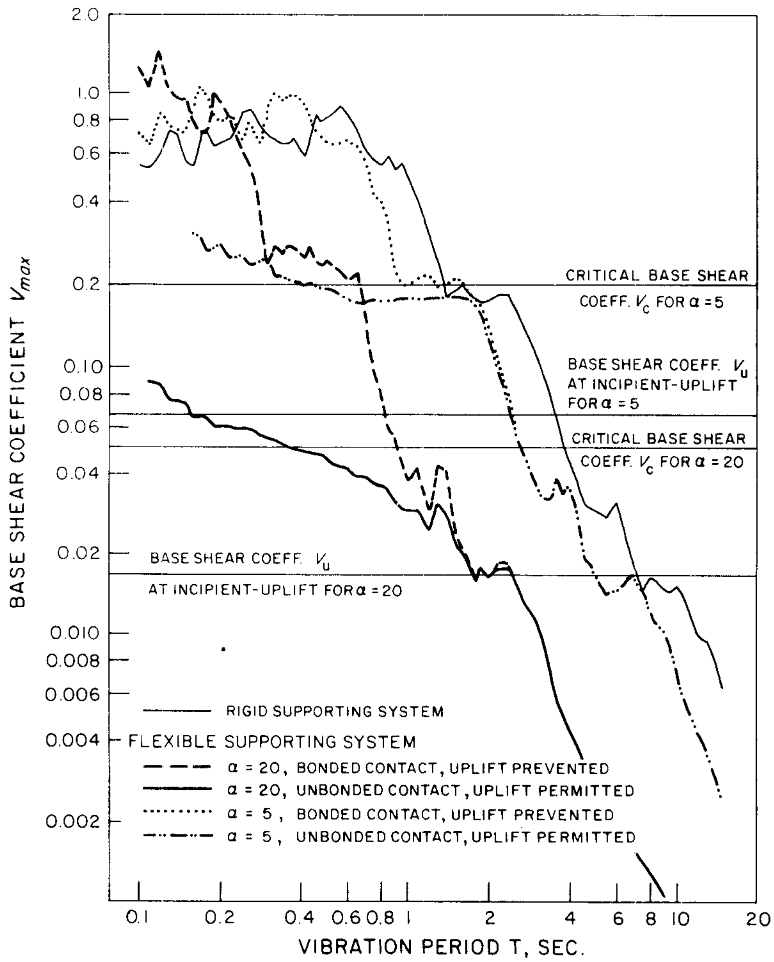

Figure 10. Response spectra for structures ($\beta = 8$, $\gamma = 0$, $\xi = 0{\cdot}05$, $\xi_v = 0{\cdot}4$) subjected to El Centro ground motion for two values of slenderness-ratio parameter $\alpha = 5$ and 20

These effects of varying parameters α, β and γ on the response of structures supported on Winkler foundation are similar to those identified from the response of structures with two supporting elements.[11]

The effects of gravity and inertia forces in the vertical direction (the so-called p–δ effect) on the dynamic response of structures are displayed in Figure 12 wherein the response spectra are presented for two cases: (a) p–δ effect neglected and (b) p–δ effect included. Over a wide range of periods the response spectrum is essentially the same, with or without p–δ effects. For a fixed frequency ratio, the supporting system becomes increasingly flexible for structures with longer vibration periods, and the structure can overturn because of p–δ effects.

SUMMARY AND CONCLUSIONS

The effects of transient foundation uplift on the earthquake response of buildings have been investigated. This study was based on structural idealizations that are relatively simple but realistic in the sense that they incorporate the most important features of foundation uplift. In its fixed base condition the structure itself was idealized as a single-degree-of-freedom system attached to a rigid foundation which is flexibly supported. The flexibility and damping of the supporting soil were represented by a Winkler foundation with spring-damper elements distributed over the entire width of the foundation mat.

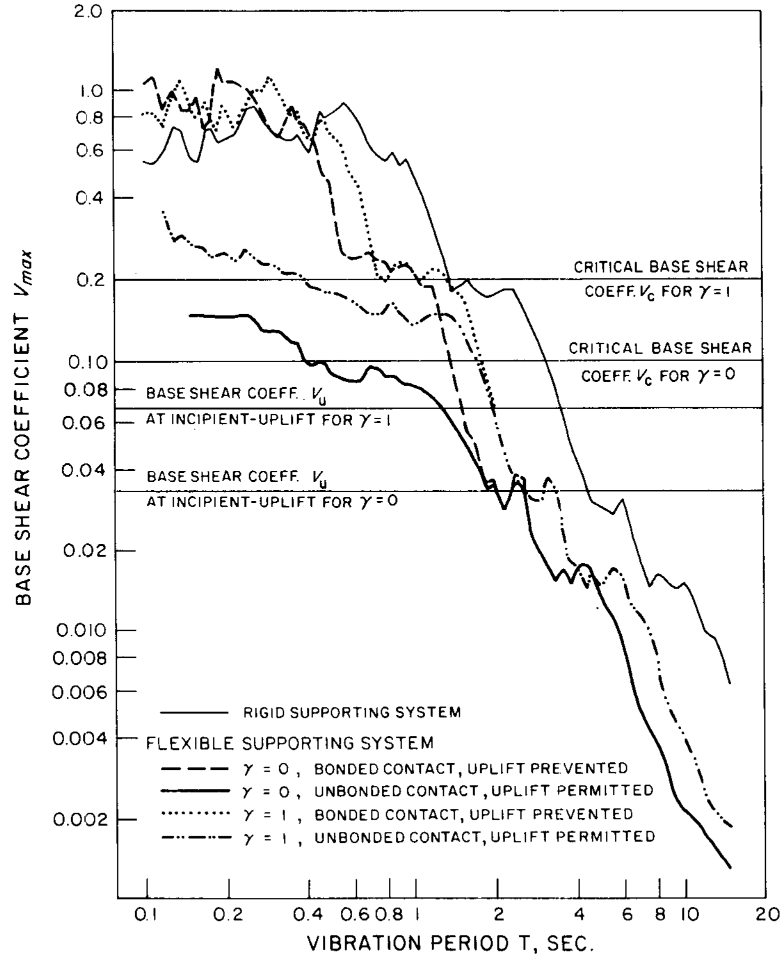

Figure 11. Response spectra for structures ($\alpha = 10$, $\beta = 8$, $\xi = 0{\cdot}05$, $\xi_v = 0{\cdot}4$) subjected to El Centro ground motion for two values of mass ratio $\gamma = 0$ and 1

Under the action of a static lateral force, uplift of the foundation mat is initiated when the base shear reaches one-third of the critical value. The critical base shear corresponds to the physically unrealizable condition of uplift of the entire foundation mat from the supporting elements except for one edge. As the base shear increases beyond the value at incipient uplift the foundation mat separates over increasing width from its supporting elements. The critical base shear depends only on the gravity force and slenderness-ratio parameter.

In order to study the effects of foundation-mat uplift on the maximum response of buildings, response spectra were presented. For each set of system parameters the maximum base shear was plotted against the natural vibration period of the corresponding rigidly-supported structure for two conditions of contact between the foundation mat and the supporting spring-damper elements: (a) bonded contact preventing uplift and (b) unbonded contact with uplift permitted. A study of these response spectra plots led to the following conclusions:

1. The base shear developed in structures with relatively long vibration periods is below the static value at incipient uplift and the foundation mat does not uplift from its supporting elements.
2. For short-period structures, the base shear exceeds the incipient-uplift value if foundation-mat uplift is prevented; for such structures, permitting uplift has the effect of reducing the base shear—to values somewhat above the incipient-uplift value and in some cases even above the critical value.
3. Because the response of a structure with foundation mat permitted to uplift is controlled by the critical

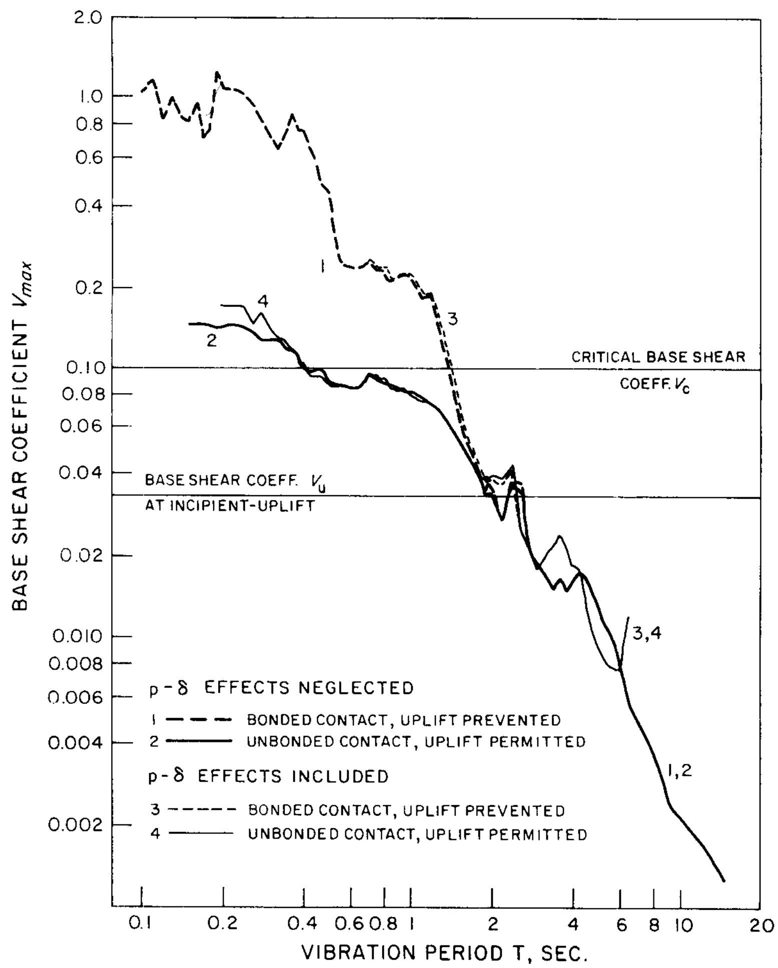

Figure 12. Response spectra for structures ($\alpha = 10$, $\beta = 8$, $\xi = 0{\cdot}05$, $\xi_v = 0{\cdot}4$) subjected to El Centro ground motion for two cases: (a) p–δ effects neglected; and (b) p–δ effects included

base shear, which is independent of the ground motion, the base shear is affected only slightly by earthquake intensity.

4. The foundation mat of a slender structure has a greater tendency to uplift, resulting in greater reductions in base shear.
5. Because the critical base shear increases with mass ratio γ so does the maximum earthquake induced base shear.
6. The shape of the response spectrum is affected little by the frequency ratio β, but it does have the effect of shifting the response spectrum to the left with larger shift for smaller β, i.e. for the more flexible supporting elements.
7. Over a wide range of structural vibration periods, the effects of gravity and inertia force in the vertical direction (the so-called p–δ effects) are insignificant in the response of uplifting structures.

Comparison of the response spectra for Winkler and two-element systems demonstrated that the two spectra agree reasonably well except for very short periods and the latter system is successful in displaying the main effects of foundation uplift on maximum structural response. Both foundation models are deficient because they do not recognize that foundation stiffness and damping parameters depend on the displacement amplitude and excitation frequency. Furthermore, the foundation parameters would be very difficult to evaluate for practical situations, because they would depend on the details of the foundation design, including the shape, degree of embedment and deformability of the foundation mat. For these reasons, both foundation models used in this work are highly simplified approximations of the real situation, and there may not be much to be gained in using the seemingly more accurate Winkler model over the equivalent two-element model. Useful estimates of the effects of foundation uplift on structural response could be obtained from the equivalent two-element model, which is much easier to analyse.

The possibility of transient uplift of a portion of the foundation mat or of a few individual footings, as the case may be, should be considered in the analysis of response of structures subjected to intense earthquake ground motion. Because foundation uplift generally has the effect of reducing the structural deformations and forces, there is no need to prevent it but, on the contrary, it is desirable to permit it. The foundation, underlying soil and the structural columns should be properly designed to accommodate the transient uplift and the effects of subsequent impact on contact.

Although the maximum base shear is reduced because of foundation-mat uplift for slender structures over a wide range of fixed-base vibration periods, very short period structures may be exceptions to this behaviour. Their response, as seen in results presented elsewhere,[12] may increase because of foundation-mat uplift. However, these vibration periods are unrealistically short for slender structures representative of medium-rise to high-rise structures with longer vibration periods. Thus, the increased computed response of very short period structures, in spite of foundation-mat uplift, is of little practical consequence.

Because they are based on the response results presented here and in the more complete report,[11] all for one selected ground motion, strictly speaking the above-mentioned conclusions may not be valid for other excitations or range of structural parameters. However, as observed elsewhere,[12] the response behaviour of structures, excluding those with very short fixed base vibration periods, is simple when their foundation mat is permitted to uplift even though the system is non-linear. In a sense the behaviour seems to be even simpler than that of the corresponding linear system—the structure with its foundation mat bonded to the supporting soil. In particular, the maximum base shear of uplifting structures does not depend significantly on the details or intensity of the ground acceleration $\ddot{u}_g(t)$, but is controlled by the critical base shear V_c (equation (1d)) for the structure. Thus it seems that the above-mentioned conclusions should be also valid for a variety of ground motions.

ACKNOWLEDGEMENTS

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APPENDIX

Notation

- b half width of foundation mat
- c lateral damping coefficient of superstructure
- c_w damping coefficient per unit width of Winkler foundation
- g acceleration of gravity
- h height of superstructure
- k lateral stiffness of superstructure
- k_w stiffness per unit width of Winkler foundation
- m mass of superstructure
- m_0 mass of foundation mat
- M static base moment applied at c.g. of foundation mat
- M_c maximum static base moment at full uplift of foundation mat with contact reduced to one edge
- M_u static base moment at incipient uplift of foundation mat supported on Winkler foundation
- T natural vibration period of the rigidly supported structure
- u structural deformation
- u_c maximum structural deformation under the action of static lateral force
- $\ddot{u}_g(t)$ horizontal ground acceleration
- u_{max} maximum structural deformation during an earthquake

u_u static structural deformation at incipient uplift of foundation mat on Winkler foundation
v vertical displacement of c.g. of foundation mat
v_l vertical displacement of left edge of foundation mat
v_r vertical displacement of right edge of foundation mat
v_s vertical displacement of foundation mat under the action of gravity forces
V_{max} a base shear coefficient $= \mathbf{V}_{max}/w$
V_c a base shear coefficient $= \mathbf{V}_c/w$
V_u a base shear coefficient $= \mathbf{V}_u/w$
$\mathbf{V}_c$ maximum base shear that can be developed under the action of static lateral force
$\mathbf{V}_u$ static base shear at incipient uplift of foundation mat on Winkler foundation
$\mathbf{V}_{max}$ maximum base shear
w weight of superstructure
x lateral displacement of structure relative to supporting foundation $= u + h\theta$
$\dot{x}(0)$ initial velocity of structure
α slenderness-ratio parameter
β $= \omega_v/\omega$
γ ratio of foundation mass to superstructure mass
θ rotation of foundation mat
θ_c static rotation of foundation mat at full uplift with contact reduced to one edge
θ_u static rotation of foundation mat on Winkler foundation at incipient uplift
ξ damping ratio of the rigidly supported structure
ξ_v damping ratio in vertical vibration of the system of Figure 1 with its foundation mat bonded to the Winkler foundation
ψ_i ith vibration mode of system after uplift
ω natural vibration frequency of the rigidly supported structure
ω_v vertical vibration frequency of the system with its foundation mat bonded to the supporting elements

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