Nonlinear Impact and Chaotic Response of Slender Rocking Objects[a]

Discussion by W. J. Stronge[3]

A few slender columns, obelisks, and stelae remain standing after thousands of years in earthquake prone regions: this has prompted analyses of rocking and possible overturning of slender blocks in response to horizontal periodic motion of a level base. For planar motion of a rigid prismatic block, the authors performed an analysis to obtain boundaries between regions of periodic, overturning, and chaotic behaviors. They analyze equations of motion derived originally by Aslam et al. (1980) and used subsequently by Spanos and Koh (1984).

Eqs. (1*a*) and (1*b*) assume that a rectangular block rocks successively about two opposite edges of the base without sliding. Furthermore, there is an implicit assumption that the block does not bounce; i.e. each collision is perfectly inelastic, so at least one of the edges is always in contact with the supporting foundation. The equations neglect energy dissipation ΔK due to each inelastic collision; for each collision this neglected dissipation is a part of the kinetic energy immediately before impact $K(t^-)$

$$\frac{\Delta K}{K(t^-)} = \frac{3\left(\frac{B}{H}\right)^2\left[4 + \left(\frac{B}{H}\right)^2\right]}{4\left[1 + \left(\frac{B}{H}\right)^2\right]} \qquad (23)$$

where B/H = ratio of width to height. A block that does not bounce must be very slender for this source of dissipation to be negligible. If dissipation due to the normal impulsive reaction at the contact point is negligible, however, what are the sources of energy loss during impact?

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The authors define a ratio e between the angular speeds before and after each collision $e = \dot{\theta}(t^+)/\dot{\theta}(t^-)$ and presume that each impact dissipates a part $1 - e^2$ of the kinetic energy $K(t^-)$. They use the term "coefficient of restitution" for e and consider that this is a parameter. The authors neglect to point out that this assumption generally requires that each collision generates an impulsive reaction moment about the colliding edge. In fact, this moment does work on the rocking block of width h; it adds energy if $e = 1$ and dissipates energy for $e << 1$. If this impulsive moment is not present and the block does not bounce, the angular speeds before and after each collision are uniquely related

$$\frac{\dot{\theta}(t^+)}{\dot{\theta}(t^-)} = \frac{1 - 0.5\left(\frac{B}{H}\right)^2}{1 + \left(\frac{B}{H}\right)^2} \qquad (24)$$

Any other ratio for the angular speeds requires an impulsive moment.

Thus, the equations of motion considered by the authors are not representative of rocking blocks, because they overconstrain the problem and introduce an artificial impulsive reaction moment about the contact point. Further, the effects of bounce and friction during collision have been neglected, although these are likely sources of complex behavior (Stronge 1987). Some bounce is inevitable if the only nonnegligible deformation is in a very small region around the contact point; but if the block bounces there will generally be initial slip for subsequent collisions—this tangential relative motion develops while the block is airborne. During each collision, any slip is opposed by friction, and this friction dissipates energy. Methods for analyzing the effects of friction during impact have been published by Stronge (1991).

Appendix. References

Stronge, W. J. (1987). "The domino effect: A wave of destabilizing collisions." *Proc., Royal Soc. London*, A409, 199–208.
Stronge, W. J. (1991). "Rigid body impact with friction." *Proc., Royal Soc. London*, 431, 169–181.

Closure by Solomon C. S. Yim,[4] Member, ASCE, and Huan Lin[5]

The writers wish to thank the discusser for his interest and constructive comments on the paper.

The physics of impact-energy dissipation associated with the no-bouncing, no-slipping assumption pointed out by the discusser had been examined previously by the writers. Expressions for energy loss and the corresponding coefficient of restitution e relating the angular velocity before and after a perfect impact based on conservation of momentum were stated in (6)–(9) in Yim et al. (1980). However, the writers felt that it was appropriate to continue treating the coefficient of restitution e as a system parameter in

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this study [as in Aslam et al. (1980), Yim et al. (1980), and Spanos and Koh (1984)] for the following reasons.

The use of coefficient of restitution e is conceptually simple, although (based on the writers' calculations) the e-values calibrated from the experiments in the studies by Aslam et al. (1980) and Wong and Tso (1989) were higher than those predicted by the perfect-impact, no-bouncing, no-slipping assumption. Complex behavior was observed in the rocking response, while no bouncing was reported (Wong and Tso 1989). These phenomena indicate that the actual energy loss was less than that predicted by the perfect-impact assumption and that bouncing (albeit not perceivable) might have occurred. However, both Aslam et al. (1980) and Wong and Tso (1989) were able to obtain good agreement between analytical and experimental results by using higher e-values than the analytical ones with no bouncing and no slipping. It appears that the use of a higher e-value (which is crucial) can adequately account for the possible influence of bouncing and other unobservable behaviors.

In the writers' study, the special case with $e = 1$ yields a simple Hamiltonian system, which was useful for demonstrating the sensitivity of the system response. [Housner (1963) also used the same assumption ($e = 1$) in his probabilistic analysis to demonstrate intrinsic characteristics of the system behavior.]

The Melnikov analysis technique presented in the writers' paper is applicable for general e. Therefore, it can be applied to cases with e-values consistent with perfect impact and/or other assumptions suggested by the discusser.

It may be interesting in future studies to derive an analytical expression for the coefficient of restitution e taking into account a certain level of bouncing in the dynamic analysis.

Appendix. Reference

Housner, G. W. (1963). "The behavior of inverted pendulum structures during earthquakes." *Bull. Seismo. Soc. Am.*, 53, 404–417.