Stochastic Response of Offshore Structures Excited by Drag Forces

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Abstract: This paper describes a new method for representing hydrodynamic drag forces on a marine structure subjected to random waves. It is shown that the obtained force representation has the correct statistical properties to good approximation. A fair agreement can also be obtained between the spectral density of the force representation and that of the original drag force. The distinct advantage of the proposed representation is that it becomes possible to carry out a dynamic analysis of a linear structure in the frequency domain using available techniques, which are part of a larger packet that includes efficient tools for providing detailed statistical information about the response process. This has implications toward more efficient analyses of, for example, long-term fatigue of offshore structures.

INTRODUCTION

A new method for representing the drag forces on a marine structure in random waves has recently been proposed (Naess 1993). The reason for seeking a new way of representing the drag forces is that the traditional representation of such forces makes it difficult to develop efficient response analysis methods for dynamically sensitive structures subjected to random drag forces. Today, the only practical way to carry out a stochastic dynamic analysis of a drag-dominated structure without linearizing the drag force is to apply the technique of time domain Monte Carlo simulation. This entails an integration in time of the equation of motion for each realization of the force in the generated sample of force time histories. By this, a sample of response time histories is obtained, and various statistical descriptors may be estimated depending on the specific purpose of the analysis. However, the computational burden involved in application of this procedure for the estimation of, for example, long-term fatigue is almost prohibitive at present for a detailed analysis. Even though the computational capacity of computers is increasing at a rapid pace, it is still considered to be of interest to investigate the possibility of developing a more efficient method of analysis.

The most extensively used representation of drag forces on structures is obtained by linearization. For structures modeled by linear equations of motion it is then possible to carry out the dynamic analysis in the frequency domain, which is computationally much more efficient than having to solve the problem in the time domain. A serious drawback of the linearization approach is that the statistics of the linear force term deviates substantially from that of the standard drag force model (Pierson and Holmes 1965; Tickell 1977). This deviation propagates through to the calculated dynamic response, but it is usually somewhat reduced. The extent of the reduction depends on the dynamic properties of the structure. Broadly speaking, the lower the damping the more Gaussian the dynamic response. However, extensive Monte Carlo simulations have shown that the dynamic responses of drag-dominated structures like offshore jacket platforms are distinctly non-Gaussian (Karunakaran et al. 1992). This may have significant implications for the estimation of extreme responses and fatigue life predictions. Some recent efforts to investigate purpose-specific linearization procedures with a potential to improve linearization-based predictions of extremes and fatigue may be noted (Naess et al. 1992; Naess and Winterstein 1993).

Another technique that has been proposed recently is the method of statistical quadratization (Donley and Spanos 1990; Spanos and Donley 1991), which includes terms up to second order in a Volterra series expansion of the drag force. In its standard form, it reduces to the method of statistical linearization when the current speed is zero. The method will also miss third (and higher) harmonics of wave frequencies present in the drag force spectrum. To improve on these aspects, it was suggested by Spanos and Donley (1991) to extend their methodology to include a statistical cubicization method. This approach has been implemented by Kareem and Zhao (1994).

The statistical cubicization approach has certain similarities with previous efforts directed toward developing a higher order representation theory for the drag force based on a simple polynomial expansion [cf., e.g., Hu and Lutes (1987)]. In its simplest form, this theory leads to estimates of the spectral density of the drag force that contains spectral components at higher harmonics of the frequencies in the wave spectrum. This feature is clearly important in cases where eigenfrequencies of a dynamic structure may fall within the range of these higher harmonics. The main drawback of such higher order representations of the drag force is that dynamic analyses beyond the spectral level becomes rather complicated, and the accuracy of the analyses have not been satisfactorily verified.

In the present paper an alternative approach is presented. For the case of zero current, it will be shown that it is possible to construct a genuinely quadratic representation of the drag force that reproduces the statistical properties of the standard formulation of the drag force very closely, and which has a spectral density that approximates the desired force spectrum reasonably well. As is demonstrated, a distinct advantage of this representation is that it can bring dynamic analysis back into the frequency domain, as was also achieved for the linearized force representation. As could be expected, the calculations required when applying the quadratic force representation are more extensive than for the linearized force. However, the analysis is still much more efficient than a time domain solution.

A considerable amount of work has already been devoted to the development of statistical analyses of quadratic systems, and it is shown that the frequency domain analysis related to the quadratic representation of the drag force essentially reduces to the solution of an eigenvalue problem. It is then straightforward to calculate detailed statistical information related to the dynamic response like the probability density function of the response quantity in question, as well as an estimate of its extreme value distribution. The requirement for appli-

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cation of a frequency domain analysis is that the equations of motion of the structure are time invariant and linear.

DRAG FORCE AND DYNAMIC MODEL

For the subsequent developments it is expedient to adopt the following simple single-degree-of-freedom (SDOF) equation of motion describing the displacement response of an offshore structure or structural element

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F(t) \quad (1)$$

where M = total mass, including added mass effects; C and K = damping and stiffness parameters, respectively; $F(t)$ = hydrodynamic loading; and $X(t)$ = the corresponding displacement response.

In the context of offshore structures, a frequently used environmental force model that includes the drag force is the Morison type wave loading, which is adopted here. In accordance with this model, the external force $F(t)$ is assumed to be of the form

$$F(t) = k_m\dot{U}(t) + k_d[U(t) - \dot{X}(t)]|U(t) - \dot{X}(t)| \quad (2)$$

where k_m and k_d = appropriate constants; and $U(t)$ = water particle velocity in the given direction at a specified location. Since the purpose of the present paper is to study dynamic structures, the relative velocity $U(t) - \dot{X}(t)$ has been used in the formulation of the drag force. The effect of relative acceleration has been accounted for in the mass term M. It is assumed that for most wave loading conditions, the water particle velocity will substantially exceed the displacement velocity of the structure. Hence the following approximation is adopted:

$$[U(t) - \dot{X}(t)]|U(t) - \dot{X}(t)| \approx U(t)|U(t)| - 2|U(t)|\dot{X}(t) \quad (3)$$

The term $2|U(t)|\dot{X}(t)$ can be viewed as a time variant damping contribution. Since it can contribute substantially to the overall damping, it cannot in general be neglected. For practical calculations, it is advantageous to replace this damping with a time invariant term. This can be achieved in various ways. The simplest procedure is to replace $|U(t)|$ by its mean value $E[|U(t)|]$ (Penzien 1976). In the case of prediction of extreme responses, a different choice would in general be appropriate. Strategies for such replacement are not pursued here; it is assumed that such a choice has been made. In the subsequent developments, the equation of motion is therefore expressed by (1), where the hydrodynamic loading is of the form

$$F(t) = k_m\dot{U}(t) + k_dU(t)|U(t)| \quad (4)$$

In the present paper the focus is on slender, drag-dominated structures and the effect of the inertia force is neglected, leaving the drag force $F_d(t) = k_dU(t)|U(t)|$ as the loading process to be considered.

The ocean surface elevation at a specified reference point is assumed to be well represented by a zero-mean, stationary Gaussian process, $\eta(t)$ say. The water particle velocity $U(t)$ is assumed to be linearly related to $\eta(t)$. $U(t)$ is therefore also stationary and Gaussian. It will also be assumed that $E[U(t)] = 0$, that is, no current is present.

DRAG FORCE REPRESENTATION

The task still ahead is to formulate the drag force representation. For this purpose we shall introduce an auxiliary, zero-mean Gaussian process $P(t)$, corresponding in a sense to the water particle velocity process $U(t)$. $P(t)$ can be assumed given in discretized form as follows:

$$P(t) = \sum_{j=-n}^{n}{}_0 \sqrt{S(\omega_j)\Delta\omega}\, e^{i\omega_j t}B_j \quad (5)$$

where the two-sided spectral density $S(\omega)$ is as yet unknown; the index zero on the summation sign indicates that the summation index omits zero; $0 \le \omega_1 < \omega_2 < \ldots < \omega_n$ is a discretization, assumed equidistant for simplicity, of a pertinent part of the positive frequency axis; $\omega_{-j} = -\omega_j$ and $\Delta\omega = \omega_{j+1} - \omega_j$; and $\{B_j\}$ is a set of independent, complex Gaussian $N(0, 1)$-variables with independent, identically distributed real and imaginary parts ($j = 1, \ldots, n$), $B_{-j} = B_j^*$ (where the asterisk signifies complex conjugation).

To achieve the desired representation, a process $N(t)$ associated with $P(t)$ is required. It is defined as follows:

$$N(t) = \sum_{j=-n}^{n}{}_0 \sqrt{S(\omega_j)\Delta\omega}\,\hat{\chi}(\omega_j) e^{i\omega_j t}B_j \quad (6)$$

where the transfer function $\hat{\chi}(\omega) = -i$ for $\omega > 0$, $\hat{\chi}(0) = 0$; and $\hat{\chi}(\omega) = i$ for $\omega < 0$. That is, $\hat{\chi}(\omega)$ is the transfer function for the Hilbert transform (Cramér and Leadbetter 1967). Hence the process $N(t)$ becomes the Hilbert transform of $P(t)$. We are now in a position to formulate the representation of the drag force $F_d(t) = k_dU(t)|U(t)|$. To this end, the process $\Phi(t)$ is defined by the following equation:

$$\Phi(t) = k_d[P(t)^2 - N(t)^2] \quad (7)$$

An underlying assumption in this paper is that there is no current. This limitation had to be imposed here due to the symmetric character of representation process $\Phi(t)$ as defined by (7). To lift the restriction of zero current, the force representation has to be made nonsymmetrical. The best way to achieve this is not clear at present.

Before we enter a discussion of the statistical properties of $\Phi(t)$, it is convenient to make a few comments on the spectral properties of the obtained force representation. By substituting into (7) from (5) and (6), it can be shown that

$$\Phi(t) = k_d \sum_{i=-n}^{n}{}_0 \sum_{j=-n}^{n}{}_0 \sqrt{S(\omega_i)S(\omega_j)}\Delta\omega[1 - \hat{\chi}(\omega_i)\hat{\chi}(\omega_j)^*]e^{i(\omega_i-\omega_j)t}B_iB_j^* \quad (8)$$

Hence, $\Phi(t)$ is obtained as a quadratic transformation of the Gaussian process $P(t)$. The quadratic transfer function $\hat{K}_2(\omega, \omega')$ of the force representation $\Phi(t)$ can be identified from this equation. It takes the form

$$\hat{K}_2(\omega, \omega') = k_d[1 - \hat{\chi}(\omega)\hat{\chi}(\omega')] \quad (9)$$

Invoking the definition of the transfer function $\hat{\chi}(\omega)$ of the Hilbert transform, it is recognized from (8) and (9) that the frequency content of the force representation $\Phi(t)$ consists of precisely those frequencies that can be expressed as the sum of two frequencies in the spectrum of $P(t)$, that is, in $S(\omega)$. It is shown in Appendix I that the (two-sided) spectral density $S_\Phi(\omega)$ of $\Phi(t)$ is given as follows:

$$S_\Phi(\omega) = k_d^2 \int_{-\infty}^{\infty} 2S(\omega')S(\omega - \omega')|1 - \hat{\chi}(\omega')\hat{\chi}(\omega - \omega')|^2\, d\omega'$$

$$= 8k_d^2 \int_0^{|\omega|} S(\omega')S(|\omega| - \omega')\, d\omega' \quad (10)$$

A problem we have to address in the following is how to determine $S(\omega)$ so that the resulting spectral density $S_\Phi(\omega)$ given by (10) is a good approximation to the desired spectrum of the drag force. However, before embarking on this problem, we demonstrate the most important advantage of the proposed drag force representation. It will be shown that the probability distribution as well as the mean level-crossing rates of $\Phi(t)$ closely follow those of the drag force $F_d(t)$. This is a strong indication that $\Phi(t)$ has the correct statistical properties; hence, the proposed method is a substantial improvement compared to other available representation strategies.

By straightforward calculations from the given expressions, it follows that $E[F_d(t)] = E[\Phi(t)] = 0$, $\sigma_{F_d}^2 = \mathrm{Var}[F_d(t)] = 3k_d^2\sigma_U^4$, and $\sigma_\Phi^2 = \mathrm{Var}[\Phi(t)] = 4k_d^2\sigma^4$, where σ_U denotes the standard deviation of $U(t)$ and σ that of $P(t)$. By requiring that $\sigma_\Phi = \sigma_{F_d}$, a relation is established between σ_U and σ. The expression for the probability density function (PDF) $f_{F_d}(\ )$ of $F_d(t)$ can be gleaned from the literature (Borgman 1967a; Grigoriu 1984)

$$f_{F_d}(\phi) = \frac{1}{2\sqrt{2\pi k_d\sigma_U^2}}\left(\frac{|\phi|}{k_d\sigma_U^2}\right)^{-1/2}\exp\left(-\frac{|\phi|}{2k_d\sigma_U^2}\right) \quad (11)$$

To calculate the PDF of $\Phi(t)$, it is first noted that $(s > 0)$

$$f_{P^2}(s) = f_{N^2}(s) = \frac{1}{\sqrt{2\pi}\sigma\sqrt{s}}\exp\left\{-\frac{s}{2\sigma^2}\right\} \quad (12)$$

It is easily verified that $P(t)$ and $N(t)$ as random variables are uncorrelated. Since they are Gaussian variables, they are therefore statistically independent. Hence, it follows that

$$f_\Phi(\phi) = k_d^{-1}f_{P^2}*f_{-N^2}(|\phi|/k_d) = \int_{|\phi|/k_d}^{\infty}\frac{1}{2\pi k_d\sigma^2}\cdot\frac{1}{\sqrt{s(s-|\phi|/k_d)}}\exp\left[-\frac{1}{2\sigma^2}(2s-|\phi|/k_d)\right]ds$$
$$= \frac{1}{2\pi k_d\sigma^2}\exp\left(\frac{|\phi|}{2k_d\sigma^2}\right)\int_1^\infty\frac{1}{\sqrt{u(u-1)}}\exp\left(-\frac{|\phi|}{k_d\sigma^2}u\right)du$$
$$= \frac{1}{2\pi k_d\sigma^2}K_o\left(\frac{|\phi|}{2k_d\sigma^2}\right) \quad (13)$$

where $K_o(\ )$ denotes a Bessel function of imaginary argument [cf. Gradshteyn and Ryzhik (1965)]. From the asymptotic behaviour of this Bessel function, it can be shown that

$$f_\Phi(\phi) \underset{|\phi|\to\infty}{=} \frac{1}{2\sqrt{\pi k_d\sigma^2}}\left(\frac{|\phi|}{k_d\sigma^2}\right)^{-1/2}\exp\left(-\frac{|\phi|}{2k_d\sigma^2}\right) \quad (14)$$

This expression clearly bears a close resemblance with $f_{F_d}(\phi)$. For a more detailed comparison, we have plotted both PDFs as given by (11) and (13) in Figs. 1 and 2 under the condition that $\sigma_\Phi = \sigma_{F_d}$. Since the PDFs are symmetrical, they are plotted only for positive values of the arguments. It is seen that the agreement between the two PDFs is indeed very good.

To be able to assert practical equivalence for our purposes between $\Phi(t)$ and $F(t)$, it is in addition necessary to compare the level-crossing rates of the two processes. Agreement also between these quantities indicates very similar range distributions and extreme values provided that the spectral densities of the two processes are approximately equal. The mean level-upcrossing rate $\nu_{F_d}^+(\ )$ for the drag force $F_d(t)$ is given by (Moe and Crandall 1977)

$$\nu_{F_d}^+(\phi) = \frac{\dot\sigma_U}{2\pi\sigma_U}\exp\left(-\frac{|\phi|}{2k_d\sigma_U^2}\right) \quad (15)$$

where $\dot\sigma_U^2 = \mathrm{Var}[\dot U]$.

It can be shown (Naess 1992) that the corresponding mean upcrossing rate $\nu_\Phi^+(\ )$ of $\Phi(t)$ is given to good approximation by

$$\nu_\Phi^+(\phi) = \frac{\dot\sigma\rho}{\pi\sigma}\exp\left(-\frac{|\phi|}{2k_d\sigma^2}\right) \quad (16)$$

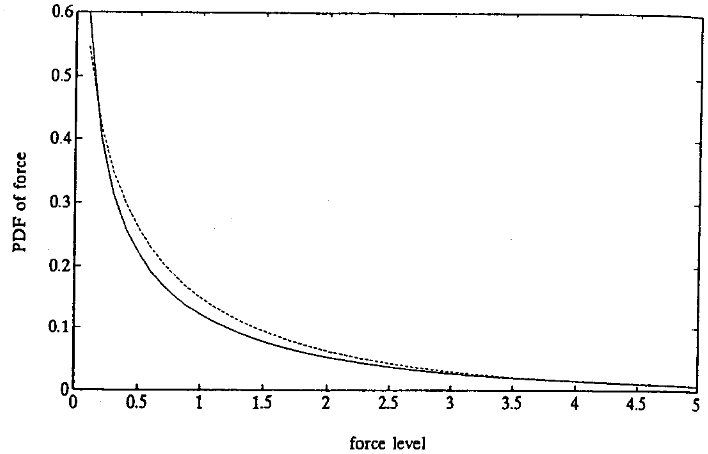

FIG. 1. Linear Plots of PDFs of $F_d(t)$ and $\Phi(t)$ for $\sigma_{F_d} = \sigma_\Phi = \sqrt{3}$ ($\sigma_U = 1$) and $k_d = 1$[—— for $f_{F_d}(\phi)$; - - - - for $f_\Phi(\phi)$]

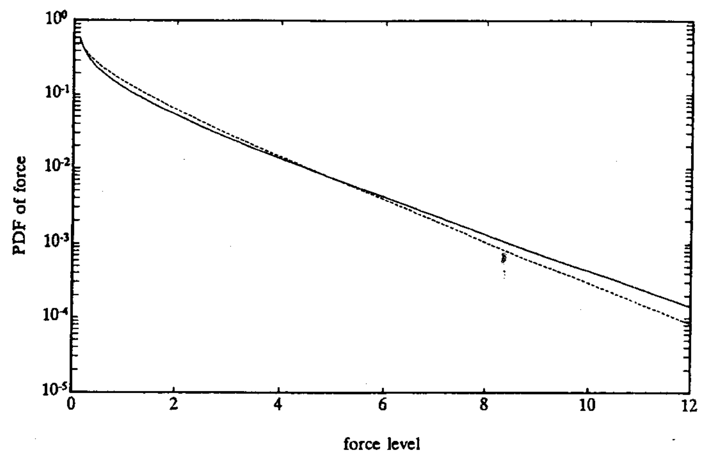

FIG. 2. Logarithmic Plots of PDFs of $F_d(t)$ and $\Phi(t)$ for $\sigma_{F_d} = \sigma_\Phi = \sqrt{3}$ ($\sigma_U = 1$) and $k_d = 1$[—— for $f_{F_d}(\phi)$; - - - - for $f_\Phi(\phi)$]

In this equation, $\rho = E[\dot P(t)N(t)]/(\dot\sigma\sigma)$, where $\dot\sigma^2 = \mathrm{Var}[\dot P(t)]$. Since $N(t)$ is the Hilbert transform of $P(t)$, it is in general a very good approximation to assume that $\rho = 1$. Within this approximation, obviously $2\nu_P^+(0) = \nu_\Phi^+(0)$. For a reasonable agreement between the spectral densities of $\Phi(t)$ and $F_d(t)$, it also follows that $\nu_\Phi^+(0) \approx \nu_{F_d}^+(0) = \nu_U^+(0)$. From this it can now be concluded by comparing (15) and (16) that there is also good agreement between the mean level-upcrossing rates of $\Phi(t)$ and $F_d(t)$. Hence, from a practical statistical point of view, the two processes are almost identical. The remaining problem is therefore to investigate to what extent it is possible to approximate the spectral density of the drag force $F_d(t)$.

SPECTRAL DENSITY APPROXIMATIONS

The mathematical problem of determining a spectral density $S(\omega)$ so that $S_\Phi(\omega)$ given by (10) is equal to $S_{F_d}(\omega)$ does not have an exact solution in general. This is demonstrated in Appendix II. However, it is shown that a fairly good approximation can be obtained by a rather simple procedure. To this end it is convenient to rewrite (10) in terms of one-sided spectra. The generic definition is as follows: $S^+(\omega) = 2S(\omega)$ for $\omega \ge 0$, and $S^+(\omega) = 0$ for $\omega < 0$. The superscript + is used to indicate that the spectral density is one-sided. Eq. (10) can then be rewritten in integral form as

$$S_\Phi^+(\omega) = 4k_d^2\int_{-\infty}^{\infty}S^+(\omega')S^+(\omega-\omega')\,d\omega' = 4k_d^2S^+*S^+(\omega) \quad (17)$$

where the asterisk signifies convolution. Hence, a genuine convolution equation is obtained, which is mathematically advantageous.

The approach to the stated problem chosen here should be considered as a preliminary procedure adopted mainly to demonstrate the feasibility of obtaining a practical solution. The method of approach is based on the underlying property of infinitely decomposable distribution laws (Loève 1977), of which the Gaussian law is a special case, which assures in particular that the sum of two independent Gaussian variables is again a Gaussian variable (Cramér 1946; Papoulis 1965). An alternative way of stating this property is that the convolution of two Gaussian densities is again a Gaussian density. The idea behind our procedure is to exploit this fact and approximate the given spectral density $S^+_{F_d}(\omega)$ by a sum of suitably chosen Gaussian densities. The objective then becomes that of representing $S^+(\omega)$ by a sum of suitably chosen Gaussian densities so that the autoconvolution of this sum, that is $S^+_\Phi(\omega)$, approximates the sum previously obtained for $S^+_{F_d}(\omega)$. Strictly speaking, no Gaussian density belongs to the space of one-sided spectral densities. But from a practical point of view, by choosing the right balance between the mean value and standard deviation, the values of the PDF for negative values of the argument can be neglected. For the sake of the discussion, it is convenient to start with the representation of $S^+(\omega)$. It is accordingly assumed that

$$S^+(\omega) = \sum_{i=1}^{n} \frac{\alpha_i}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(\omega - \omega_i)^2}{2\sigma_i^2}\right] \quad (18)$$

The corresponding expression for the spectrum of the force representation then becomes

$$S^+_\Phi(\omega) = 4k_d^2 S^+ * S^+(\omega)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{4k_d^2 \alpha_i \alpha_j}{\sqrt{2\pi(\sigma_i^2 + \sigma_j^2)}} \exp\left\{-\frac{[\omega - (\omega_i + \omega_j)]^2}{2(\sigma_i^2 + \sigma_j^2)}\right\} \quad (19)$$

It is in the interplay between these two equations that the desired approximation is worked out. Given a representation of $S^+_{F_d}(\omega)$ as a sum of Gaussian PDFs consisting of say m terms, it is recognized from (18) and (19) that the number of terms n in the representation of $S^+(\omega)$ should be around $m/2$. This clearly shows that in general, only an approximate solution can be obtained.

A way of increasing the accuracy of the spectral representation $S^+_\Phi(\omega)$, is to augment the force representation model itself. This is done by introducing a force process $\Phi(t) = \Phi_1(t) + \cdots + \Phi_l(t)$, where the $\Phi_j(t)$, $j = 1, \ldots, l$ are independent processes similar to that defined by (7). By choosing l large enough, the corresponding spectral representation can be made as accurate as desired. The drawback of this approach is that the statistics of the obtained force representation may deteriorate somewhat relative to the target statistics. Also, part of the dynamic analysis has to be carried out l times.

We shall not embark on a detailed study of the approximation process here, mainly because this has to be done only once. To each wave condition, that is, to each particular (normalized) drag force spectrum, a corresponding spectral density $S^+(\omega)$ can be specified. For a given model of the spectral density, a more systematic procedure can be developed using the system identification methodology proposed by Bendat et al. (1992) in conjunction with standard optimization techniques. In this paper we shall limit ourselves to illustrate the relation between (18) and (19) for the specific case of a wave force spectrum reported by Borgman (1967b), indicating the possibility of reproducing a reasonably accurate force spectrum. Strictly speaking, the reported force spectrum also contains an inertia force term, but that is not important for our discussion here. Plots of $S^+(\omega)$ and the corresponding $S^+_\Phi(\omega)$ are presented in Fig. 3. It is seen that a fair agreement has been obtained between the spectrum $S^+_\Phi(\omega)$ and the obtained wave force spectrum presented by Borgman (1967b).

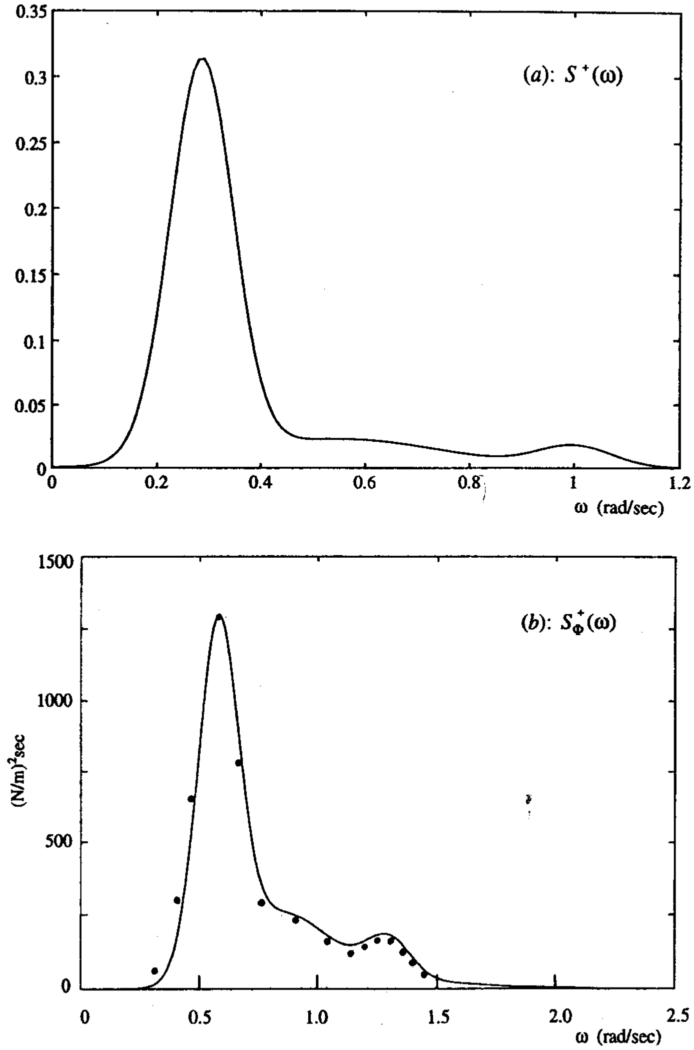

FIG. 3. Plots of Spectral Density: (a) $S^+(\omega)$ and Resulting (b) $S^+_\Phi(\omega)$ [• = Experimental Drag Force Spectrum (Borgman 1967b)]

It is emphasized, however, that in a practical application the starting point is a model wave spectrum, for example, a JONSWAP spectrum (Sarpkaya and Isaacson 1981), from which an appropriate water particle velocity spectrum is derived. The drag force spectrum is then calculated by invoking the standard drag force model. Depending on the dynamic characteristics of the structure to be analyzed, this may lead to either a linearized force spectrum or a more complete expression containing self-convolutions of the water particle velocity spectrum in addition to the linearized term (Borgman 1967b).

To make practical applications to extensive structures simple, an issue that has to be addressed is the problem of transformation of the fitted spectrum $S^+(\omega)$ of the auxiliary velocity process $P(t)$ for different locations on the structure relative to a reference point. The brute force method is simply to determine the drag force spectrum for each location according to standard transformation rules of the water particle velocity spectrum (Sarpakaya and Isaacson 1981), and then do the fitting of the spectral density of the auxiliary velocity process for each separate location, and at the same time introduce an appropriate correlation structure between the location specific auxiliary velocity processes. Clearly, a preferable procedure is a direct transformation of the auxiliary velocity process associated with a chosen reference point. This approach is pursued in subsequent work.

DYNAMIC RESPONSE

From (1) it is found that the linear transfer function $\hat{L}_1(\omega)$ of the dynamic model is given by

$$\hat{L}_1(\omega) = (-\omega^2 M + i\omega C + K)^{-1} \quad (20)$$

Adopting the approximation $\Phi(t) = F_d(t)$, and invoking (8), it can then be shown that the dynamic response $X_d(t)$ to the drag force can be represented by the equation (Naess and Ness 1992)

$$X_d(t) = \sum_{i=-n}^{n}{}_{0} \sum_{j=-n}^{n}{}_{0} Q_{ij} e^{i(\omega_i - \omega_j)t} B_i B_j^* \quad (21)$$

where

$$Q_{ij} = k_d \sqrt{S(\omega_i)S(\omega_j)} \hat{L}_1(\omega_i - \omega_j)[1 - \hat{\chi}(\omega_i)\hat{\chi}(\omega_j)^*] \quad (22)$$

This shows that the dynamic response is given by a quadratic transformation of the underlying auxiliary process $P(t)$. The corresponding quadratic transfer function $\hat{H}_2(\omega, \omega')$ is identified as

$$\hat{H}_2(\omega, \omega') = k_d \hat{L}_1(\omega + \omega')[1 - \hat{\chi}(\omega)\hat{\chi}(\omega')] \quad (23)$$

Following the developments in Naess and Ness (1992), it can now be shown that the dynamic response can be expressed by the relation

$$X_d(t) = \sum_{j=1}^{n} \lambda_j [W_j(t)^2 - W_{-j}(t)^2] \quad (24)$$

where the λ_j = the positive eigenvalues of the matrix $\mathbf{Q} = (Q_{ij})$. For each positive eigenvalue there is a corresponding negative eigenvalue of equal magnitude. The $W_j(t)$ are real, Gaussian $N(0, 1)$-processes defined as follows:

$$W_j(t) = \sum_{i=-n}^{n}{}_{0} v_{j,i} e^{i\omega_i t} B_i \quad (25)$$

where the $\mathbf{v}_j = (v_{j,-n}, \ldots, v_{j,n})^T$ are the orthonormal eigenvectors corresponding to the λ_j. It can be shown (Naess and Ness 1992) that $W_{-j}(t)$ is the Hilbert transform of $W_j(t)$ for every j.

The representation given by (24) is very convenient for calculating the statistics of the dynamic response. The first four cumulants, k_j, $j = 1, \ldots, 4$, are given by (Naess 1987)

$$E[X_d(t)] = k_1 = 0 \quad (26)$$

$$\mathrm{Var}[X_d(t)] = k_2 = 4 \sum_{j=0}^{n} \lambda_j^2 \quad (27)$$

$$k_3 = 0 \quad (28)$$

$$k_4 = 96 \sum_{j=0}^{n} \lambda_j^4 \quad (29)$$

No closed-form expression for the PDF of $X_d(t)$ is known, but an accurate numerical method for calculating this PDF has been developed. This method is described in detail by Naess and Ness (1992). Calculation of upcrossing rates can be carried out as shown by Naess (1987). This is exemplified in the next section by calculation of statistical moments and PDF of the dynamic response of a compliant drag-dominated structure subjected to random waves.

NUMERICAL EXAMPLE

To illustrate the results of a dynamic analysis, we calculate the statistics of the displacement response of a slender drag-dominated offshore structure subjected to random waves. Specifically, it is assumed that the drag force is characterized by the spectral density described previously in this paper. This corresponds to the numerical value k_d = 170. Further, it is assumed that the structure can be represented as a linear SDOF dynamical system with the following parameters: structural and added mass M = 500 kg/m; total damping ratio ξ = 0.10; natural period T_e = 5 s.

By this, all the required information to determine the matrix $\mathbf{Q} = (Q_{ij})$ is available. A standard library routine is used to calculate the positive eigenvalues of the obtained matrix, and the first four cumulants of the displacement response are found to be $k_1 = k_3 = 0$, $k_2 = 0.2827 \cdot 10^4$ mm^2 and $k_4 = 0.3322 \cdot 10^8$ mm^4. This leads to a kurtosis $\gamma_2 = 7.16$, indicating that the response statistics deviate significantly from the Gaussian case, which has $\gamma_2 = 3$.

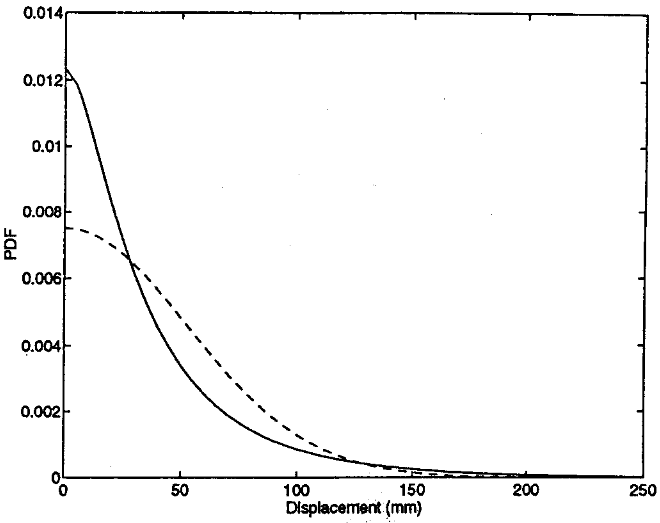

FIG. 4. Linear Plot of PDF of Displacement Response (—— = Exact PDF; - - - - = Gaussian PDF)

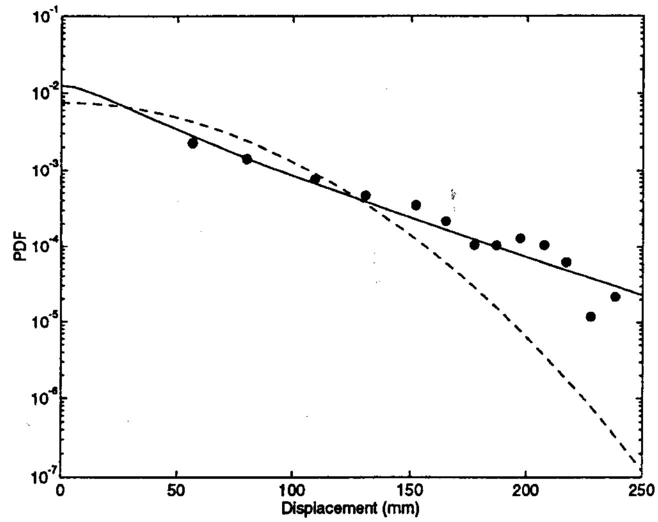

FIG. 5. Logarithmic Plot of PDF of Displacement Response (Key as in Fig. 4; • = Simulation Results)

The PDF of the displacement response is shown in Figs. 4 and 5. The corresponding Gaussian PDF with the same mean value and standard deviation is also shown. As expected, the Gaussian response assumption leads to a substantial underestimation of large responses.

To verify the general features of the calculated response statistics, we have plotted the results of Monte Carlo simulations in Fig. 5. The plotted simulation results have been obtained for a nodal displacement response of a jack-up structure specified by a linear dynamic model having a damping ratio of 10%, as in our example, which was subjected to pure drag forces with zero current. The kurtosis of the empirical distribution was found to be 7.26, which is indeed very close to the theoretical value obtained earlier. By scaling the PDF derived from the simulation results to have the same standard deviation as in our example, one would expect the two PDFs to agree. This is clearly borne out by Fig. 5.

CONCLUSIONS

A new representation of the drag forces on an offshore structure in the absence of current is proposed. It is demonstrated that the proposed force representation to a large extent has the same statistical properties as the original drag force. The spectral density of the force representation can be made to fit that of the drag force reasonably well.

The main advantage of the proposed force representation is that it becomes possible to carry out a dynamic analysis in the frequency domain without having to linearize the drag force. The only limitation that has to be imposed, consistent with other state-of-the-art nonlinear second-order stochastic analysis methods, is that the equations of motion are linear, or have been linearized. This ensures that the response process can be expressed as quadratic transformations of Gaussian processes. The techniques for the frequency domain analysis of such transformations have been developed extensively over the last few years, and are fairly well established. For example, both PDFs and extreme value distributions can be calculated.

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APPENDIX I. SPECTRAL DENSITY OF $\Phi(t)$

To determine the spectral density of the drag force representation $\Phi(t)$, we start by finding the autocorrelation function $R_\Phi(\tau) = E[\Phi(t)\Phi(t + \tau)]$. From (8) it follows that

$$R_\Phi(\tau) = k_d^2 \sum_{i=-n}^{n}{}_0 \cdots \sum_{l=-n}^{n}{}_0 \sqrt{S(\omega_i)S(\omega_j)S(\omega_k)S(\omega_l)}\hat{K}_2(\omega_i, -\omega_j)$$

$$\cdot \hat{K}_2(\omega_k, -\omega_l) \cdot e^{i(\omega_i-\omega_j)t} e^{i(\omega_k-\omega_l)(t+\tau)} E[B_i B_j^* B_k B_l^*](\Delta\omega)^2 \quad (30)$$

Taking advantage of a well-known result from the theory of Gaussian variables (Lin 1967), it is found that

$$E[B_i B_j^* B_k B_l^*] = E[B_i B_j^*]E[B_k B_l^*] + E[B_i B_k]E[B_j^* B_l^*]$$

$$+ E[B_i B_l^*]E[B_j^* B_k] = \delta_{i,j}\delta_{k,l} + \delta_{i,-k}\delta_{-j,l} + \delta_{i,l}\delta_{j,k} \quad (31)$$

Substituting this expression into (30), we get

$$R_\Phi(\tau) = \left\{ k_d \sum_{i=-n}^{n}{}_0 S(\omega_i)\hat{K}_2(\omega_i, -\omega_i)\Delta\omega \right\}^2$$

$$+ k_d^2 \sum_{i=-n}^{n}{}_0 \sum_{j=-n}^{n}{}_0 2S(\omega_i)S(\omega_j)|\hat{K}_2(\omega_i, -\omega_j)|^2 e^{i(\omega_i-\omega_j)\tau}(\Delta\omega)^2 \quad (32)$$

The first term on the right-hand side of this equation is zero since $\hat{K}_2(\omega, -\omega) = 0$.

Writing (32) in integral form leads to the following equation:

$$R_\Phi(\tau) = k_d^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 2S(\omega)S(\omega')|\hat{K}_2(\omega, -\omega')|^2 e^{i(\omega-\omega')\tau}\, d\omega\, d\omega'$$

$$= k_d^2 \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} 2S(\omega)S(\Omega - \omega)|\hat{K}_2(\omega, \Omega - \omega)|^2\, d\omega \right\} e^{i\Omega\tau}\, d\Omega \quad (33)$$

where the variable substitution $(\omega, \omega') \rightarrow (\omega, \Omega) = (\omega, \omega - \omega')$ has been implemented.

From (33) we can now identify the spectral density $S_\Phi(\omega)$ of $\Phi(t)$ as

$$S_\Phi(\Omega) = k_d^2 \int_{-\infty}^{\infty} 2S(\omega)S(\Omega - \omega)|\hat{K}_2(\omega, \Omega - \omega)|^2\, d\omega \quad (34)$$

Invoking (9) and using the properties of $\hat{\chi}(\omega)$, the validity of (10) follows immediately.

APPENDIX II. SPECTRAL DENSITY INEXACTITUDE

It is shown that the problem of finding a one-sided spectral density whose autoconvolution equals a given one-sided spectrum cannot in general be solved exactly.

To this end, assume that a one-sided spectrum $S_0^+(\omega)$ is given. We want to find a one-sided spectrum $S^+(\omega)$ that satisfies the equation

$$S_0^+(\omega) = S^+ * S^+(\omega) \quad (35)$$

The (inverse) Fourier transform, denoted by a tilde, of a spectral density is defined by

$$\tilde{S}^+(t) = \int_{-\infty}^{\infty} S^+(\omega)e^{i\omega t}\, d\omega \quad (36)$$

By application of the (inverse) Fourier transform to (35), it is found that

$$\tilde{S}_0^+(t) = \tilde{S}^+(t)^2 \quad (37)$$

which leads to the equation

$$\tilde{S}^+(t) = \sqrt{\tilde{S}_0^+(t)} \quad (38)$$

If (35) has a solution, it is given by the direct Fourier transform

$$S^+(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{S}^+(t)e^{-i\omega t}\, dt \quad (39)$$

From (36) it can be seen that $\tilde{S}_0^+(t)^* = \tilde{S}_0^+(-t)$, and (38) then implies that the same relation is satisfied by $\tilde{S}^+(t)$. Taking account of this, it follows from (39) that $S^+(\omega)$ is a real function. However, it is not in general nonnegative. In addition, it may even be nonzero for negative values of the argument.

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