PARAMETRIC IDENTIFICATION AND SENSITIVE STUDY OF A NONLINEAR MOORED STRUCTURE

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ABSTRACT

The highly nonlinear responses of compliant ocean structures characterized by a large-geometry restoring force and a coupled fluid-structure interaction excitation are of great interest to ocean engineers. Practical modeling, parameter identification, and incorporation of the inherent nonlinear dynamics in the design of these systems are essential and challenging. An experimental mooring system exhibiting nonlinear behavior due to geometric nonlinearity of mooring line angles and the complexity of hydrodynamic excitations is chosen for the study. An Independent-Flow-Field (IFF) model based on Morison equation and its associated nonlinear system identification algorithm is used to evaluate the system parameters of an experimental submerged mooring system. With the input wave and output system response data known, based on multiple input/single-output linear analysis of reverse dynamic system, the methodology identifies the linear and nonlinear system properties. A sensitive analysis is conducted to investigate the coupled hydrodynamic forces modeled by the Morison equation, nonlinear stiffness from mooring lines and nonlinear response.

INTRODUCTION

Complex nonlinear responses including harmonics, sub- and super-harmonics and chaos, have been observed and demonstrated in various compliant ocean systems characterized by large-geometry nonlinear mooring restoring force and coupled fluid-structure interaction exciting force. When examining the responses of these highly nonlinear systems, it is important to develop sophisticated analytical models that the details of the nonlinear responses can be captured accurately. However, at the same time the models have to be sufficiently simple that modern geometrical analysis techniques and efficient computer simulations can be performed. Deterministic analysis theories and numerical prediction techniques of relatively simple models have been developed to analyze the complex nonlinear phenomena for single-point mooring systems (Gottlieb et al 1992), ships (Bishop and Virgin 1988), and multi-point mooring systems (Bernitsas and Chung 1990, Gottlieb and Yim 1993). Lin and Yim (1996 and 1997) developed stochastic extensions of these techniques and corresponding analyses. They provided guidelines for interpreting field and experimental observations where randomness cannot be neglected.

Experiments on a single-degree-of-freedom (SDOF) and a multidegree-freedom (MDOF) nonlinear multi-point moored submerged sphere subject to wave excitations have been conducted at the O.H.Hinsdale Wave Laboratory at Oregon State University (Yim et al 1993, Narayanan and Yim, 2001). Measured results for both systems indicated that various types of nonlinear responses including harmonic, sub- and super-harmonics and chaotic responses were present. In this study, the wave input and the system responses measured during the test are employed for parameter identification.

The applicability of two different models, (A) relative-velocity (RV) and (B) independent flow field (IFF) models and their corresponding algorithms have been examined and IFF model with the nonlinear-structure nonlinearly-damped (NSND) algorithm is determined to be the most suitable analytical model for the experimental system (Narayanan and Yim, 2001). In this paper, the resulting system using the identified parameters obtained based on the NSND algorithm is employed to predict the responses of the fluid-structure interaction of the SDOF, symmetric spherical mooring system.

Using the measured wave excitation and response data together with the identified system parameters, a detailed study is performed on the response behavior of the system under consideration. A sensitivity analysis is conducted to determine the optimal range of system parameters and understand the effect of varying the stiffness and damping coefficients on the system response.

The independent flow field (IFF) model requires the knowledge of inertia and drag coefficients, C_m and C_d respectively for the evaluation of hydrodynamic force. Theoretical studies of unsteady motions involving a sphere in a real fluid have so far been restricted to small Reynolds numbers (Wang 1965, Hjelmfelt *et al* 1967). The C_m for fixed spheres was found to vary between 1.43 and 1.73 within the range of $0.2 \le KC \le 3.2$ (Harleman and Shapiro 1958). For a pilot study in the ocean on wave-induced forces on a fixed sphere with the inertia forces dominating the total force and Re ranging from 10^5 to 5 x 10^5 , Grace and Zee (1978) found the average inertia coefficient to be 1.21 and the C_d to be 0.4. With the coefficients dependent on KC and Re, reasonable estimates of the hydrodynamic coefficients for a sphere are within the following bounds, $0.1 \le C_d \le 1.0$ and $1.0 \le C_m \le$ 1.5 (Grace and Casino 1969, Grace and Zee 1978). In this study, NSND algorithm is also employed to evaluate the effects of hydrodynamic coefficients on system response by varying C_m and C_d within a range.

EXPERIMENTAL DATA

An experiment was performed at the O. H. Hinsdale Wave Laboratory at Oregon State University on a multi-point moored submerged sphere subject to wave excitations. The experimental model consists of a submerged moored neutrally buoyant sphere excited by regular and random waves. Springs were attached to the sphere to provide the restoring force at an angle of 90° (four-point system). The sphere was restricted to move only in the surge direction by passing a rigid steel rod through the center of the sphere. The equations of motion for this SDOF moored structural system subject to excitations consist of periodic waves perturbed with random noise has been derived (Yim et al 1993, Narayanan and Yim, 2001).

Eight tests were conducted on the sphere with periodic plus white noise excitations (Yim et al 1993). All the experimental data have wave period of T = 2 seconds with varying wave heights and noise/signal ratio. The wave displacement and surge response of the sphere were measured and the wave velocity and acceleration were numerically evaluated using a central-difference method (Gerald and Wheatley 1989). Each of the tests displays a certain degree of subharmonics in the sphere movement. The data sets SL1, SL2, SM1, SM2, SM3, SH1, SH2 and SH3 are grouped according to wave excitation amplitudes, where 'S' stands for single-degree-freedom, and 'L', 'M' and 'H' represents low, medium and high wave amplitudes, respectively. A typical segment of the wave time series and its corresponding spectra, and a typical segment of the response time series and its corresponding spectra for the high wave amplitude data sets grouped are given in Fig.1. The mean spectra, SH is also shown in the figure and is considered to be representative of the group. The input wave characteristics such as wave height (H), C_m, C_d, Keulegan Carpenter number (KC) and Reynolds number (Re) are shown in the Table 1.

Data	H (m)	Cm	C _d	KC	Re
SL1	0.17	1.4	0.1-0.9	0.56	5.70E4
			(0.5)		
SL2	0.24	1.4	0.1-0.9	0.79	7.80E4
			(0.5)		
SM1	0.35	1.3	0.1-0.9	1.18	1.20E5
			(0.5)		
SM2	0.36	1.3	0.1-0.9	1.18	1.20E5
			(0.5)		
SM3	0.49	1.3	0.1-0.9	1.57	1.60E5
			(0.5)		
SH1	0.66	1.1	0.1-0.9	2.16	2.20E5
			(0.5)		
SH2	0.66	1.1	0.1-0.9	2.18	2.22E5
			(0.5)		
SH3	0.67	1.1	0.1-0.9	2.20	2.30E5
			(0.5)		

Table 1 Input wave characteristics of the SDOF subharmonic data





The sampling interval used in the experiment was 0.0625 second (16 Hz), which yields a Nyquist frequency of 8 Hz. The total number of samples of the excitation and response time histories for spectral simulations is 8192 (512 seconds), with sub-record lengths of 1024 for the Fourier transforms (64 seconds).

IFF Model Algorithm

By considering surge as the generalized displacement coordinate, the governing equation of motion for the SDOF mooring system can be written as

$$m\ddot{x}(t) + C_s \dot{x}(t) + R(x(t)) = f(t)$$
 (1)

where m = mass of the sphere, f(t) = hydrodynamic force acting on the sphere, $C_s =$ linear structural damping coefficient, R(x(t)) =nonlinear restoring force, $x(t), \dot{x}(t), \ddot{x}(t)$ are the system displacement, velocity and acceleration respectively.

The restoring force can be approximated by a third order polynomial obtained through a least square approximation (Narayanan and Yim, 2001). The polynomial is expressed as

$$R'(x(t)) \cong a_1 x(t) + a_2 x(t)^2 + a_3 x(t)^3$$
⁽²⁾

For the Independent Flow Field (IFF) model, nonlinear interaction between the fluid and structural velocities is decoupled and the hydrodynamic force is evaluated using Eq.3. Nonlinear structural damping force and the wave excitation drag force can then be treated separately.

$$f(t) = \rho \forall C_{m} \dot{u}(t) - m_{a} \ddot{x}(t) + \frac{\rho}{2} A_{p} C_{d} u(t) |u(t)|$$

$$-\frac{\rho}{2} A_{p} C_{d} \dot{x}(t) |\dot{x}(t)|$$
where
$$\forall = \frac{\pi}{2} D^{3}$$
(4a)

$$A = \frac{\pi D^2}{(4b)}$$

$$m_a = \frac{\pi}{\epsilon} D^3 C_a \tag{4c}$$

 ρ = mass density, D = diameter of sphere, u, \dot{u} (t) is the water particle velocity and acceleration respectively in surge direction C_a = added mass coefficient, C_d' = nonlinear structural damping coefficient, C_m = hydrodynamic inertia coefficient and C_d = hydrodynamic drag coefficient. The values of $C_{\rm m}$ and $C_{\rm d}$ may be obtained from wave experiments while the coefficients Ca and Cd' are derived from oscillating sphere in otherwise calm water. Also

$$C_{m}, C_{d} = f\left(Re_{F} = \frac{u_{o}D}{\upsilon}, KC_{F} = \frac{u_{o}T}{D}\right)$$
(5a)

$$C_a, C_d' = f\left(Re_N = \frac{\dot{x}_o D}{\upsilon}, KC_N = \frac{\dot{x}_o T_o}{D}\right)$$
 (5b)

where u_0 , \dot{x}_0 = amplitudes of the water particle and structure velocity, respectively, T and $T_o =$ periods of oscillation of water particle and structure, respectively (they are often equal), $\upsilon =$ viscosity of the fluid, Re = Reynolds number, KC = Keulegan-Carpenter number. Note that suffix 'F' refers to far field and suffix 'N' to near field (Chakrabarti 1987).

The IFF assumption results in the following nonlinear equation of motion given by

$$\begin{pmatrix} m + m_{a} \\ \dot{x}(t) + C_{s} \\ \dot{x}(t) + a_{1} \\ x(t) + a_{2} \\ x^{2}(t) + a_{3} \\ x^{3}(t)$$

$$+ \rho C_{d} \\ \frac{\pi D^{2}}{4} \\ \dot{x}(t) \\ \dot{x}(t) \\ = f_{1}(t)$$
(6a)
(6b)

where,

$$f_{1}(t) = \rho \frac{\pi}{6} D^{3} C_{m} \dot{u}(t) + \rho C_{d} \frac{\pi D^{2}}{4} u(t) |u(t)|$$

$$C_{s} = 2\zeta_{1} \sqrt{(a_{1}(m + m_{a}))}$$
(6c)
$$\zeta = domning coefficient$$

 $\zeta_1 = \text{damping coefficient}$

The corresponding single-input/single-output nonlinear forward model with feedback is shown in Fig.2a. The nonlinear forward model is converted to reverse dynamic model by applying the Reverse-Multiple-Input/single-Output (R-MI/SO) procedures (Bendat 1998). The corresponding reverse dynamic four-input/single-output nonlinear model without feedback is shown in Fig.2b. Using the frequency response functions, linear and nonlinear system parameters are identified (Narayanan and Yim, 2001).





b)



Fig.2: The nonlinear-structure nonlinearly-damped (NSND) model a) with feedback b) without feedback

RESULTS

The nonlinear system identification algorithm. NSND for the IFF model is applied to all the data sets using the R-MI/SO technique and system parameters are identified. Using the identified parameters, the response is evaluated for the model using a fourthorder Runge-Kutta method (Gerald and Wheatley 1989).

A comparison of time series and spectra between the identified response using the IFF model and the experimental response for a typical experimental data is shown in Fig.3. The system parameters, a_1 , a_2 , a_3 , ζ_1 and C_{d1} identified for all the test data using the IFF model are given in Table 2.

a)



Fig.3 Comparison of simulated response using NSND model with the experimental response: a) time series, b) spectra

Data	a ₁	a ₂	a ₃	C _{d1}	$\zeta_1(\%)$	f _{n1} (Hz)
	(N/m)	(N/m^2)	(N/m^3)			
SL1	128.8	315.6	721.3	2.5	3.5	0.22
SL2	125.6	280.1	814.7	3.5	3.4	0.23
SM1	128.8	260.8	863.0	3.0	3.0	0.23
SM2	132.0	257.6	769.6	1.5	2.9	0.24
SM3	125.6	206.1	689.1	1.0	2.8	0.23
SH1	128.8	209.3	689.1	0.8	3.0	0.23
SH2	128.8	209.3	689.1	0.2	3.2	0.23
SH3	125.6	190.0	627.9	0.3	3.1	0.22

Table 2 Identified system parameters of the experimental data

SENSITIVITY ANALYSIS

A parametric study is performed to determine the sensitivity of the system to variations in the parameters. Specifically, each parameter is varied in prescribed increments while keeping all other identified parameters constant (Table 2) and the surge response is computed for each variation by solving (Eq.6a). The simulated responses using the identified parameters are compared against each other in both the time and frequency domains.

From the parametric study, an optimal range and the most suitable value of the system parameters are obtained and tabulated in Table 3. Because the data sets belong to L, M and H groups exhibit similar behavior; only the mean of the resulting spectra for each variation is discussed in the following paragraphs.

The effect of varying linear stiffness coefficient, a_1 on SL, SM and SH are demonstrated in Fig.4. The spectral density normalized with the variance of experimental wave data (S_{xxn}) is plotted against frequency for a_1 from 58.0 to 202.9 N/m or a1n (the ratio of instantaneous value of a_1 to the best value of a_1 as given in Table 3a from 0.5 to 1.6. It can be observed that there is a slight increase in the primary resonance response as a_1 increases. The subharmonic resonance region shifts towards the right with increasing a_1 . The trend can be observed more clearly (from SL to SH) as the wave amplitude increases.

r			2		1	
Data	$a_1 (N/m)$	$a_2(N/m^2)$	$a_3 (N/m^3)$	C_{d1}	ζ_1	f _n
					(%)	(Hz)
SL1	122.4-32.0	48.3-	157.8-	1.5-	1.0-	0.23
	(128.8)	378.0	1410.4	2.5	4.0	
		(215.7)	(772.8)	(2.0)	(3.0)	
SL2	119.1-	48.3-	157.8-	1.5-	1.0-	0.23
	132.0	380.0	1410.4	2.5	4.0	
	(125.6)	(215.7)	(772.8)	(2.0)	(3.0)	
SM1	122.4-	48.3-	157.8-	1.5-	1.5-	0.23
	132.0	380.0	1255.8	2.5	4.0	
	(128.8)	(215.7)	(708.4)	(2.0)	(3.3)	
SM2	122.4-	141.7-	470.1-	0.3-	2.0-	0.24
	135.2	286.6	933.8	0.7	4.0	
	(128.8)	(215.7)	(708.4)	(0.5)	(3.0)	
SM3	122.4-	141.7-	470.1-	0.3-	2.0-	0.23
	135.2	286.6	933.8	0.7	4.0	
	(125.6)	(215.7)	(708.4)	(0.5)	(3.0)	
SH1	122.4-	167.4-	550.6-	0.1-	2.5-	0.23
	138.5	286.6	933.8	0.2	4.0	
	(132.0)	(225.4)	(740.6)	(0.15)	(3.3)	
SH2	122.4-	190.0-	627.9-	0.1-	2.0-	0.23
	135.2	238.3	772.8	0.2	4.0	
	(128.8)	(215.7)	(708.4)	(0.15)	(3.0)	
SH3	122.4-	199.6-	660.1-	0.1-	2.0-	0.22
	135.2	238.3	772.8	0.2	4.0	
	(132.0)	(219.0)	(708.4)	(0.15)	(3.0)	

Table 3 Identified system parameters from the sensitivity analysis of the SDOF subharmonic data

When a_2 is increased from 0 to 476.6 N/m², there is no significant change in the data group SL as shown in Fig.5a. However, the response in the secondary resonance region increases from $a_2n = 0$ to 2.5 for SM and SH, and the effects are more pronounced for the latter (Fig.5 b and c). The total energy of the response in the primary resonance region is affected by changing a_2 .





4c)



Fig.4 Effect of a1 on SDOF system behavior: a) SL, b) SM c) SH



Fig.5 Effect of a2 on SDOF system behavior: a) SL b) SM c) SH

Fig.6 shows that varying a_3 from 0 to 1568.1 N/m³ or a3n from 0 to 2.5 affects only the response in the secondary resonance region, which decreases as a_3 increases. The variation is most noticeable for SH in Fig.6c.





Fig.6 Effect of a₃ on SDOF system behavior: a) SL b) SM c) SH

With regards to varying the linear structural damping coefficient ζ_1 from 0 to 0.1, it is observed that the response in the subharmonic region decreases with increasing damping while the primary resonance region remains unaffected as demonstrated in Fig.7. This result indicates that the subharmonic response is sensitive to structural damping. This phenomenon is often observed in responses of nonlinear systems.

The effects of varying C_{d1} on the identified response are demonstrated in Fig.8. It shows that the secondary resonance region generally decreases with increasing Cdl. However, the optimum range that identify response comparable to the experimental response differs for the data groups SL, SM and SH. The most suitable value goes as high as 2 for SL and it decreases to 0.5 for SM and 0.15 for SH. This apparent behavior is probably caused by the inability of the model to approximate accurately the actual nonlinear behavior of the complex damping mechanism of the SDOF configuration. In the physical system, with the rod passing through the center of the sphere (to restrict vertical and rotational motions), the Coulomb frictional component is proportional to the magnitude of the normal reaction force between the sphere and the supporting rod. Because the sphere is neutrally buoyant, this normal force is proportional to the magnitude of the oscillatory lift force. The nonlinear effects become more severe at the lower wave amplitudes prominent due to the sticky (stop and go, highly nonlinear) motion of the sphere, thus affecting the response prediction capability of the model.



7b)



Fig.7 Effect of ζ_1 on SDOF system behavior: a) SL b) SM c) SH



Fig.8 Effect of C_{d1} on SDOF system behavior: a) SL b) SM c) SH

EFFECTS OF HYDRODYNAMIC COEFFICIENTS ON SYSTEM RESPONSE

The IFF model requires the knowledge of C_d and C_m for the evaluation of hydrodynamic force on the sphere. As mentioned earlier, the effect of C_m and C_d on the nonlinear response has not been studied before according to the authors' knowledge. In order to

investigate the response behavior of the system, C_m is varied within the range of 1 - 1.5 and the NSND algorithm is then applied. The identified properties are tabulated for different C_m in Table 4a. From the table, magnitudes of C_a , a_1 , a_2 , a_3 , C_d , ζ_s and C_s increase with increasing C_m . The natural frequency identified is constant for all the cases. The responses simulated using the parameters are compared with the measured response in Fig.9a. The primary resonance energy of all the predicted responses is practically constant and agrees favorably with that of the measured response. Note that the subharmonic energy of the predicted response decreases with increasing values of inertia coefficient and $C_m = 1.3$ matches well with the experimental response.

The drag coefficient C_d is varied between 0.2 -- 1.0 and the properties are identified in Table 4b. The parameters remain consistent for different values of C_d . The responses simulated using the parameters are compared with the measured response in Fig.9b, and it can be observed that the response does not change significantly with varying values of C_d . Based on the water depth to wavelength (h/L) and diameter to wave height (D/H) ratios (Nath and Harleman 1970), the inertia effects dominate the total forces and the response, as expected, is found to be relatively insensitive to changes in C_d .

4a)

Cm	Ca	Cd	a ₁	a ₂	a ₃	Ċ	ζ1	f _n
			N/m	N/m ²	N/m ³			
1.10	0.11	1.0	119.1	167.4	911.3	0.18	0.02	0.24
1.20	0.21	1.0	122.4	180.3	924.1	0.18	0.03	0.24
1.30	0.32	1.0	132.0	215.7	1020.7	0.18	0.03	0.24
1.40	0.42	1.0	151.3	235.1	1175.3	0.19	0.03	0.24
1.50	0.51	1.0	161.0	244.7	1284.8	0.19	0.03	0.24

4b)

C _d	Cm	C _d	a ₁	a ₂	a ₃	Ċ,	ζ_1	f _n
			N/m	N/m ²	N/m ³			
0.20	1.3	0.3	128.8	215.7	972.4	0.18	0.021	0.24
0.50	1.3	0.3	128.8	219.0	988.6	0.18	0.033	0.24
0.80	1.3	0.3	132.0	219.0	1004.7	0.19	0.032	0.24
1.00	1.3	0.3	132.0	219.0	1004.7	0.19	0.035	0.24
1.20	1.3	0.3	132.0	222.2	1020.7	0.19	0.035	0.24

Table 4 Identified system parameters using NSND model by varying hydrodynamic coefficients: a) C_m , b) C_d

9a)



9b)



Fig.9 Comparison of identified response using NSND model with the measured response by varying hydrodynamic coefficients: a) $C_{m_{\rm c}}$ b) C_d

CONCLUDIG REMARKS

The independent flow field (IFF) model and its corresponding algorithm, nonlinear-structure linearly-damped (NSLD), has been examined and used to evaluate the linear and nonlinear system parameters of the experimental system. The sensitivity analysis of the SDOF system presented here reveals that the effects of variations in system parameters on the predicted responses become more significant with increasing wave excitation amplitude. Three groups are established among the tests depending on low, medium or high wave excitation amplitude based on the response behavior. The response variation becomes more significant with increasing wave amplitude. The optimal value and range of nonlinear structural damping coefficient varies among the tests. This apparent behavior is probably caused by the inability of the model to approximate accurately the actual nonlinear behavior of the complex damping mechanism of the SDOF configuration as the Coulomb frictional component is not included in the mathematical model. For the set of experimental data considered, when C_m is varied between 1.0 –1.5, the subharmonic energy of the predicted response decreases with increasing values of inertia coefficient. Because the experimental wave-structure interaction characteristics fall within the inertia regime, it is not possible to accurately evaluate the drag coefficients. Indeed, the response is observed to be insensitive to variations in C_d.

ACKNOWLEDGEMENT

Financial support from the United State Office of Naval Research, Grant No. N00014-92-J-1221, is gratefully acknowledged.

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