A METHODOLOGY FOR *ANALYSIS AND DESIGN* OF SENSITIVE NONLINEAR OCEAN SYSTEMS

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Abstract. A methodology for systematic investigation and incorporation of highly nonlinear sensitive behaviors into reliability analysis and design of ocean structural systems using modern geometric and stochastic techniques is summarized in this paper. The representative systems examined are characterized by a nonlinear restoring force and a coupled fluid-structure interaction exciting force. Regular wave excitations are first examine to demonstrate the complex nature of nonlinear responses. A deterministic analysis procedure which includes the method of harmonic balance to solve for approximate solutions, a variational method for their stability analysis, and a Melnikov approach (global stability) to identify the existence of complex nonlinear (and possibly chaotic) motions is demonstrated. Locations of nonlinear primary and secondary resonances are identified in the parameter space. Highly nonlinear phenomena are identified and routes of response transition to chaos are demonstrated. The presence of random excitation components is next taken into account to assimilate nearly periodic wave excitations. A stochastic analysis procedure which includes a stochastic Melnikov approach and the Fokker-Planck formulation is then presented. A criterion based on the stochastic Melnikov process demonstrating the noise effects on chaotic response is developed. The Fokker-Planck equation is derived and solved for response probability density functions to interpret the response behavior from an ensemble perspective. The systematic stochastic analysis procedure is also applicable for response analysis to purely random excitations. An experimental study assimilating nonlinear behavior of an ocean system is briefly summarized and its results are compared against both deterministic and stochastic analytical predictions. Experimental results from deterministic models verify the existence of the primary and secondary resonances as analytically predicted. Nonlinear responses, e.g., harmonic, subharmonic and ultrasubharmonic, are also observed. Experimental results from stochastic model demonstrate noise-induced transitions between distinct response modes, e.g., harmonic and subharmonic. Engineering applications, e.g., system reliability, are demonstrated using probability density functions as a measure. The systematic analysis and design methodology provides a means to integrate existing seemingly independent deterministic and stochastic design procedures.

1. Introduction

With the demand for economical rapid transportation, installation and deployment of strategic facilities in increasingly shallow water and difficult to maneuver territories, the overall physical sizes of a new generation Naval and industrial ocean systems have been decreasing steadily. These systems include moored and free-floating ships, barge, buoys and platforms. Large system response often result under moderate to high sea states and the behavior of these systems is highly nonlinear and sensitive in nature. Examples of sources of severe nonlinearity of these systems include the relatively weak lateral restoring force of tension-leg platforms (TLPs), nonlinear upright moments of ship roll motions and light mooring lines on buoys which can become slack during dynamic motions of moored structures, giving rise to a discontinuity in stiffness (Thompson et al. 1984). Another sources of nonlinearity arises from the quadratic nature of the hydrodynamic loading effects and parametric couplings in the inertia forcing (Gottlieb and Yim 1993).

Because of the complexity of these nonlinear system behavior in general, a great number of studies have been concentrated on deterministic response to periodic wave excitations. In particular, the intricate nature of system behavior has been thoroughly investigated. These studies assume that responses to periodic excitations are periodic and concentrate on developing solutions that take into account the contribution of resonance, ultraharmonic and subharmonic components. In addition to the nonlinear periodic responses identified by classical analyses, recent studies discovered the possible existence of another class of nonlinear response behavior of offshore structural systems to deterministic excitations, called chaotic response. In spite of being subjected to the deterministic forcing, the chaotic motion possesses random-like characteristics. The "steady-state" behavior of these responses is non-periodic and is unpredictable in a deterministic sense.

Under moderate to high sea states, unexpected and undesired large transient extreme responses, large periodic response and even complex stochastic (possibly chaotic) response can occur in the nonlinear ocean structural systems. In practice, some potentially dangerous nonlinear resonances have been reported in these systems, e.g. tethered buoyant platforms and articulated moored towers. Experiments on a common type of articulated moored tower revealed the existence of unexpected subharmonic resonances, leading to large deflections (Thompson et al. 1984). In addition, chaotic behavior has been observed in numerical simulations of the response of full-scale models subjected to "100-year" waves using large-scale finite-element analysis programs in the course of detailed design of some commercial platforms. These nonlinear responses may induce human discomfort, hinder critical maneuver, and lead to unexpected dynamic instability and possible catastrophic failure, thus compromising operation safety and system reliability.

An in-depth understanding of these nonlinear and sensitive response behaviors of the ocean structural systems is essential to develop new concepts of structural analysis, design, monitoring and control. Investigations in characteristic nonlinear response phenomena to deterministic wave excitations, e.g. bifurcations, resonances, and chaotic motions, in ocean structural systems have been conducted for single-degree-of-freedom models (considering the dominant motion only). The ***systems*** investigated include articulated moored towers (Gottlieb et al. 1992), multi-point moored structures (Gottlieb and Yim 1992, 1993; Gottlieb et al. 1997) and free-standing offshore equipments (Yim and Lin 1991a-b, 1992). Employing classical stability analysis techniques, primary and secondary resonances can be identified in the parameter space. By varying the system parameters, transition in response characteristics (in amplitude and period) classified as bifurcation is analytically predicted and numerically identified. Chaotic motions are also observed in these systems via local and global stability analyses. Chaotic response is found to be highly sensitive to perturbations in initial condition and excitation details. Perturbed chaotic response trajectory diverges from the unperturbed trace, and yet still resides in a bounded sub-space called, "strange attractor", as assessed in previous studies. The random-like characteristics are also examined from a probabilistic perspective. With a selected sampling rate (sampled every forcing period), some invariant statistical properties, e.g. stationarity and ergodicity, are found in chaotic time histories (Yim and Lin 1992).

Experimental studies have been carried out to verify the nonlinear behavior predicted analytically and numerically in ocean structural systems. An experiment has been conducted at Oregon State University to assimilate the behavior a multi-point moored structure subjected to ***(regular and random)*** wave excitations (Yim et al. 1993). The primary goal of this experiment is to identify the existence of nonlinear resonances, bifurcations and even chaotic motions in the system response to periodic wave excitations. Being aware of the existence of uncontrollable noise perturbations in the wave flume, additional noise perturbations are later added to examine their effects on system responses. Tests to spectrum-specified random waves are also conducted to assimilate and analyze structural responses in random wave fields.

For the tests excited by deterministic periodic waves, experimental observations confirm the existence of ultraharmonic and subharmonic responses. Locations of primary and secondary resonances are also identified. Existence of higher-order nonlinear responses are yet to be identified via more advance time series analysis techniques. However, there are experimental observations which can not be interpreted from a conventional deterministic perspective. Transitions in distinct characteristic motions, e.g. harmonic and sub-harmonic, are observed in a realization of the system response. The unexpected transitions are conjectured to be caused by the presence of uncontrollable noise perturbations in the wave flume.

Thus, an analysis procedure needs to be developed to take into account random perturbations.

Taking into the presence of random perturbations, global response stability analysis is carried out via a stochastic Melnikov process, and response probabilistic characteristics can be governed by the Fokker-Planck equation (Lin and Yim 1995, 1996a-d, 1997; Yim and Lin 1997b). Criteria derived from the stochastic Melnikov process (or function) indicate the presence of random perturbations enlarges the potential chaotic domain, in other words, chaotic motions may be observed in an expanded parameter space where they does not occur otherwise. Evolution of distributions of response characteristics (displacement and velocity in a 2-D space) is governed by a deterministic partial differential equation called Fokker-Planck equation. Resulting probability density functions, sampled on phase plane, provide the information of global dynamical system response behavior. With the presence of weak random perturbations, multiple steady-state responses are demonstrated, which explains the unexpected transitions between distinct characteristic responses observed in the experiment. Intensity of random noise is considered as a parameter to examine its effects on the nonlinear system responses. The Fokker-Planck formulation can be applied to solve for probability density functions of system response to purely random wave excitations. Time-average probability density functions are found to be a measure of system response to all possible forcing excitations, i.e. periodic, periodic with perturbations and purely random. With the measure, engineering applications such as extreme distributions can be carried out (Lin and Yim 1995).

This paper summarizes the authors' recent investigations in nonlinear response behavior of a multi-point-moored structural system. Deterministic and periodic wave excitations are first considered and stability analysis procedures are employed to demonstrate the intricate nature of system responses. More realistic wave excitation models, e.g., nearly periodic and purely random, are later introduced to be incorporated in stochastic analysis and the system responses are interpreted from a probabilistic perspective. Results of the corresponding experimental studies in both deterministic and stochastic cases are briefly described and compared against the analytical predictions. Potential applications in reliability based design using probability density functions as a measure are elaborated.

2. System Description

A submerged, neutrally buoyant multi-point moored ocean system shown in Fig.1 is modeled as a single-degree-of-freedom rigid body in surge, hydrodynamically damped and excited nonlinear oscillator.

2.1 *Equation of Motion*

The equation of motion is derived based on equilibrium of geometric restoring forces and dynamic forces induced by body motion under wave excitation. The governing equation is given by (Gottlieb and Yim 1992)

$$(M+\rho\forall C_A)\ddot{x}_1 + C\dot{x}_1 + 4K\left[x_1 - \frac{l_c}{2d}\left\{\frac{\beta+x_1}{\sqrt{1+(\beta+x_1)^2}} - \frac{\beta-x_1}{\sqrt{1+(\beta-x_1)^2}}\right\}\right] = F_D + F_I$$

$$= \frac{\rho}{2}C_D A_p(u_1-\dot{x}_1)\,|u_1-\dot{x}_1| + \rho\forall(1+C_A)\left[\frac{\partial u_1}{\partial t} + (u_1-\dot{x}_1)\frac{\partial u_1}{\partial x_1}\right] \qquad (1a)$$

where M is the body mass; C, system damping coefficient; K, elastic coefficient of mooring line. β denotes the degree of geometric nonlinearity; l_c, initial length of mooring lines (see Fig.1 for d); C_D and C_A, hydrodynamic viscous drag and added mass coefficients, respectively; A_P, projected drag area; $\forall$, displace volume; and ρ, water mass density. x_1 represents the surge displacement, and u_1 is the fluid particle velocity. Moreover, u_1 and β are given by

$$u_1 = U_0 + \omega a\frac{\cosh kh}{\sinh kh}\cos(kx_1 - \omega t)\ ; \qquad \beta = \frac{2b-L}{2d} \qquad (1b)$$

where U_0 denotes colinear current magnitude. a,ω and k are wave amplitude, frequency and number, respectively. L is the diameter of the sphere (see Fig.1 for b), and h is water depth. Sources of system nonlinearities are further elaborated in the following section.

2.2 *Identification of Nonlinearities*

The Hamiltonian (corresponding to the undamped and unforced) system associated with Eq.(1) is obtained by setting C, F_D and F_I to zero. Response (natural) frequency of the Hamiltonian system can be computed by directly integrating the Hamiltonian phase plane. It is found that the response natural frequencies are closely related to the degree of geometric nonlinearity due to mooring line configurations. The strongest nonlinearity is obtained for right angle mooring ($\beta=0$) whereas the weakest nonlinearity is found for small angles ($\beta \gg 1$).

Identification of the generic exciting force nonlinearities is conveniently described by scaling system displacement by the wave number k and approximating the restoring force by polynomials. It is found that the bias and parametric excitation can be only neglected (e.g., equivalent linearization) for very small hydrodynamic excitation. A bias and parameter excitation have been found to be a precursor of response symmetry breaking leading to period doubling and generating a mechanism for system instabilities even for small amplitude response (Miles 1988). Thus, the moored system is a coupled nonlinear parametrically excited system and is expected to exhibit complex dynamics (Troger and Hsu 1977) and chaotic motions (HaQuang et al. 1987).

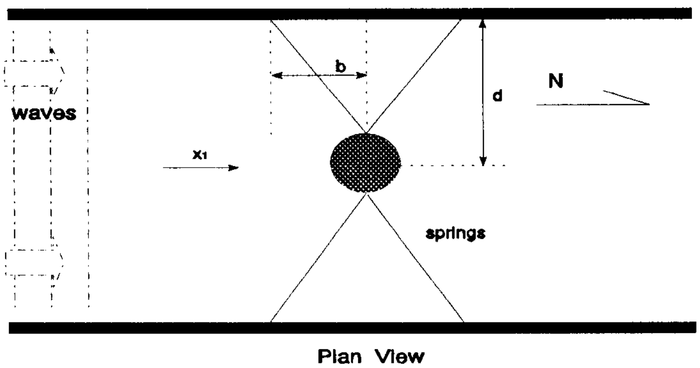

Fig.1 A submerged, hydrodynamically damped and excited nonlinear ocean structural system

3. Deterministic Analysis

Although waves are random in nature under field conditions, idealized (deterministic) regular waves are first employed to demonstrate the intrinsic complex behavior of highly nonlinear offshore structures. Existence of highly nonlinear phenomenon in system responses, including sensitive (chaotic) behaviors, is identified using stability analyses (local and global) and is demonstrated via numerical simulations.

3.1 Deterministic Model

Assuming convective effects in the inertia force negligible and normalizing the governing equation w.r.t. the total mass $(M+\rho\forall C_A)$, Eq.(1) can be rewritten as

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\gamma x_2 - \delta x_2|x_2| - R(x_1) + A\cos\omega t \qquad (2)$$

where x_1 $(= x)$ and x_2 are nondimensionalized surge displacement and velocity, with γ and δ denoting the structural and hydrodynamic damping coefficients, respectively. The wave excitation is assumed to be (purely deterministic and) harmonic, typical of specified maximum "100-year" waves of (sufficiently long, though finite) duration. Because of the complexity of Eq.(2), approximate solutions need to be developed and their stability analysis follows.

3.2 Approximate Solution

Periodic solutions to Eq.(2) are approximated by a sum of harmonic functions in the method of harmonic balance (Gottlieb and Yim 1992b). The approximate solution is then substituted back in the Eq.(2), and the nonlinear differential equation is converted to a set of simpler nonlinear algebraic equations. The approximate solution form is assumed as

$$
\begin{aligned}
x_{0(n/m)} &\cong A_{0(n/m)} + \sum_{i}^{I} A_{i(n/m)} \cos\left[i\,\frac{n}{m}\theta + \Psi_{i(n/m)} \right] \\
y_{0(n/m)} &\cong -\,\omega\frac{n}{m} \sum_{i}^{I} i\, A_{i(n/m)} \sin\left[i\,\frac{n}{m}\theta + \Psi_{i(n/m)} \right]
\end{aligned}
\qquad (3)
$$

where $A_{0(n/m)}, A_{i(n/m)}, \Psi_{i(n/m)}$ are solution amplitudes and phases, I is the order of approximation ($i=1,2,3,\ldots,I$), and n/m is the order of ultrasubharmonics.

3.3 Local Stability Analysis

Stability of the approximate solutions is examined to determine their physical realizability. A stable solution can be physically realized, and an unstable solution indicates the existence of other forms of stable solutions. Varying system parameters in the parameter space, the phenomenon of response gaining and losing response can be observed and called stability bifurcations.

Local stability of the approximate solution can be determined by considering a perturbed solution, $x(t)=x_0(t)+\epsilon(t)$, which upon substitution in Eq.(2), results in a nonlinear variational equation. Linearizing the variational equation yields a set of linear ordinary differential equations with periodic coefficient functions $(H_{1,2}[x_0(\theta),y_0(\theta)]= H_{1,2}[x_0(\theta+2\pi),y_0(\theta+2\pi)])$. A generalized Hill's variational equation is obtained by substituting the approximate solutions (Eq.(3)) to $H_{1,2}$ and is given by

$$
\begin{aligned}
\dot{\epsilon} &= \eta \\
\dot{\eta} &= H_1(\theta)\,\eta + H_2(\theta)\,\epsilon
\end{aligned}
\qquad (4a)
$$

where $H_{1,2}$ are

$$H_1 = \xi_{0(n/m)} + \sum_j \xi_{Cj(n/m)} \cos\left[j\frac{n}{m}\theta\right] + \xi_{Sj(n/m)} \sin\left[j\frac{n}{m}\theta\right] \tag{4b}$$

and

$$H_2 = \zeta_{0(n/m)} + \sum_j \zeta_{Cj(n/m)} \cos\left[j\frac{n}{m}\theta\right] + \zeta_{Sj(n/m)} \sin\left[j\frac{n}{m}\theta\right] \tag{4c}$$

with (ξ_{Cj},ζ_{Cj}) and (ξ_{Sj},ζ_{Sj}) the Fourier coefficients calculated from $H_{1,2}$. The particular solution to Eq.(4a), $\epsilon = \exp(\nu t)Z(t)$, with the Floquet theory (Ioos and Joseph 1981) can be used to identify the stability regions for symmetric and unsymmetric responses.

A comprehensive numerical study and presented example of various types of nonlinear responses collaborating the analytical predictions have been conducted (Gottlieb and Yim 1992b). These examples include pitchfork bifurcation, dynamic symmetry breaking, multiple occurrence of unsymmetric subharmonics, period doubling in subharmonic domains, period doubling in ultraharmonic domains, and coexistence of harmonic and multiple subharmonics of different orders (Gottlieb and Yim 1992b).

3.4 Identification of Bifurcation Superstructure

As indicated in the stability analysis, the response gains and loses stability when the system parameters are varied. A universal underlying pattern (superstructure) about the stability gaining and loosing near resonances of the moored system has been demonstrated (Gottlieb et al. 1997).

Numerical search for the existence of various types of responses and identifying the underlying global pattern is guided by the stability regions in parameter space near low-order and higher-order resonances. A sample superstructure is presented in Fig.2. The pattern consists of intersecting "resonance horns" that portray asymptotic behavior for large excitation.

A resonance number $R_{(n/m,j,d)}$ describing a repeating global bifurcation pattern is defined here to classify the bifurcation pattern of the subharmonic, ultraharmonic and ultrasubharmonic solutions for the submerged moored system. Index [*n/m*] describes the nonlinear resonance relationship $n\omega \approx m\sqrt{\alpha_1}$ (where ω - wave excitation frequency; α_1 - coefficient of the linear component of the nonlinear stiffness). The second index [*j*], determines the order of ratios with non-common factors. The third index determines the dimension [*d*] of the response (i.e. integer deterministic versus fractal chaotic) (Feder 1989).

Knowledge of the intricate superstructure enables identification of coexisting solutions and pitchfork or period doubling bifurcations. Coexistence found by local analysis, can be determined by resonance numbers $R_{(n/m,j)}$ with similar n/m ratios (e.g. $R_{1/2,1}$ and $R_{3/5,1}$). Also note that ultraharmonic solutions described by an even descriptor (n or m) are unsymmetric whereas odd descriptors describe symmetric or self-similar solutions. Numerical examples of coexisting ($n/m=1/2$) and ($n/m=4/5$) and singly existing self-similar ($n/m=3/5$) can be found in Gottlieb and Yim (1992b), respectively.

Stability loss of a symmetric solution which evolves in parameter space to two partner orbits (Gottlieb and Yim 1992b) is described by the ordering index j: 1,2 for $n/m=2/1$ (e.g. $R_{2/1,1} \rightarrow R_{2/1,2}$) in the ultraharmonic domain. Similarly, the period doubling bifurcation (Gottlieb and Yim 1992b) is described by the ordering index in the subharmonic domain j: 1,2 for $n/m=1/2$ (e.g. $R_{1/2,1} \rightarrow R_{1/2,2}$). Note that period doubling in the ultraharmonic domain (Gottlieb and Yim 1992b) is described by j: 2,3 for $n/m=2/1$ (e.g. $R_{2/1,2} \rightarrow R_{2/1,3}$).

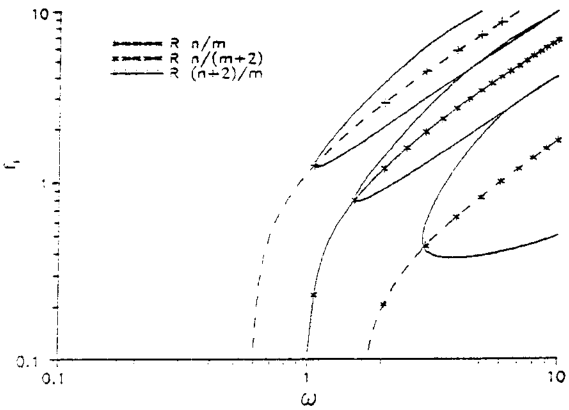

Fig.2 **Bifurcation superstructure**

3.5 Routes to Chaotic Responses

Possible routes to a strange attractor can be described by the evolution of unsymmetric and symmetric solutions as is evident by the spectral content of the pre-chaotic and post-chaotic motions. One possible route is through smooth, continuous period multiplying. This route includes period doublings and can be traced in the superstructure by the ordering index j: 2,4,8...(e.g. $R_{1/2,1} \rightarrow R_{1/2,2} \rightarrow R_{1/2,4} \rightarrow R_{1/2,8}$). The period doubling route to chaotic motions is observed with the appearance of additional even harmonics. Similarly, a period tripling route with the appearance of additional odd harmonics j: 3,9 (e.g. $R_{1/3,1} \rightarrow R_{1/3,3}$ ...) has been identified and verified numerically. Thus, the period multiplying scenario describes an accumulation of internal resonance horns in the bifurcation sets. Note that when the multiplying sequence is infinite, the dimension index [d], describing the number of systems degrees-of-freedom, does not retain its integer value and is replaced by a characteristic fractal dimension (e.g. $R_{"1/2",\infty,"2.31"}$).

Based on our detailed numerical study, another route to chaotic motions is found in the abrupt change to and from neighboring periodic motions (e.g. $R_{1/1,1} \rightarrow R_{1/3,1}$). This occurs near the local tangent bifurcation values and is associated with contraction of the $2m\pi/n$ ultrasubharmonic. This route is found to be short lived in parameter space and culminates in a strange attractor when a "collision" occurs between two neighboring attractors separated by a saddle (i.e. bifurcation defined as a heteroclinic tangency).

3.6 Global Stability Analysis (Melnikov Approach)

The Melnikov approach is utilized to examine the global stability and determine the chaotic domain in the parameter space. When perturbed stable and unstable manifolds transversely intersect, chaotic motions may exist. Existence of homoclinic or heteroclinic connection is essential for applying the Melnikov method. The Hamiltonian system corresponding to (1a) has only one fixed point, a center, and no homoclinic or heteroclinic orbits. Nevertheless, homoclinic connections may exist near the primary resonance in the associated averaged system (Gottlieb and Yim 1993), which is demonstrated in the following section.

3.6.1 Existence of homoclinic orbits

By treating time as a normalized state variable, θ, employing a polynomial approximation for restoring force ($\alpha_1 x_1 + \alpha_3 x_1^3$) (Gottlieb and Yim 1993), Eq.(1a) can be rewritten in an autonomous form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\gamma x_2 - \alpha_1 x_1 - \alpha_3 x_1^3 - \mu\omega^2 a_1' \sin\theta + F_D(x_2,\omega,\theta) \\ \dot{\theta} &= \omega \end{aligned} \qquad (5)$$

A pair of homoclinic orbits within the averaged system can be determined based on the Bendixson criterion (Jordan and Smith 1987). For small structural damping, the system near primary resonance contains homoclinic loops defined by the stable and unstable manifolds of the saddle. The pair of homoclinic orbits are identified to exist. Based on the existence of homoclinic connections, the Melnikov function is formulated in the following section.

3.6.2 *Melnikov function*

The Melnikov function estimating the distance between the stable and unstable manifolds is given by (Guckenheimer and Holmes 1983)

$$M(\theta_o) = \int_{-\infty}^{\infty} F(q_{\pm}^{0}(\theta)\wedge G(q_{\pm}^{0}(\theta),\theta+\theta_o)d\theta = \int_{-\infty}^{\infty} (F_1G_2-F_2G_1)d\theta \qquad (6)$$

where F_1 and F_2 denote the components of a 2-D Hamiltonian vector field, and G_1 and G_2 represent the perturbations. When $M(\theta_o)=0$ in Eq.(6), chaotic response may exist and a criterion for the chaotic domain can be thus defined.

It is shown numerically that when the Melnikov criterion is satisfied ($M(\theta_o)=0$), system response does exhibit transient chaotic motion. In other words, the system response behaves in a chaotic fashion for a considerably long duration of time before settles to a periodic steady state (Gottlieb and Yim 1993).

3.7 *Experimental Verification*

An experimental study was conducted at Oregon State University to assimilate nonlinear ocean structural response behavior subjected to various wave excitations, including periodic, nearly periodic and narrow-banded. The configuration and setup of the experiment are briefly described here, and the experimental results under periodic wave excitations are compared against the analytical predictions in the following section.

3.7.1 *Experiment configuration and setup*

The experimental model was situated in a two dimensional wave flume with length 342 ft., width 12ft. and depth 15ft. Waves are generated by hydraulic driven and hinged flap wave board (Yim et al. 1993). Quantitative data recorded during each test included wave profiles at several locations along the channel, current and sphere movements, and restoring forces in the springs.

3.7.2 *Observations and comparisons*

Nonlinear experimental responses, including (harmonic) resonances, subharmonic and ultraharmonic, have been observed (Lin et al. 1997). These results collaborate the complex nonlinear system behavior predicted by those of the analytical model (Eq.(2)). Frequency response curves demonstrate a nonlinear relationship between the wave excitation and system responses in parameter space (Fig.3). Both primary and secondary resonances are exhibited in the curve, and possible existence of bifurcation superstructure and routes to chaotic motions is implied. Good agreement between numerical simulations and experimental response is shown, and the validity of the analytical model is demonstrated. Besides the steady-state nonlinear responses, transitions from one response state to another, which can not be explained by deterministic analysis, were observed in the experiment (Fig.4).

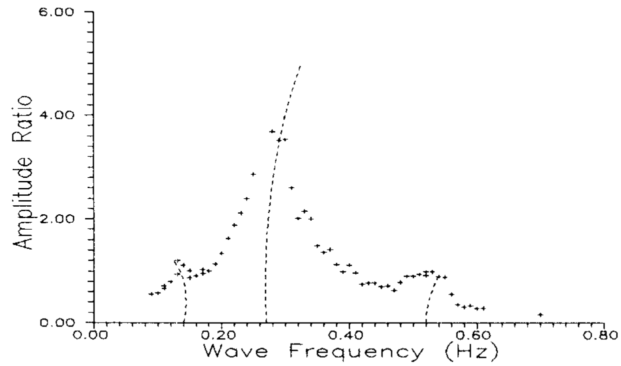

Fig.3 Frequency response ratio (response amplitude/ wave amplitude) versus wave frequency

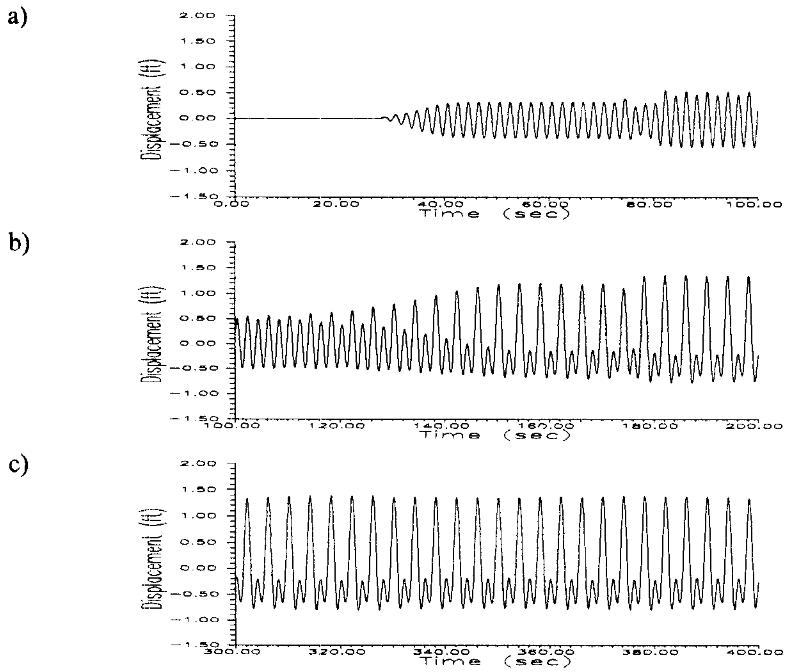

Fig.4 Transition from harmonic to subharmonic in "deterministic" moored structural response

Initially, the system response behave in a harmonic fashion for about 120 seconds (Fig.4a) and then transitioned to a subharmonic steady state (Figs.4b-c). This transition may be induced by the presence of uncontrollable noise which is caused by a combination of reflection, re-reflection, wave diffraction by the model, and imperfect energy dissipation of the testing facilities. A stochastic analysis approach accounting for these uncontrollable noise is needed to further investigate the observed nonlinear response.

4. Stochastic Analysis

Weak random perturbations are now added to the forcing excitation to assimilate nearly periodic wave conditions (periodic plus uncontrollable tank noise in the experiment), which may also occur in the field when storms are filtered by ocean topography or near-by marine structures (Jefferys 1987). Taking into account the presence of random noise, global (Melnikov) stability analysis is then employed to identify its effects on highly sensitive nonlinear (chaotic) domains in parameter space. Stochastic properties of noisy nonlinear responses are examined via the Fokker-Planck formulation. Responses to purely random wave excitations are also investigated and the associated engineering application is according conducted.

4.1 Analysis of Responses to Nearly Periodic Excitation

The stochastic model can be obtained by lumping all random perturbations and subsequently adding to the forcing excitation in Eq.(2)

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\gamma x_2 - \delta x_2 |x_2| - R(x_1) + A\cos\omega t + \xi(t) \end{aligned} \tag{7a}$$

where the lumped random excitation component is described by a zero-mean, delta-correlated white noise process, $\xi(t)$,

$$\begin{aligned} \langle \xi(t) \rangle &= 0 \\ \langle \xi(t)\xi(t') \rangle &= \kappa\delta(t-t') \end{aligned} \tag{7b}$$

with κ denoting noise intensity.

4.1.1 Global Stability Analysis (Melnikov Approach)

With the presence of random perturbations, the distance between the stable and unstable manifolds is given by (Lin and Yim 1995)

$$M(\theta_o) = \int_{-\infty}^{\infty} F(q_{\pm}^{0}(\theta) \wedge G(q_{\pm}^{0}(\theta), \theta+\theta_o) d\theta = \int_{-\infty}^{\infty} (F_1 G_2 - F_2 G_1) d\theta \tag{8}$$

where the vector G includes deterministic and random perturbation components. Noise effects on occurrence of chaotic responses are to be identified in the Melnikov sense here by isolating the damping term related to the chaos threshold and regrouping the perturbation effects into deterministic and random components. The criterion based on the stochastic Melnikov process provides a necessary condition for the existence of chaotic response (Yim and Lin 1991a), and the noise effects should be represented in a mean-square sense

$$\gamma^{*} (\int m_1^{d} d\theta)^2 = (\int m_2^{d} d\theta)^2 + \sigma_1^2 + \sigma_2^2 \tag{9}$$

where the subscript d denotes Melnikov components due deterministic perturbations, and σ_1^2 and σ_2^2 are the variances associated to random perturbations. The positive terms, σ_1^2 and σ_2^2, indicate that the presence of noise lowers the threshold for chaos and enlarges the possible chaotic domain (mean-square sense) in the parameter space.

4.1.2 Noise-induced chaos

As indicated in the criterion based on the stochastic Melnikov function, the presence of random perturbations enlarges the potential chaotic domain in the parameter space and noisy chaotic response may occur in the expanded region. Numerical verification of the analytical predication is given in Fig.5. When the excitation is deterministic, the system response behaves in a subharmonic fashion (Fig.5a). When weak random noise is present, the response is in a noisy chaotic mode (Fig.5b-c), which means the "orderness" of the chaotic motion is preserved despite the fractal boundary of the strange attractor is smoothed. When the noise intensity is further increased, the response becomes more random-like (Fig.5d).

4.1.3 Fokker-Planck formulation

The Fokker-Planck formulation is applied when random wave fields are considered. The Fokker-Planck equation (FPE), statistically equivalent to Eq.(7), governs the evolution of response probability density function (PDF) in phase space and is given by

$$\frac{\partial P(X,t)}{\partial t} = -\frac{\partial}{\partial x_1}\left[x_2 P(X,t)\right] - \frac{\partial}{\partial x_2}\left[\left(-R(x_1) - \gamma x_2 - \gamma x_2|x_2| + F_D(x_2,t) + F_I(t)\right)P(X,t)\right] + \frac{\kappa}{2}\frac{\partial^2 P(X,t)}{\partial x_2^2} \tag{10}$$

with $X=[x_1\ x_2]^T$. $P(X,t)$ is the joint PDF in (surge displacement and velocity) phase space; $(x_2,\ -R(x_1)-\gamma x_2-\delta x_2|x_2|+F_D+F_I)$ corresponds to the drift vector; and $\kappa/2$ is the only nonzero entry in the two by two diffusion matrix.

A path-integral solution (a semi-analytical iterative) procedure, is employed to solve FPE for PDFs. In the path integral solution procedure, the traveling path of the PDF in phase space is discretized into infinitesimal segments. Each segment represents a short time propagation between two consecutive states in the corresponding Markov process. The short time propagation is approximated by a time-dependent Gaussian distribution, called the short time PDF whose mean and variance are determined by the drift vector and the diffusion matrix, respectively. The PDF at the succeeding state can be determined through the propagation. Thus the probability at a desired state can be obtained by applying the short time propagation iteratively.

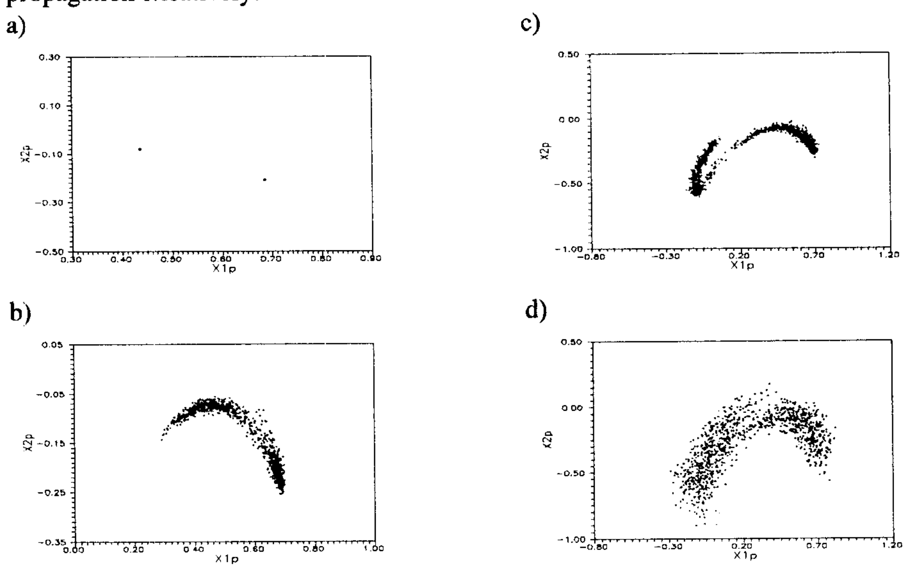

Fig.5 Noise-induced transition of system response

4.1.4 *Probabilistic interpretation of response*

Global information of system response can also be illustrated via steady-state PDF (Fig.6). Figure 6a shows coexisting periodic responses, which may be obtained near resonance with various initial conditions. Figures 6b-d show the corresponding PDF with various noise intensity. The PDFs are obtained with quiescent initial condition. When the intensity of the noise present is very low (κ=0.003), the PDF is concentrated at the domain of small-amplitude response (Fig.6b), indicating its stronger attracting strength relative to the coexisting large amplitude motions (i.e., rarely visiting the large-amplitude domain). When the noise intensity increases (κ=0.007), the domains of attraction of the coexisting responses begin to bridge as shown in Fig.6c. Characteristics of both attractors (small and large amplitudes) may exhibit in the response behavior. When the noise intensity is moderate (κ=0.02), the attractors further merge into a single attraction domain (Fig.6d), in which no obvious periodic response can be distinguished. Numerical results show that the variation of boundary of the emerged attraction domain is stabilized when the noise intensity further increases. The smooth curve and the stationary boundary of PDF implies strong randomness in the response behavior.

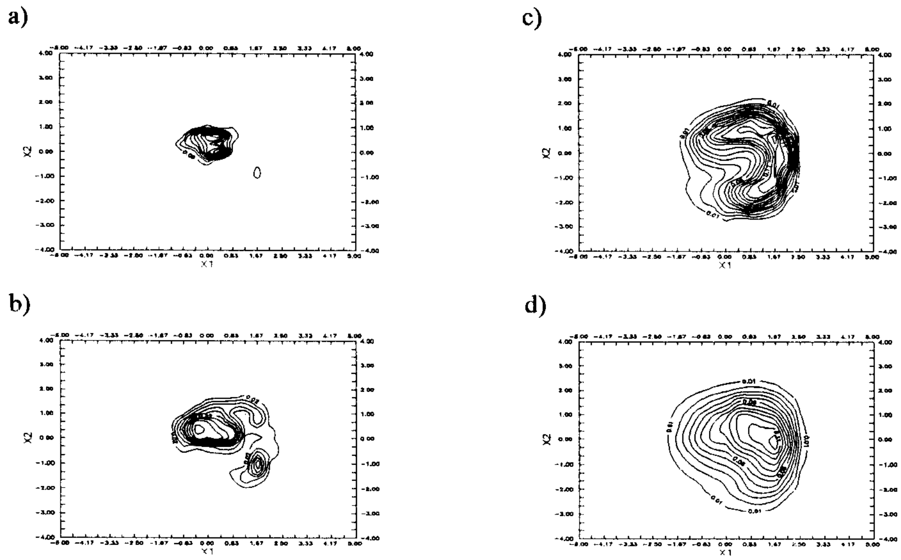

Fig.6 Coexisting periodic responses with increasing noise intensity (JPDF's)

The experimental observed "unexpected" transition from harmonic to subharmonic can be interpreted using PDF. The transition indicates the coexistence of harmonic and subharmonic response attractors. Moreover, due to the presence of weak random perturbations (caused by uncontrollable tank noise), the domains of coexisting attractors are not well bridged (Fig.6b). Because the experiment model is excited with quiescent initial condition, the response trajectories reside in the (weak) small-amplitude attractor for about 120 seconds and then transition to the (strong) large-amplitude attractor with low probability of return.

4.1.5 Experimental comparisons

Under specified wave excitation conditions described above, total of 9 tests were conducted and the model responses were recorded (Tests D4-13, see Yim and Lin (1997a) for detailed documentation). It is observed that when the wave amplitude increases from 1.5 ft. to 2.2 ft., a relatively strong subharmonic component is shown in the response in both time and frequency domains (Fig.7). It is also evident that the response oscillates between coexisting distinct response attractors, i.e., harmonic and subharmonic (Fig.7). The experimental observation can be interpreted in light of the analytical prediction. Lin and Yim (1996) suggest that the presence of random noise facilitates a global description of the system behavior in the phase space. When the random noise intensity is moderate, the attraction domains of the coexisting response are well-bridged. The steady-state PDF indicates an oscillatory shift between coexisting response attractors in an ensemble sense (Lin and Yim 1996). The experimental response drifting back and forth between the coexisting harmonic and subharmonic attractors is due to the presence of random noise, thus the combined characteristics are observed (Fig.7).

4.2 Analysis of Responses to Purely Random Excitation

The stochastic model can be obtained by employ linearly filtered white noise to approximate spectrum-specified random waves

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\gamma x_2 - \delta x_2 |x_2| - R(x_1) + f(t) \end{aligned} \tag{11a}$$

The random wave excitation $f(t)$ is obtained by

$$\ddot{f}(t) + \beta \dot{f}(t) + \Omega^2 f(t) = \xi(t) \tag{11b}$$

where $\xi(t)$ is a zero mean, delta-correlated white noise (Eq.(7b)).

The linear filter (Eq.(11b)) adds another two state variables to the Fokker-Plank formulation. The associated Fokker-Planck equation (FPE) is given by with $X=[x_1\ x_2\ x_3\ x_4]^T$.

$$\frac{\partial P(X,t)}{\partial t} = -\frac{\partial}{\partial x_1}\left[x_2 P(X,t)\right] - \frac{\partial}{\partial x_2}\left[\left(-R(x_1) - \gamma x_2 - \delta x_2 |x_2| + x_3\right)P(X,t)\right]$$
$$-\frac{\partial}{\partial x_3}\left[x_3 P(X,t)\right] - \frac{\partial}{\partial x_4}\left[\left(-\beta x_4 - \Omega^2 x_3\right)P(X,t)\right] + \frac{\kappa}{2}\frac{\partial^2 P(X,t)}{\partial x_2^2} \tag{12}$$

a)

b)

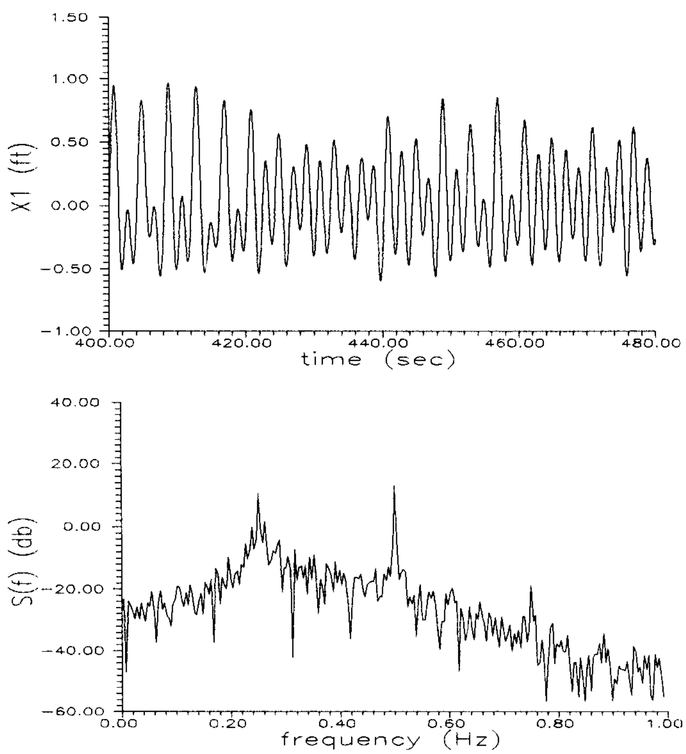

Fig.7 Noisy combined harmonic/subharmonic response: a) time history and b) energy spectrum

5. Design Applications

With the PDF as a measure, a probabilistic-based design can be carried out. Results to nearly periodic and random (e.g., JONSWAP) waves are compared to suggest the most suitable excitation for an integrated design. The distribution of large excursions in the system response is then calculated and will be used as a system performance index, based on which, recommendations to practicing engineers follow.

5.1 *Transient PDF and Failure Probability*

Evolution of response PDF is indicated by the time-dependent drift coefficient in the Fokker-Planck formulation (for nearly periodic excitation ($\kappa \to 0$)) and the examples demonstrated in previous sections. Incorporating the evolving (transient) PDF with the domain of interest ("safe" domain), the time-dependent failure probability can be calculated. The time-dependent failure probability is computed by summing up the probability out-crossing the boundaries of the "safe" domain at each time step. The boundary condition is as follows:

$$P(x_1, x_2, t) = 0 \tag{13}$$

for $x_1 > +x_{1s}$, $x_2 > 0$; $x_1 \le -x_{1s}$, $x_2 > 0$; $x_1 \ge +x_{1s}$, $x_2 < 0$; and $x_1 < -x_{1s}$, $x_2 < 0$. Variables $\pm x_{1s}$ define the upper/lower bounds of the "safe" domain.

Comparison of the time-dependent (transient) probability of failure/safety of structural responses near a highly nonlinear domain to those of nearly periodic and purely random excitations is shown in Fig.8. When the "safety" boundary is specified, the transient failure probability (P_{tf}) goes up to 0.02 in about 95 seconds for JONSWAP random waves and about 320 seconds for nearly periodic waves. Thus, system failure can be anticipated in a shorter duration to the purely random wave excitation (JONSWAP).

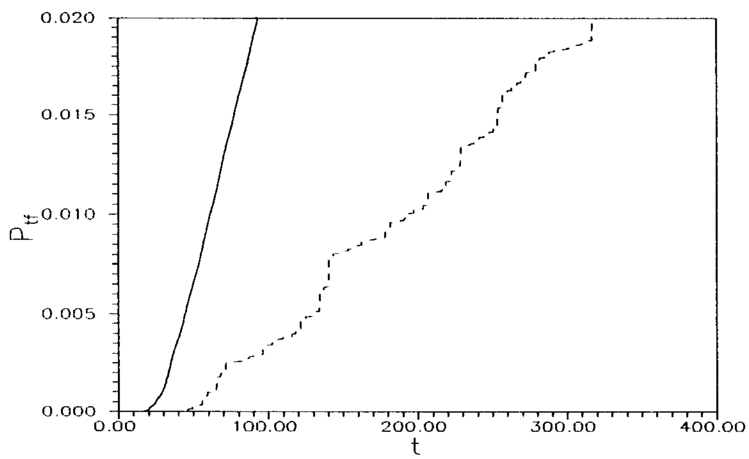

Fig.8 Transient failure probability (dashed line - nearly periodic waves and solid line - JONSWAP random waves)

5.2 *Time-Averaged Probability Density*

As noted in the FPE (Eq.(10)), the drift coefficient is periodic in time for nearly periodic case ($\kappa \rightarrow 0$). The periodicity can be suppressed by taking an average over a sufficiently long duration to form a time-averaged PDF (which is an invariant measure as shown in Lin and Yim (1995)). The TAPDF is given by

$$P_{av}(X) = \frac{1}{T_n} \int_0^{T_n} P(X,t)\, dt \tag{14}$$

Good agreement has been demonstrated between the simulated TAPDF and the theoretical result (Lin and Yim 1995). With the TAPDF, distribution of large excursions can be estimated.

5.3 *Distribution of Large Excursions*

Using the TAPDF and employing Rice's formula, the mean up-crossing frequency of the response can be evaluated as

$$\mu^{+}_{x_1}(x_d) = \int_0^{\infty} x_2 P_{av}(x_d, x_2)\, dx_2 \tag{15}$$

Moreover, by adopting the assumption of statistically independent large-amplitude up-crossings, which leads to Poisson-distribution crossing events (Naess and Johnsen 1993), the asymptotic approximation of the probability that x_1 exceeds a specified high level x_d during time T is given by

$$P_F(x_d, T) = 1 - \exp(-\mu^{+}_{x_1}(x_d) T) \tag{16}$$

Figure 8 shows the distribution of large displacements in coexisting domain over a duration of 5 minutes to nearly periodic and JONSWAP waves. It is observed numerically that regardless of whether the system response is in the chaotic or multiple steady-state response domain, results from the JONSWAP random waves provide more conservative design values. Based on numerical results, this trend is consistently observed over a wide range of parameters. Thus purely random waves are recommended for detailed designs.

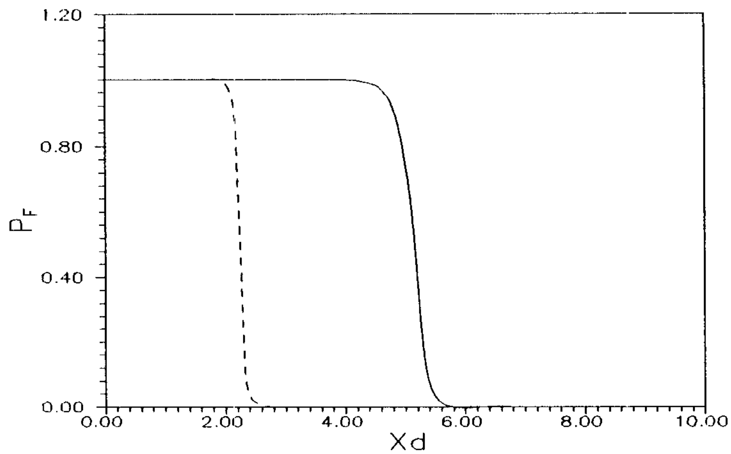

Fig.9 Distribution of large displacements; responses subjected to nearly periodic and purely random waves are denoted by dashed and solid lines, respectively

6. Concluding Remarks

Complex nonlinear and sensitive deterministic/stochastic phenomena are investigated in a general class of ocean moored systems. The system response behavior is characterized by a nonlinear restoring force and a coupled fluid-structure interaction exciting force. An analysis is carried out by first considering deterministic periodic excitation conditions to examine the complex nonlinear nature of the response behavior. Stochastic analyses concerning nearly periodic (periodic plus weak noise) and (spectrum-specified) purely random excitations are developed to examine the system response form an ensemble perspective. Experimental studies assimilating the deterministic and stochastic nonlinear ocean moored structural responses are summarized. A potential engineering application using stochastic analysis means is also demonstrated. Some pertinent conclusions are accordingly summarized as follows:

6.1 Deterministic Analysis

Rich and complex phenomena, e.g., bifurcations, harmonic resonance, subharmonic, ultraharmonic, ultrasubharmonic and even chaos, are observed in the response behavior. A bifurcation scenario in the response is analytically predicted using the method of harmonic balance for approximate solutions and a variation method for their stability analysis. A global stability analysis is carried out employing Melnikov function to identify the existence of chaotic motion. Numerical results demonstrate that organized stability transitions between resonances can be characterized by a steady state superstructure in the bifurcation sets. Routes to chaotic response are found via period doubling, period tripling, and sudden explosion.

6.2 Stochastic Analysis

Noise effects on the existence of chaotic response are demonstrated based on criteria derived from stochastic Melnikov (function) process. Analytical results, verified by numerical simulations, indicate that the presence of weak noise perturbations expedites possible occurrence of chaotic response. Stochastic properties of noisy nonlinear response are examined using the Fokker-Planck formulation. The resulting probability density function depicts global system response on the phase plane. Numerical results indicate noise intensity is a controlling parameter of transition between coexisting response attractors. When the noise intensity is low, the response trajectories reside in the stronger attractor with low probability of exiting to the other attractor. When the noise intensity is moderate, the domains of coexisting attractors are bridged and the system response exhibits combined characteristics. When the noise intensity is high, the domain of the coexisting attractors are further bridged and smoothed, and the system response appears random. Random wave excitations (JONSWAP) are approximated by linearly filter white noise, and good agreement is shown.

6.3 Experimental Comparisons

Deterministic experimental results verify the existence of harmonic resonance, ultraharmonic and subharmonic responses. These results collaborate the complex nonlinear system behavior predicted by those of the analytical model. Primary and secondary resonances are exhibited in the frequency response curves, and possible existence of bifurcation superstructure and routes to chaotic response is implied. "Unexpected" transitions from harmonic to subharmonic responses are found to be caused by the coexistence of response and the presence of weak tank noise.

Stochastic experimental results confirm that noise effects should be taken into account for the analysis and design of deterministic system. With weak noise present, response transition from the weaker attractor to reside in the stronger attractor is observed. With a moderate noise intensity, oscillations of the response between coexisting response attractors are also demonstrated. Both experimental observations confirm the analytical predictions.

Numerical results indicate the current deterministic analysis practice of using "100-year" periodic waves may be inadequate for the estimation of maximum response for highly nonlinear offshore structures. Nearly periodic waves are proposed to provide a more realistic model compared to the conventional "100-year" waves. Using probability density function as a measure, the maximum displacement and system reliability in responses to nearly periodic and JONSWAP random waves are estimated and compared. It is found that the results of the corresponding JONSWAP wave excitations provide more conservative design values than those of the nearly periodic waves. Hence, purely random waves with the stochastic analysis in the probability domain form a useful analysis procedure and are recommended for detailed designs.

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