# Simplified Earthquake Analysis of Multistory Structures with Foundation Uplift

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**Abstract:** A simplified analysis procedure is developed to consider the beneficial effects of foundation-mat uplift in computing the earthquake response of multistory structures. This analysis procedure is presented for structures attached to a rigid foundation mat which is supported on flexible foundation soil modeled as two spring-damper elements, Winkler foundation with distributed spring-damper elements, or a viscoelastic half space. In this analysis procedure, the maximum, earthquake induced forces and deformations for an uplifting structure are computed from the earthquake response spectrum without the need for nonlinear response history analysis. It is demonstrated that the maximum response is estimated by the simplified analysis procedure to a useful degree of accuracy for practical structural design.

## Introduction

The earthquake response of structures is usually analyzed under the assumption that the foundation is firmly bonded to the soil. Such analyses often predict a base overturning moment that exceeds the available overturning resistance due to gravity loads, which implies that a portion of the foundation mat or some of the individual column footings, as the case may be, would intermittently uplift during the earthquake. Several examples of towers and oil tanks uplifting from the underlying soil during Arvin Tehachapi (1952), Alaska (1964), and Imperial Valley (1979) earthquakes are cited in a recent work (8). Uplift of multistory building foundations has rarely been observed because the uplift is expected to be small and the foundation-soil interface is often inaccessible for observation.

Analytical studies (6–8) as well as experimental investigation (5) of the complicated nonlinear response of structures have demonstrated the important effects of foundation uplift. The dynamics of the structure can be profoundly modified and the structural deformations and forces are generally reduced.

Because a rigorous treatment of the nonlinear effects of foundation mat uplift is quite complicated, a simple, practical procedure to account for these effects in computing the design forces and deformations has been developed for structures which respond as single-degree-of-freedom systems in their fixed base condition (3). This procedure is extended in this paper to structures, such as multistory buildings, which respond as multi-degree-of-freedom systems in their fixed base condition, attached to a rigid foundation mat which is supported on two spring-

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damper elements, Winkler foundation with distributed spring-damper elements, or a viscoelastic half space.

## System Considered

The system considered is shown in Fig. 1, consisting of the shear-building idealization of a multistory building supported through a foundation mat on flexible foundation. In this idealization, we assume that the mass of the building is concentrated at the floor levels; the floor systems and beams are rigid so that the lateral displacements relative to the ground are entirely due to deformations in columns, thus leading to a single-degree-of-freedom per floor. Viscous damping is assumed to be of such a form that the building on rigid foundation admits decomposition into classical normal modes. The foundation mat, idealized as a rigid rectangular massless plate of negligible thickness, rests on two spring-damper elements, one at each edge of the foundation mat connected to the rigid base. It is presumed that horizontal slippage between the mat and supporting elements is not possible. The mass and damping coefficients of the foundation model are assumed constant, independent of displacement amplitude or excitation frequency.

Prior to the dynamic excitation, the foundation mat rests on the spring-damper elements only through gravity and is not bonded to these supporting elements. Thus, a supporting element can provide an upward

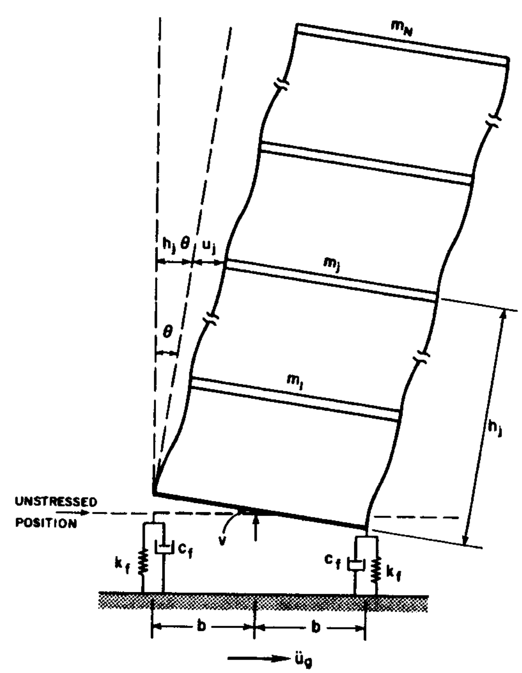


FIG. 1.—Multistory Structure Supported on Two Spring-Damper Element Foundation

reaction force to the foundation mat but not a downward pull. During vibration of the system, this upward reaction force will vary with time. At any instant of time, the total reaction force—which for a supporting element consisting of a spring and damper in parallel is the sum of the elastic and damping forces—reaches zero, one edge of the foundation mat is in the condition of incipient uplift from the supporting element. Obviously, the supporting element provides no reaction force at an uplifted edge, until contact of the edge with its supporting element is re-established. A detailed description of the properties of the two spring-damper element foundation can be found in Ref. 12.

The base excitation is specified by the horizontal ground motion with displacement $u_g(t)$ and acceleration $\ddot{u}_g(t)$. Under the influence of this excitation the foundation mat would rotate through an angle $\theta(t)$ and undergo a vertical displacement $v(t)$, defined at its c.g. relative to the unstressed position. Prior to dynamic excitation, the vertical displacement is the static displacement due to the total weight of the superstructure and foundation mat. During the dynamic excitation the vertical displacement $v$ will remain constant at the initial static value if the foundation mat is bonded to the supporting element, but it will vary with time in the unbonded case.

## Method of Analysis

**Dynamic Equilibrium.**—The displaced configuration of the structure-foundation system at any instant of time can be defined by the floor displacements $u_j(t)$ due to structural deformations, foundation mat rotation $\theta(t)$, and vertical displacement $v(t)$ at the center of gravity of the foundation mat.

In terms of these degrees-of-freedom, the equations expressing the dynamic equilibrium of the $N$ floor masses are

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{m}\mathbf{h}\ddot{\theta} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\mathbf{1}\ddot{u}_g(t) \quad \text{(1a)}$$

In addition, there are two equations expressing the equilibrium of the structure-foundation system as a whole in rotation and vertical vibration

$$\mathbf{h}^T\mathbf{m}\ddot{\mathbf{u}} + I_b\ddot{\theta} + M(t) = -\mathbf{h}^T\mathbf{m}\mathbf{1}\ddot{u}_g(t) \quad \text{(1b)}$$

$$m_t\ddot{v} + R(t) = -m_t g \quad \text{(1c)}$$

In these equations, $\mathbf{u}$ is the vector of floor displacements $u_j$; $\mathbf{m}$ is the diagonal matrix of $N$ floor masses $m_j$; $\mathbf{k}$ is the stiffness matrix of the building on rigid foundation—the standard tri-diagonal matrix in terms of story stiffnesses $k_j$ (2); $\mathbf{c}$ is the viscous damping matrix on rigid foundation; $\mathbf{h}$ is the vector of floor heights $h_j$, which denotes the height of the $j$th floor above the base; $\mathbf{1}$ is a column vector with each of the $N$ elements equal to unity; $k_f$ and $c_f$ are the stiffness and damping coefficients of each spring-damper element supporting the foundation mat; $b$ denotes the half base width; $m_t = \Sigma_{j=1}^{N} m_j$ is the total mass of the superstructure; $I_b = \Sigma_{j=1}^{N} m_j h_j^2$ is the total moment of inertia of all the masses about the base if the moment of inertia of all the masses about their respective centroidal axes is neglected; and $M(t)$ and $R(t)$ are the overturning moment and vertical force reactions provided by the supporting elements on the foundation mat.



These reaction forces are related to the vertical displacement $v(t)$ and rotation $\theta(t)$ of the foundation mat as follows:

$$M(t) = \epsilon_1 c_f b^2\dot{\theta} + \epsilon_2 c_f b\dot{v} + \epsilon_1 k_f b^2\theta + \epsilon_2 k_f bv \quad \text{(2a)}$$

$$R(t) = \epsilon_1 c_f\dot{v} + \epsilon_2 c_f b\dot{\theta} + \epsilon_1 k_f v + \epsilon_2 k_f b\theta \quad \text{(2b)}$$

where $\epsilon_1$ and $\epsilon_2$ depend on whether one or both edges of the foundation mat are in contact with the supporting elements:

$$\epsilon_1 = \begin{cases} 2 & \text{contact at both edges} \\ 1 & \text{left or right edge uplifted} \end{cases} \quad \text{(2c)}$$

$$\epsilon_2 = \begin{cases} -1 & \text{left edge uplifted} \\ 0 & \text{contact at both edges} \\ 1 & \text{right edge uplifted} \end{cases} \quad \text{(2d)}$$

A supporting spring-damper element provides an upward reaction force to the foundation mat

$$R_i(t) = -k_f v_i(t) - c_f\dot{v}_i(t); \quad i = l, r \quad \text{(3a)}$$

$$\text{where} \quad v_i(t) = v(t) \pm b\theta(t); \quad i = l, r \quad \text{(3b)}$$

As mentioned earlier, a downward reaction force cannot be developed because the foundation mat is not bonded to the supporting elements, i.e., $R_i(t) \geq 0$, $i = l, r$. If, at some instant of time, this condition is not satisfied at the left or right edge of the foundation mat, i.e.

$$R_i(t) < 0; \quad i = l \text{ or } r \quad \text{(4a)}$$

that particular edge uplifts from the supporting element which no longer provides any reaction to the foundation mat. Because the spring-damper element cannot extend above its initial unstressed position, i.e., $v_i(t) \leq 0$, $i = l, r$, an edge of the foundation mat would uplift at the instant of time this condition is not satisfied, i.e.

$$v_i(t) > 0; \quad i = l \text{ or } r \quad \text{(4b)}$$

The two uplift criteria, Eqs. 4a–b, are equivalent if each supporting element included only a spring.

The equations of motion for the system of Fig. 1 are nonlinear as indicated by the dependence of the coefficients $\epsilon_1$ and $\epsilon_2$ (Eqs. 2c–d) on whether one or both edges of the foundation mat are in contact with the supporting elements, the left edge is uplifted, or the right edge is uplifted. However, for each of the three contact conditions the governing equations are linear, but they depend on the contact condition. Thus the dynamic response of the nonlinear system of Fig. 1 can be viewed as the sequential response of three linear systems.

The equations of motion developed here are also valid for a more realistic idealization of building frames where joint rotations occur because of flexural flexibility of beams, provided the shear building stiffness matrix is replaced by the lateral stiffness matrix of the frame (Ref. 2, pp. 92–94).

**Modal Equations.**—For each of the three linear systems previously mentioned, the vector $\mathbf{u}(t)$ of floor displacements due to structural deformations is a linear combination of the conventionally defined natural modes of vibration of the building (2)

$$\mathbf{u}(t) = \sum_{n=1}^{N} Y_n(t)\boldsymbol{\phi}_n \qquad (5)$$

where $\boldsymbol{\phi}_n$ denotes the vibration modes of the building with its base fixed, i.e., with the foundation mat in the system of Fig. 1 bonded to rigid supporting elements. The fixed-base natural vibration frequencies and modes of the building are solutions of the eigenvalue problem

$$\mathbf{k}\boldsymbol{\phi}_n = \omega_n^2 \mathbf{m}\boldsymbol{\phi}_n \qquad (6)$$

Introducing the transformation of Eq. 5 into Eq. 1*a*, premultiplying the resulting equation by $\boldsymbol{\phi}_n^T$, and utilizing the orthogonality properties of modes we obtain the equation of motion for the *n*th vibration mode of the building

$$M_n(\ddot{Y}_n + 2\xi_n\omega_n\dot{Y}_n + \omega_n^2 Y_n) + L_n^r\ddot{\theta} = -L_n^h\ddot{u}_g(t)$$

$$n = 1, 2, \ldots, N \qquad (7a)$$

where $\xi_n$ is the damping ratio; $M_n = \Sigma_{j=1}^N m_j\phi_{jn}^2$ is the modal mass; $L_n^h = \Sigma_{j=1}^N m_j\phi_{jn}$ is the excitation coefficient associated with base translation; and $L_n^r = \Sigma_{j=1}^N m_j\phi_{jn}h_j$ is the excitation coefficient associated with base rotation. With the transformation of Eq. 5, Eq. 1*b* becomes

$$\sum_{n=1}^{N} L_n^r\ddot{Y}_n + I_b\ddot{\theta} + M(t) = -L_o^r\ddot{u}_g(t) \qquad (7b)$$

where $L_o^r = \Sigma_{j=1}^N m_j h_j$.

These equations in terms of modal coordinates $Y_n$ are valid for arbitrary normalization of the mode shapes. We shall later see that it is useful to re-normalize the mode shapes in a special manner and express

$$\mathbf{u}(t) = \sum_{n=1}^{N} Y_n'(t)\boldsymbol{\phi}_n' \qquad (8)$$

In order to introduce this re-normalization, we first introduce the effective mass $m_n^*$ and effective height $h_n^*$ for the *n*th mode (Ref. 2, pp. 84), two quantities that are independent of how $\boldsymbol{\phi}_n$ is normalized

$$m_n^* = \frac{(L_n^h)^2}{M_n} \quad \text{and} \quad h_n^* = \frac{L_n^r}{L_n^h} \qquad (9)$$

Defining the re-normalized mode shape as

$$\boldsymbol{\phi}_n' = \frac{L_n^h}{M_n}\boldsymbol{\phi}_n \qquad (10)$$

Eqs. 7*a*–*b* can be expressed as

$$m_n^*(\ddot{Y}_n' + 2\xi_n\omega_n\dot{Y}_n' + \omega_n^2 Y_n' + h_n^*\ddot{\theta}) = -m_n^*\ddot{u}_g(t); \quad n = 1, 2, \ldots, N \qquad (11a)$$

$$\sum_{n=1}^{N} m_n^* h_n^* \ddot{Y}_n' + I_b\ddot{\theta} + M(t) = -L_o^r\ddot{u}_g(t) \qquad (11b)$$



whereas Eq. 1*c* remains unchanged

$$m_t\ddot{v} + R(t) = -m_t g \qquad (11c)$$

By transferring the term involving foundation-mat rotation to the right hand side, Eq. 11*a* represents the equation of motion for a single-degree-of-freedom system with mass $m_n^*$ located at height $h_n^*$ above the base, natural vibration frequency $\omega_n$, and damping ratio $\xi_n$ subjected to horizontal acceleration $\ddot{u}_g(t)$ and rotational acceleration $\ddot{\theta}(t)$ of the base. However, the modal equations are coupled through the unknown base rotation $\theta$, and the $N + 2$ equations (Eq. 11) must be solved simultaneously to determine $Y_n'(t)$, $\theta(t)$ and $v(t)$.

**Numerical Integration.**—The response of the multistory structural system of Fig. 1 to specified horizontal ground motion can be evaluated by numerical solution of Eq. 11. These equations are nonlinear as indicated by the dependence of coefficients $\epsilon_1$ and $\epsilon_2$ in $M(t)$ and $R(t)$ on whether one or both edges of the foundation mat are in contact with the supporting elements. For each of the three contact conditions—contact at both edges, left edge uplifted, and right edge uplifted—the contact coefficients $\epsilon_1$ and $\epsilon_2$ are constant and the governing equations are linear. Thus, during the time duration that one particular contact condition is valid, the system behaves as a linear system, but vibration properties of the linear system are different for the three contact conditions. At any time, the numerical analysis of response over the next time-increment consists of two steps: (1) Determining the contact condition valid for that time increment; and (2) solution of the equations of motion (Eq. 11) with the corresponding values for $\epsilon_1$ and $\epsilon_2$.

The results to be presented later were obtained by numerically integrating the system of $N + 2$ equations (Eq. 11), subject to the transition criteria of Eq. 4, using an implicit Bossak-Newmark method (11) with proper choice of time-step $\Delta t$. This second order method which is unconditionally stable provides a convenient way to eliminate the negligible contribution of a very high frequency mode corresponding to the very small rotational inertia of the rigid base plate (12). Within each $\Delta t$, the instant of transition when the foundation mat begins or ceases to uplift is obtained by an iterative process. The chosen time step $\Delta t$ did not exceed one-tenth of the shortest natural vibration period of the building with fixed base.

**Structural Response.**—Once the modal displacements $Y_n'(t)$ have been determined by numerical integration of the equations of motion, the structural displacements and forces can be readily determined. The displacement of the *j*th floor is given by

$$u_j(t) = \sum_{n=1}^{N} Y_n'(t)\phi_{jn}' \qquad (12)$$

The internal forces such as story shears, story moments, etc., associated with deformations of the multistory building can be conveniently determined by first introducing the concept of equivalent lateral forces. These are external forces $\mathbf{f}$ which, if applied as static forces, would cause structural displacements $\mathbf{u}$. With the aid of Eq. 6, these forces can be expressed as

$$\mathbf{f}(t) = \sum_{n=1}^{N} \omega_n^2 \mathbf{m}\boldsymbol{\phi}_n' Y_n'(t) \qquad (13a)$$

from which the force at the $j$th floor is

$$f_j(t) = \sum_{n=1}^{N} \omega_n^2 m_j \phi_{jn}' Y_n'(t) \qquad (13b)$$

Any internal force can be determined by static analysis of the structure subjected to the equivalent lateral forces. In particular, the shear and moment at the base of the building can be expressed as

$$V(t) = \sum_{n=1}^{N} \omega_n^2 m_n^* Y_n'(t) \qquad (14a)$$

$$M(t) = \sum_{n=1}^{N} \omega_n^2 m_n^* h_n^* Y_n'(t) \qquad (14b)$$

where the effectives mass $m_n^*$ and height $h_n^*$ for the $n$th mode were defined in Eq. 9.

## Fundamental Mode System

Before examining various aspects of the earthquake response of multistory buildings with their foundation mat permitted to uplift, we introduce the structural system shown in Fig. 2. A critical base shear and

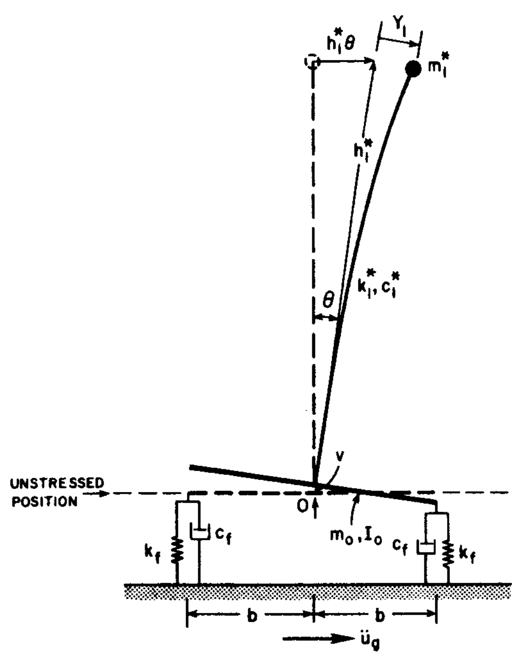


FIG. 2.—Fundamental Mode System



critical base moment defined for this system will be useful in interpretation of response results. Later, analysis of this system will provide a basis for developing a simplified analysis procedure for multistory buildings.

The superstructure in the system of Fig. 2 represents a single-degree-of-freedom model for the fundamental natural mode of vibration of the building on fixed base, with mass $m_1^*$, stiffness $k_1^* = \omega_1^2 m_1^*$, damping coefficient $c_1^* = 2\xi_1 m_1^* \omega_1$, and height $h_1^*$. The mass and mass moment of inertia of the foundation mat in this system are defined by Eq. 15, leading to the same total mass and mass moment of inertia as the multistory system of Fig. 1.

$$m_o = m_t - m_1^* = \sum_{n=2}^{N} m_n^* \qquad (15a)$$

$$I_o = I_b - m_1^* h_1^{*2} = \sum_{n=2}^{N} m_n^* h_n^{*2} \qquad (15b)$$

If the foundation mat is bonded to the supporting elements, a lateral force $f_s$ applied to the mass $m_1^*$ can increase beyond limit if the overturning effects of gravity are neglected. However, if the mat is not bonded to the supporting elements, one edge of the foundation mat is at incipient uplift when the lateral force reaches $f_{sc} = m_t g b / h_1^*$. Thus, the maximum base shear that can be developed under the action of static forces is

$$V_c = m_t g \frac{b}{h_1^*} \qquad (16a)$$

and the corresponding overturning moment is

$$M_c = m_t g b \qquad (16b)$$

## Earthquake Responses

**System Properties.**—The earthquake response results presented in this section are for a shear building idealization (Fig. 1) of a ten-story building with the following properties: All the floor masses are equal, $m_j = m$; all the story-stiffnesses are equal, $k_j = k$; mass of the foundation mat $m_o = 0$; all stories have the same inter-floor height $h$, selected to provide a slenderness ratio for the first mode $h_1^*/b = 10$; and the damping ratios for all vibration modes are the same, $\xi_n = 5\%$. Analysis of Eq. 6 for this system leads to natural frequencies of vibration $\omega_n/\omega_1 = 1, 2.98, 4.89, 6.69, 8.34, \ldots$, and the vibration modes. The effective modal masses $m_n^*$ computed from Eq. 9 and expressed as ratios to the total mass are $m_n^*/m_t = 0.844, 0.091, 0.031, 0.014, 0.007, \ldots$; and the effective modal heights $h_n^*$ computed from Eq. 9 and expressed as slenderness ratios are $h_n^*/b = 10, -3.4, 2, -1.5, 1.2, \ldots$. All these values for $\omega_n/\omega_1$, $m_n^*/m_t$ and $h_n^*/b$ are independent of the fundamental natural vibration frequency $\omega_1$. Because the combined effective mass of the first five vibration modes is 98.7% of the total mass $m_t$ of the building, the contributions of modes 6–10 would be negligibly small. Thus, the response computed by considering the first five modes in Eqs. 11–14 should provide essentially the exact response. Unless otherwise mentioned, responses presented subsequently were computed on this basis.

The stiffness $k_f$ and damping $c_f$ for each of the two supporting elements for the foundation mat are selected to provide $\omega_v/\omega_1 = 8$ and $\xi_v = 0.4$, where $\omega_v = \sqrt{2k_f/m_t}$ and $\xi_v = 2c_f/2m_t\omega_v$ are, respectively, the natural frequency and damping ratio in vertical vibration of the system of Fig. 1 with its foundation mat bonded to the supporting elements.

**Effects of Foundation-Mat Uplift.**—The maximum responses of ten-story buildings with aforementioned properties to the north-south component of El Centro (1940) ground motion are summarized in Figs. 3–4 in the form of response spectra, wherein the maximum value of a selected response quantity is plotted as a function of the fixed-base, fundamental vibration period $T_1 = 2\pi/\omega_1$ of the building. Such response spectra plots for the maximum base shear coefficient $V_{o,\max} = \mathrm{V}_{o,\max}/m_t g$

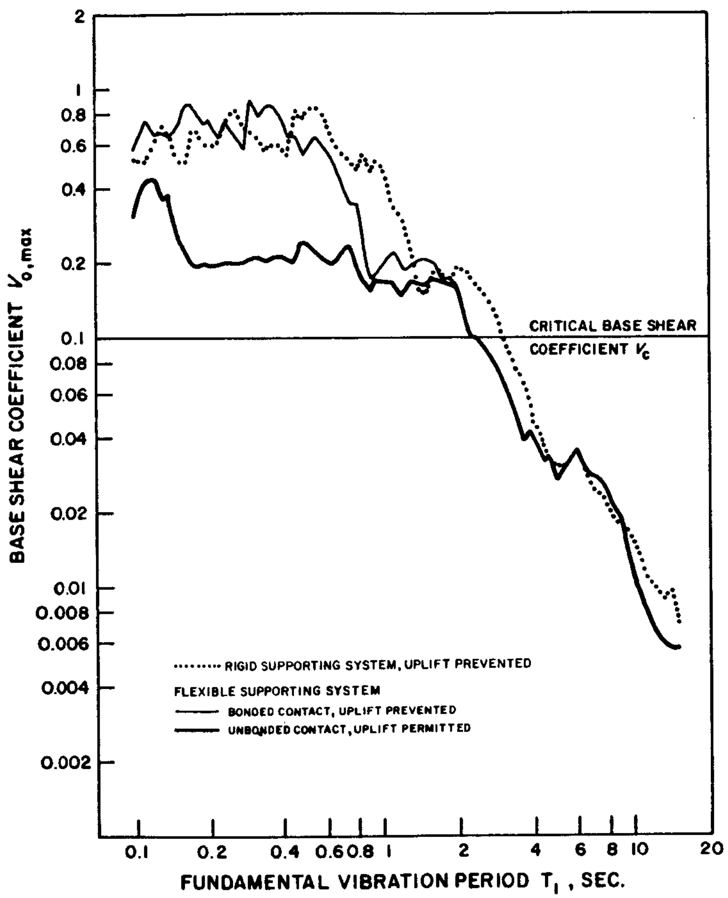


**FIG. 3.—Base Shear Response Spectra for Multistory Structures Subjected to El Centro Ground Motion for Three Support Conditions**



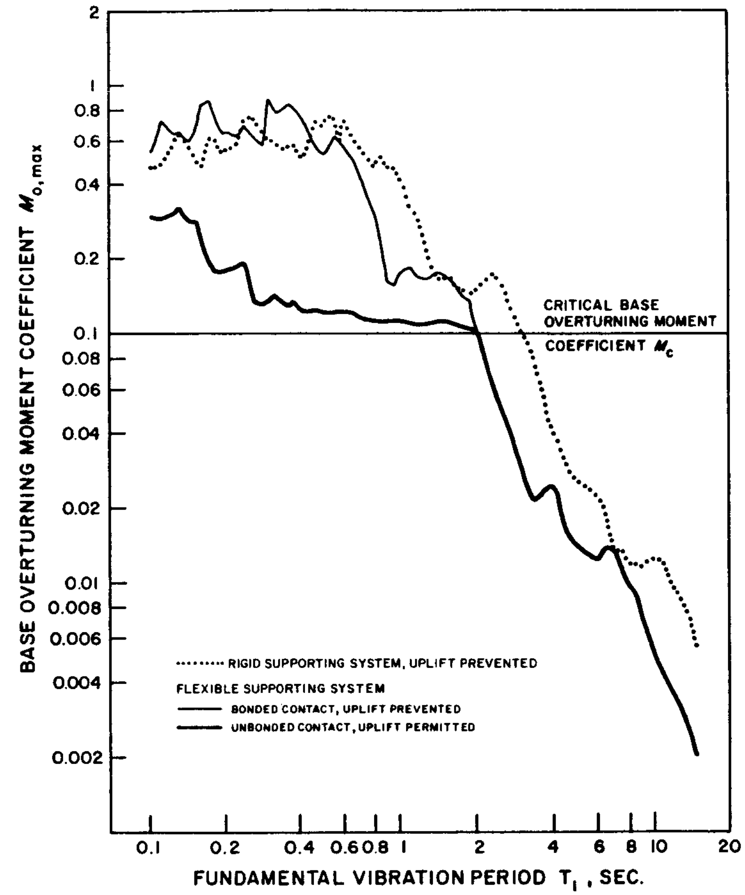


**FIG. 4.—Base Overturning Moment Response Spectra for Multistory Structures Subjected to El Centro Ground Motion for Three Support Conditions**

and maximum base overturning moment coefficient $M_{o,\max} = \mathrm{M}_{o,\max}/m_t g h_1^*$ are presented in Figs. 3–4 for two conditions of contact between the foundation mat and the supporting spring-damper elements: (1) Bonded contact preventing uplift; and (2) unbonded contact only through gravity with uplift permitted. Also presented are the results for the corresponding rigidly supported structure with uplift of the foundation mat prevented. For a selected fundamental vibration period $T_1$, the complete history of base shear $\mathrm{V}_o(t)$ and overturning moment $\mathrm{M}_o(t)$ were determined by the numerical procedures described earlier. The maximum absolute value of the response quantity provides one point on its response spectrum. Included in the response spectra plots are the critical base shear coefficient $V_c = \mathrm{V}_c/m_t g = b/h_1^*$ and the critical base overturning moment coefficient $M_c = \mathrm{M}_c/m_t g h_1^* = b/h_1^*$.

Support flexibility has the effect of modifying the vibration periods and damping ratios of the building and hence its earthquake response.

These modifications can be observed by comparing the response spectra for rigidly and flexibly supported buildings with bonded foundation mat (Figs. 3–4). Because several vibration modes may contribute significantly to the base shear response of long-period buildings, these modifications are complicated, involving crossover of the two response spectra in the long-period range, in contrast to the simple period shift in the response spectra for systems with one-degree-of-freedom in their fixed-base condition (12).

The base shear and moment developed in long-period buildings are below their respective critical values, and the foundation mat, even if it rests on the supporting soil only through gravity, does not uplift throughout the earthquake. If foundation-mat uplift is prevented, the maximum base shear and moment in short-period buildings exceeds the critical value. If the foundation mat of such a structure rests on the supporting soil only through gravity, uplift of the foundation mat may occur during the earthquake, and this generally has the effect of reducing the base shear and moment. However, these responses are not reduced to as low as the respective critical values $V_c$ and $M_c$ based on static considerations because of two factors. The first of these, associated with the dynamic effects in the problem, was discussed earlier in studies of systems with one-degree-of-freedom in their fixed base condition (12). The second factor has to do with the contributions of the higher vibration modes in earthquake response of buildings. Because the critical base shear $V_c$ and moment $M_c$ were defined for the fundamental mode of vibration of the building (Fig. 2), the response contributions of the higher vibration modes cause a further increase beyond the critical values. This increase is significant in the case of base shear, but almost negligible in base moment, which is consistent with the fact that, for a particular building, the significance of the contributions of higher vibration modes depends on the response quantity, being more significant for base shear than for moment (4,9).

**Effects of Foundation Flexibility and Uplift on Modal Responses.**— It has been demonstrated (10) that in analyzing the response of multistory structures supported through a foundation mat bonded to a viscoelastic half space, a reasonable approximation to the maximum response may be obtained by assuming that soil-structure interaction influences only the contribution of the fundamental mode of vibration to the response. Thus, the contributions of the higher modes can be computed by standard procedures ignoring interaction effects. Similar, definitive conclusions regarding the effects of foundation uplift on the response contributions of the various modes have not been reached, although it has been suggested that the vibration properties of the higher vibration modes are not affected by foundation uplift (8). It is well-known that the base overturning moment is almost entirely due to the first mode contribution; in particular, the contributions of the higher modes to the base overturning moment identically vanish if the fundamental vibration mode is linear (1). Because the uplifting tendency of the foundation mat is caused by the overturning moment at the base of the structure, it would seem that higher modes would contribute very little to this tendency. Conversely, foundation uplift should have little influence on higher mode responses.



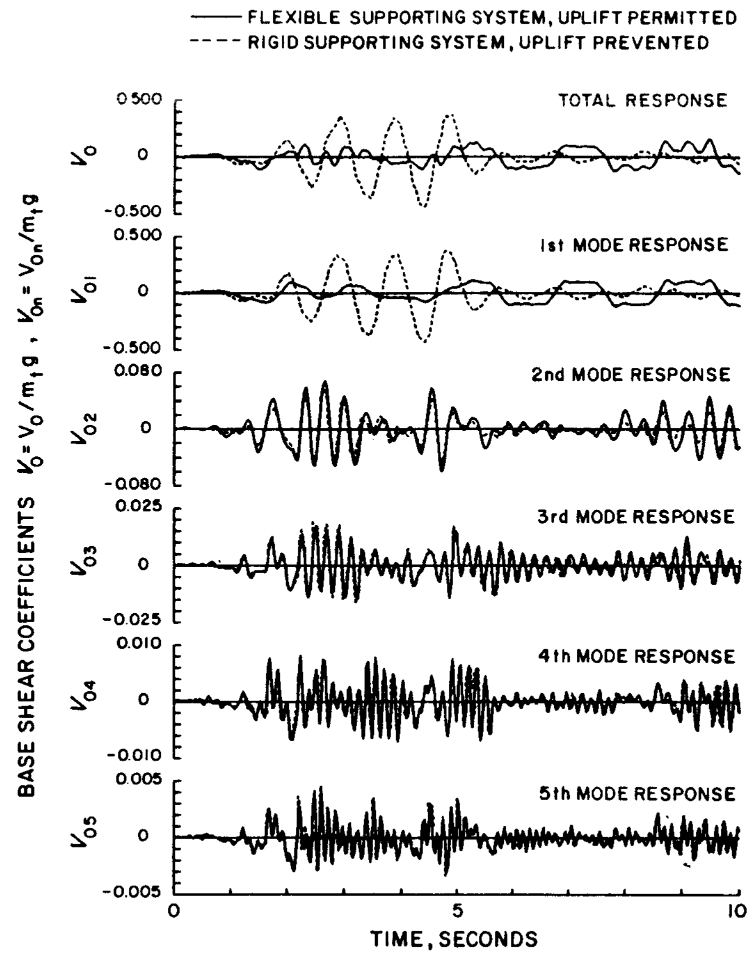


**FIG. 5.—Base Shear Response of Multistory Structure ($T$ = 1 sec) to El Centro Ground Motion for Two Support Conditions**

This expectation is confirmed by the results presented in Fig. 5 wherein the base shear for the ten-story building defined earlier with fundamental period $T_1$ = 1 sec is presented. The response results were computed by the analysis procedures presented earlier for two conditions: (1) Considering support flexibility and permitting uplift of the foundation mat; and (2) assuming that the support system is rigid and the foundation mat is bonded to the supporting elements. Presented for each of the two cases are the individual contributions of the first five modes of vibration to the base shear and the combined response. Note that the higher mode contributions have been plotted to an amplified scale. These results demonstrate that just like the total base shear, the first mode contribution is also significantly affected by the foundation mat uplift and support flexibility; however, the contributions of the higher modes are es-

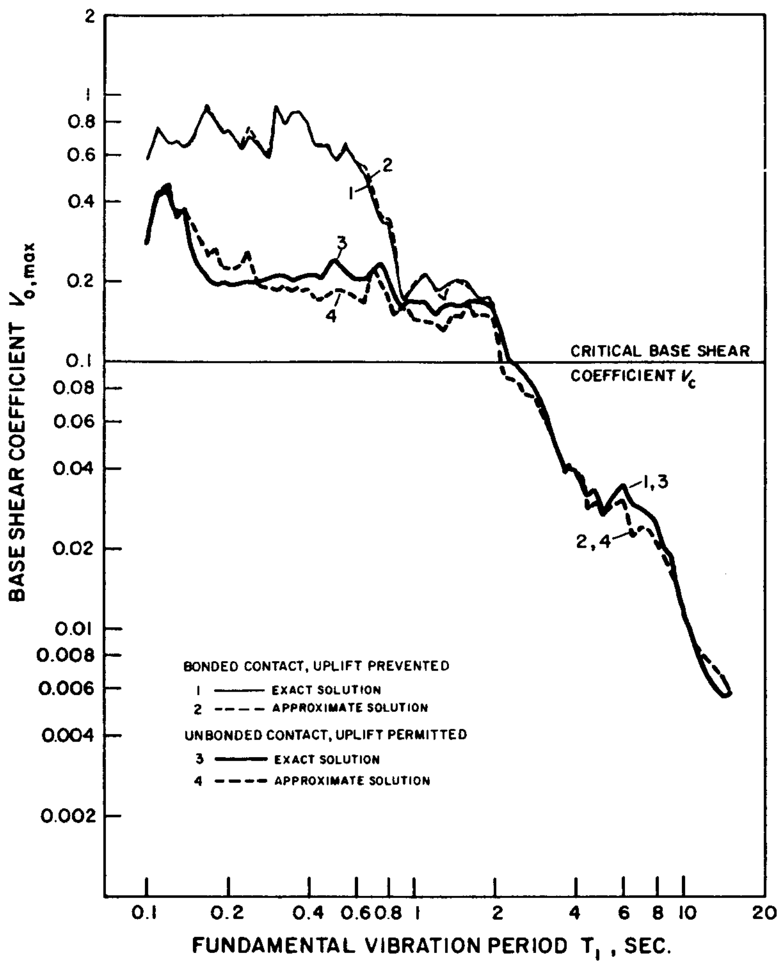


**FIG. 6.—Comparison of Base Shear Response Spectra, Computed by Exact and Approximate Analysis Procedures, for Multistory Structures**

sentially unaffected by either of these two factors. Thus, the analysis of a multistory structure could be simplified by assuming that soil-structure interaction, arising from support flexibility, and foundation-mat uplift influence only the contribution of the fundamental mode of vibration to the response; and the contributions of the higher modes can be computed by standard procedures disregarding the effects of interaction and foundation-mat uplift.

The accuracy of this approach is shown in Fig. 6, in which are presented the response spectra for base shear of the ten-story building frame defined earlier when excited by the El Centro ground motion. The approximate solutions were obtained by computing the first mode contribution to base shear by simultaneously solving Eq. 11*a* only for $n = 1$,



Eq. 11*b* with only the $n = 1$ term, and Eq. 11*c*; the higher mode contributions by standard procedures neglecting the effects of interaction and foundation-mat uplift; and exactly combining the instantaneous values of the modal contributions. The agreement in Fig. 5 between the exact and approximate solutions is indeed excellent if foundation-mat uplift is not permitted, a result that reconfirms the conclusions of earlier studies on the dynamics of structures with foundation mat firmly bonded to the supporting viscoelastic half space (10). The agreement between the two solutions deteriorates if the foundation mat is supported on the soil only through gravity forces and uplifts during the earthquake, as it does for fundamental periods less than about 2 sec. However, the quality of the approximate solution seems to be good enough for many practical applications.

**Higher Mode Response Contributions.**—The three response spectra of Fig. 3 for base shear in ten-story buildings with aforementioned properties are separately presented in Figs. 7(*a–c*). In each case, the response spectra for the contributions of the fundamental mode, and the combined contributions of the first two modes to the base shear are also presented.

Consistent with earlier studies on response of structures on rigid foundation without uplift (4,9), the significance of the response contributions of the vibration modes higher than the fundamental mode increases with increasing fundamental vibration period $T_1$ [Fig. 7(*a*)]. Because the mode shapes and ratios of natural vibration periods do not change with fundamental period $T_1$, the increased contribution of higher modes is due only to the relative values of the response spectrum ordinates, which in turn depend on the spacing of the vibration periods and on the shape of the pseudo-acceleration response spectrum. For the spectrum of the selected ground motion, as the fundamental period increases, the ordinate for a higher vibration mode generally increases relative to that for the fundamental mode, resulting in increased response contributions of the higher modes.

As seen in Fig. 7(*b*), support flexibility has the effect of increasing the significance of the contributions of higher vibration modes, an observation consistent with earlier results on soil-structure interaction effects (10). The fundamental vibration period lengthens because of support flexibility, which, for the ground motion considered, leads to a decrease in the spectral ordinate; whereas the higher mode periods and hence their spectral ordinates are essentially unaffected. Thus the increased significance of the higher mode response is because of the reduction in fundamental mode response due to support flexibility.

As discussed in the preceding sections, foundation mat uplift significantly affects the response of short-period buildings. As a result, the base shear due to the fundamental mode of vibration is significantly reduced, although it is not reduced to as low as the critical value $V_c$ based on static consideration; but the base shear response in the higher vibration modes is essentially unaffected by foundation mat uplift. Thus, the contributions of the higher modes, especially the second mode, becomes relatively more significant, as seen in Fig. 7(*c*). In the long-period range, the foundation mat does not uplift and the results in Fig. 7(*c*) are the same as in Fig. 7(*b*).

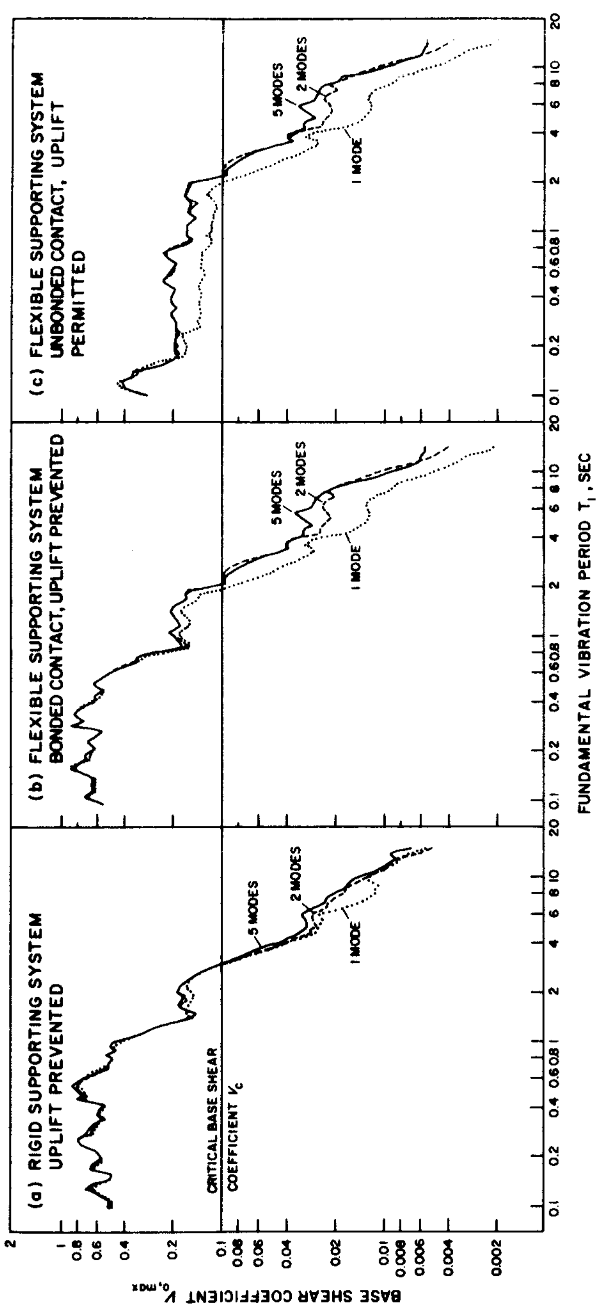


**FIG. 7.—Modal Contributions to Maximum Base Shear for Multistory Structures Subjected to El Centro Ground Motion for Three Support Conditions**



Whereas the contributions of higher vibration modes need not be considered in the analysis of short-period buildings if the foundation mat is bonded to the supporting soil [Figs. 7(*a*–*b*)], they should be considered if the foundation mat is permitted to uplift [Fig. 7(*c*)]. Thus, the higher mode contributions become significant over the entire period range if the foundation mat is permitted to uplift [Fig. 7(*c*)]. Generally, it is necessary to consider the contributions of only the first two modes, for they provide an accurate description of the response over a wide range of vibration periods [Fig. 7(*c*)].

**Effects of Higher Mode Coupling on First Mode Response.**—Much of the discrepancy in the approximate solution presented in Fig. 6 is associated with the contribution of the fundamental mode of vibration

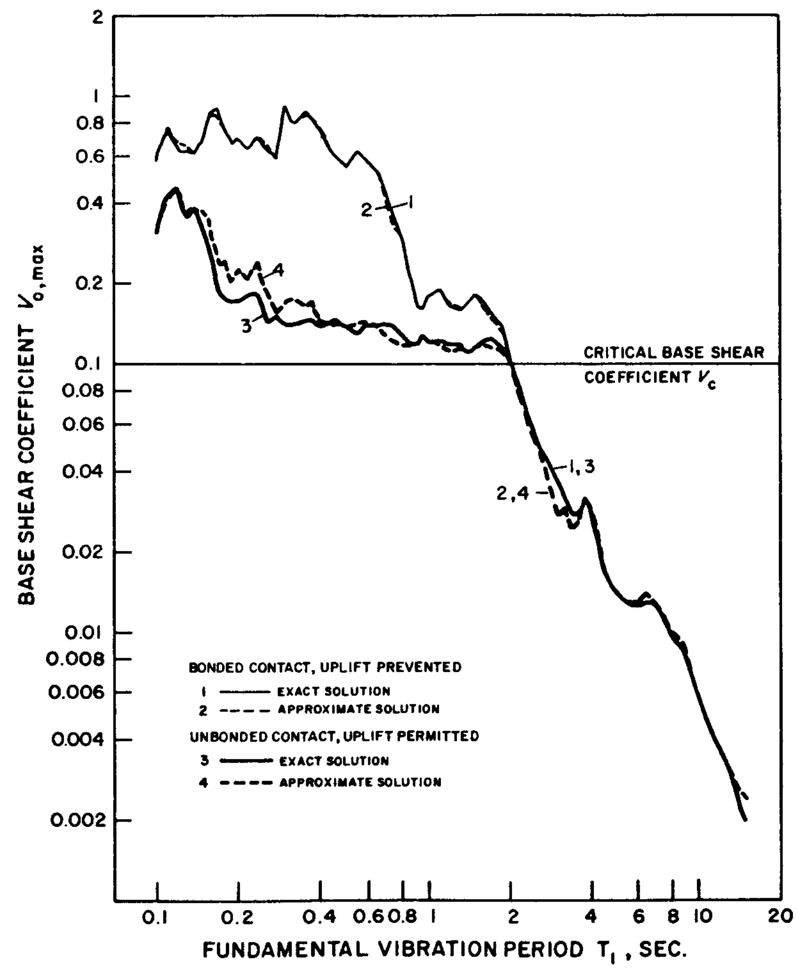


**FIG. 8.—Comparison of Fundamental-Mode Base Shear Response Spectra, Computed by Exact and Approximate Analysis Procedures, for Multistory Structures**

to the response. This modal contribution is obtained in the exact solution by solving Eq. 11 considering five vibration modes simultaneously, but in the approximate solution it is obtained by considering only the $n = 1$ term in Eq. 11, thus eliminating the coupling between the fundamental mode and higher modes. The response spectra for the fundamental mode contribution to the base shear, obtained from approximate and exact analyses, are presented in Fig. 8. It is apparent that the coupling between the fundamental mode and higher modes because of support flexibility, considered in the exact solution, has little influence on the first mode response, for the approximate and exact solutions are essentially identical when foundation mat uplift is prevented. However, modal coupling apparently increases when foundation-mat uplift is permitted, leading to noticeable errors in the approximate solution. These errors are the major contributors to the discrepancy in the approximate solution presented in Fig. 6 and discussed earlier.

## Simplified Fundamental Mode Analysis

The approximate solution for the first mode response discussed in the preceding section, which was presented in Fig. 8 and also entered into the results of Fig. 6, was obtained by solving the one-mode version of the equations of motion (Eq. 11)

$$m_1^*(\ddot{Y}_1' + 2\xi_1\omega_1\dot{Y}_1' + \omega_1^2 Y_1' + h_1^*\ddot{\theta}) = -m_1^*\ddot{u}_g(t) \quad (17a)$$

$$m_1^* h_1^* \ddot{Y}_1' + I_b\ddot{\theta} + M(t) = -L_o^r\ddot{u}_g(t) \quad (17b)$$

$$m_t\ddot{v} + R(t) = -m_t g \quad (17c)$$

where, as defined earlier, for lumped mass systems

$$m_t = \sum_{j=1}^{N} m_j; \quad L_o^r = \sum_{j=1}^{N} m_j h_j; \quad \text{and} \quad I_b = \sum_{j=1}^{N} m_j h_j^2 \quad (18)$$

Alternatively, these quantities can be expressed as modal summations

$$m_t = \sum_{n=1}^{N} m_n^*; \quad L_o^r = \sum_{n=1}^{N} m_n^* h_n^*; \quad \text{and} \quad I_b = \sum_{n=1}^{N} m_n^* h_n^{*2} \quad (19)$$

For multistory buildings, a good approximation for $m_t$, $L_o^r$ and $I_b$ is provided by the contributions of the fundamental mode of vibration, i.e.

$$m_t \approx m_1^*; \quad L_o^r \approx m_1^* h_1^*; \quad \text{and} \quad I_b \approx m_1^* h_1^{*2} \quad (20)$$

Among the three quantities, the approximation is worst for $m_t$ and progressively improves for $L_o^r$ and $I_b$, becoming essentially exact with errors generally less than 3% for the latter two. Thus $L_o^r$ and $I_b$ are approximated by Eq. 20, but not $m_t$, because it enters into the critical base shear $V_c$ (Eq. 16*a*), which has a dominating influence on the response of uplifting systems. With this approximation, Eqs. 17*a*–*c* become the governing equations of motion for the system of Fig. 2 with $m_o$ defined by Eq. 15*a* and $I_o = 0$.

Starting with the earthquake response spectrum, a simplified analysis procedure has recently been developed to determine the maximum response of the system of Fig. 2, but with massless foundation mat (3).



This procedure is based on relating the maximum response of the superstructure supported through a massless foundation mat on flexible foundation soil to the response of the corresponding rigidly supported structure, both responses computed under the assumption that the foundation mat is firmly bonded to the supporting soil. For this condition of no foundation mat uplift, the structural response is unaffected by the mass of the foundation mat. Thus, the simplified procedure seems to be extendable to the system of Fig. 2 with uplift of the foundation mat permitted, provided the mass of the foundation mat is considered in computing the critical base shear $V_c$ (Eq. 16*a*).

The maximum base shear in the system of Fig. 2 may then be determined by an extension of the simplified analysis procedure of Ref. 3, as follows:

1. Under the assumption that the foundation mat is bonded to the supporting soil, determine the effective natural vibration period of the system $\tilde{T}_1$ and the effective overall damping ratio $\tilde{\xi}_1$ of the flexibly supported system from Eqs. 21–25

$$\tilde{T}_1 = T_1\sqrt{1 + \frac{k_1^* h_1^{*2}}{K_\theta}} \quad (21)$$

where $K_\theta$ is the rocking stiffness of the foundation mat, defined by

$$K_\theta = 2k_f b^2 \quad (22)$$

and $$\tilde{\xi}_1 = \frac{\xi_1}{\left(\dfrac{\tilde{T}_1}{T_1}\right)^3} + \tilde{\xi}_0 \quad (23)$$

where $$\tilde{\xi}_0 = \frac{1}{2m_1^*\omega_1}\frac{C_\theta}{h_1^{*2}}\frac{\left[\left(\dfrac{\tilde{T}_1}{T_1}\right)^2 - 1\right]}{\left(\dfrac{\tilde{T}_1}{T_1}\right)^3} \quad (24)$$

and the rocking damping of the foundation mat

$$C_\theta = 2c_f b^2 \quad (25)$$

2. Under the assumption that the foundation mat does not uplift from the supporting soil, compute the maximum base shear

$$V_{1,\max} = m_1^* g\left(\frac{\tilde{S}_{a1}}{g}\right) \quad (26)$$

where $\tilde{S}_{a1}$ is the ordinate of the pseudo-acceleration spectrum corresponding to $\tilde{T}_1$ and $\tilde{\xi}_1$.

3. If the computed $V_{1,\max}$ is less than the critical base shear evaluated from Eq. 16*a*, the foundation mat will not uplift during the earthquake, and the computed base shear is satisfactory. Otherwise, proceed with the following steps.

4. Determine the damping ratio $\zeta$ of the system during uplift from

$$\zeta = \bar{\xi}_1 \left(\frac{r}{b}\right) \quad (27)$$

where $r^2 = h_1^{*2} + b^2$.

5. Compute the maximum base shear from

$$V_{1,\max} = V_c \left\{ \frac{h_1^{*2}}{r^2} + e^{-\zeta\phi} \sqrt{\frac{b^4}{r^4} + \frac{b^2}{r^2}\left[\left(\frac{\bar{S}_{a1}}{g}\right)^2 \left(\frac{h_1^*}{b}\right)^2 e^{\xi\pi} - 1\right]} \right\} \quad (28a)$$

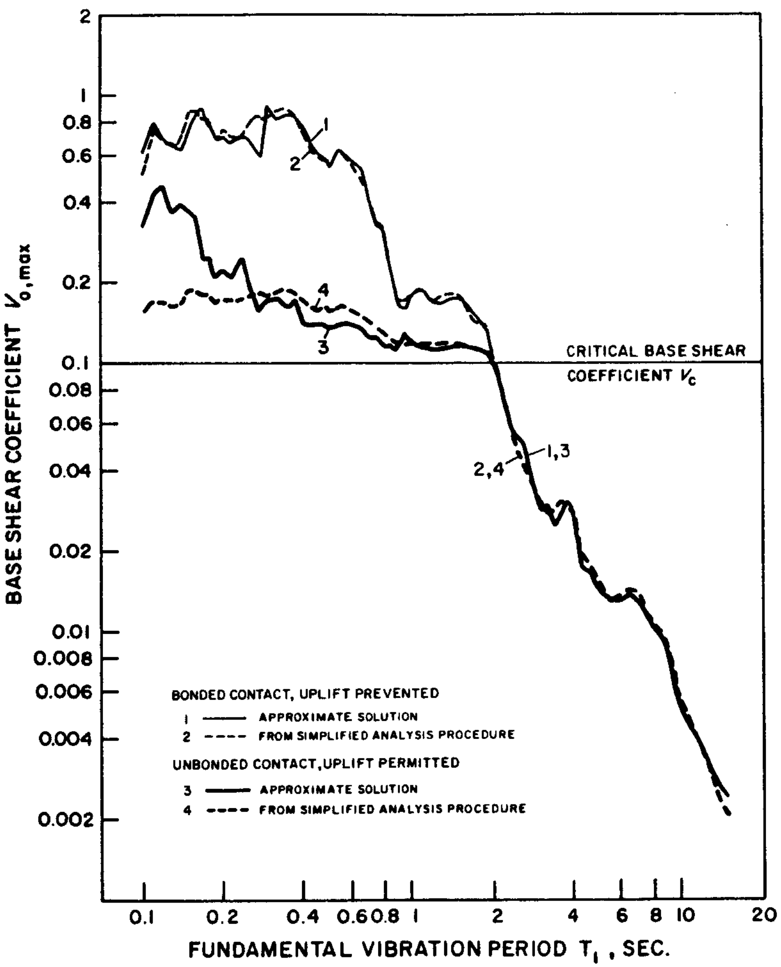


**FIG. 9.—Comparison of Fundamental-Mode Base Shear Response Spectra, Computed by Approximate and Simplified Analysis Procedures, for Multistory Structures**



where

$$\phi = \frac{\pi}{2} - \tan^{-1}\left\{\frac{b}{r}\left[\left(\frac{\bar{S}_{a1}}{g}\right)^2 \left(\frac{h_1^*}{b}\right)^2 e^{\xi\pi} - 1\right]^{-1/2}\right\} \quad (28b)$$

For slender structures, with $h_1^*/b \geq 5$, Eq. 28 can be simplified without introducing any significant error to

$$V_{1,\max} = V_c \left\{1 + \frac{b}{r} e^{-\zeta(\pi/2)} \sqrt{\left(\frac{\bar{S}_{a1}}{g}\right)^2 \left(\frac{h_1^*}{b}\right)^2 - 1}\right\} \quad (29)$$

The accuracy of this simplified analysis procedure is shown in Fig. 9 in which are presented response spectra for the base shear of the ten-story building frame described earlier when excited by the El Centro ground motion. The results are compared with the approximate fundamental mode solution previously presented in Fig. 8, which was obtained by simultaneously solving Eq. 11*a* for $n = 1$, Eq. 11*b* with only the $n = 1$ term, and Eq. 11*c*. For reasons discussed elsewhere (3), the simplified analysis procedure provides excellent results for a wide range of values for the fixed-base fundamental vibration period $T_1$ of the structure, but these results deteriorate for short-period structures if the foundation mat is permitted to uplift. That the simplified analysis procedure fails to accurately predict the response of short-period structures is not a serious limitation because such structures are usually squatty and not likely to uplift during earthquakes (3,12).

The simplified procedure just presented to analyze the maximum response in the fundamental mode of vibration was developed for structures supported through a foundation mat resting on flexible soil idealized as two spring-damper elements one at each edge of the foundation mat. As demonstrated in Ref. 3, with appropriate changes in the definitions of rocking stiffness and damping of the foundation mat (Eqs. 22 and 25), this procedure is also applicable if the flexible soil is idealized by a Winkler model with spring-damper elements distributed over the entire width of the mat.

As presented in Ref. 3, the simplified procedure may also be applied to uplifting structures supported on a viscoelastic half space provided $\tilde{T}_1$ and $\tilde{\xi}_o$ are computed by the procedures developed by Veletsos (10) instead of Eqs. 21 and 24.

## Simplified Modal Analysis Procedure

We have now seen that the maximum value of the contribution of the fundamental mode of vibration to the total response, considering the effects of soil-structure interaction and foundation-mat uplift, can be determined to a useful degree of accuracy by the simplified analysis procedure presented in the preceding section. In this procedure, the maximum response is determined directly from the response spectrum for the ground motion without the need to determine the complete history of response. Furthermore, it was demonstrated earlier that the contributions of the higher modes of vibration to the response history can be accurately computed by standard procedures disregarding the effects of interaction and foundation-mat uplift. In particular, the maximum response in each of the higher modes of vibration can be obtained by standard procedures directly from the earthquake response spectrum. The total response can then be estimated by combining the modal maxima according to the square-root-of-the-sum-of-the-squares (SRSS) formula, which is satisfactory for lateral analysis of buildings as the modal frequencies are well separated.

The resulting simplified modal analysis procedure can be implemented as follows (Ref. 2, p. 88):

1. Determine the response spectrum for the ground motion if not already available.
2. Define structural properties
   a. Compute the fixed-base mass matrix **m** and lateral stiffness matrix **k**.
   b. Estimate the fixed-base modal damping ratios $\xi_n$.
3. Solve the eigen problem

$$\mathbf{k}\boldsymbol{\phi}_n = \omega_n^2 \mathbf{m}\boldsymbol{\phi}_n \quad (30)$$

to determine the few lower natural frequencies $\omega_n$ (natural periods $T_n = 2\pi/\omega_n$) and modes $\boldsymbol{\phi}_n$ of vibration.

4. Compute the maximum base shear $V_{1,\max}$ in the fundamental mode of vibration, considering the effects of soil-structure interaction and foundation mat uplift, by the simplified analysis procedure presented in the preceding section.
5. Compute the maximum base shear $V_{n,\max}$ in the $n$th mode of vibration ($n = 2, 3, \ldots$) by standard procedures, disregarding the effects of interaction and uplift using

$$V_{n,\max} = m_n^* g \left(\frac{S_{an}}{g}\right) \quad (31)$$

where $S_{an} = S_a(T_n, \xi_n)$.

6. Compute the maximum response in individual modes of vibration by repeating the following steps for the lower modes of vibration:
   a. Compute the equivalent lateral forces at all the floor levels from the modal base shears computed in steps 4 and 5, as appropriate, using Eq. 32. The lateral force $f_{jn,\max}$ at the $j$th floor in the $n$th mode of vibration is given by

$$f_{jn,\max} = V_{n,\max} \frac{m_j \phi_{jn}}{\sum_{i=1}^{N} m_i \phi_{in}} \quad (32)$$

   b. Compute internal forces (story shears, story moments and member forces) by static analysis of the structure subjected to the equivalent lateral forces.
   c. Compute the displacements of all the floors from

$$u_{jn,\max} = \frac{1}{\omega_n^2} \frac{1}{m_j} f_{jn,\max} \quad (33)$$

   by repeating its application for all $j$.
   d. Compute the drift (or deformation) in each story from the floor displacements using Eq. 34. The deformation in the $j$th story is

$$\Delta_{jn,\max} = u_{jn,\max} - u_{j-1,n,\max} \quad (34)$$

   As shown in Fig. 7, it would generally be sufficient to compute the response in only the first two modes of vibration.

7. Determine an estimate of $r_{\max}$, the maximum of any response quantity $r$ (displacement of a floor, deformation in a story, shear or moment in a story, force in a member, etc.), by combining the modal maxima $r_{n,\max}$ computed in step 6 for the response quantity, according to the SRSS formula

$$r_{\max} = \sqrt{\Sigma r_{n,\max}^2} \quad (35)$$

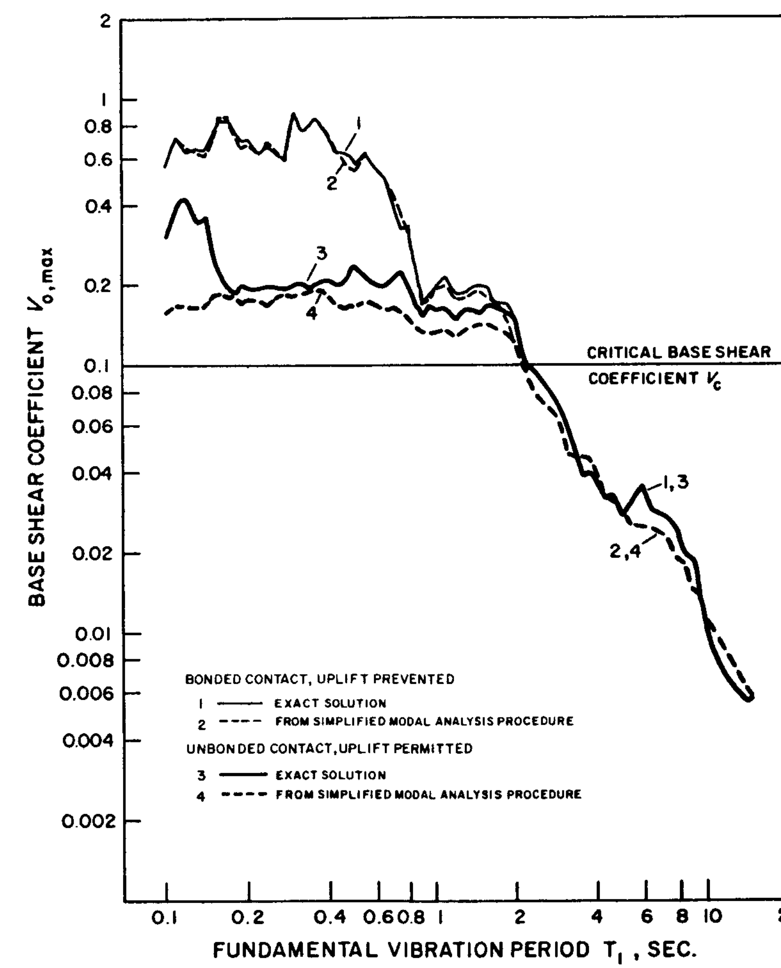


**FIG. 10.—Comparison of Base Shear Response Spectra, Computed by Exact and Simplified Modal Analysis Procedures, for Multistory Structures**

The accuracy of this simplified modal analysis procedure is shown in Fig. 10 wherein response spectra are presented for the base shear of the ten-story building frame when excited by the El Centro ground motion. The exact solution was obtained as described earlier by numerical solution of Eq. 11 and using Eq. 14*a*. The agreement in Fig. 10 between the exact and simplified solution is not as good as when only the fundamental mode response was analyzed (Fig. 9), indicating the additional errors introduced in combining the modal maxima by the SRSS formula. However, these errors in the SRSS procedure will be smaller when the earthquake excitation is characterized by a smooth design spectrum, instead of the irregular response spectrum of a single ground motion as in Fig. 10. The errors in the results of response spectrum analysis of building frames supported through a foundation mat bonded to a rigid foundation soil are known to increase with the fundamental vibration period because of the increasing significance of the higher mode contributions (4). When uplift of the foundation mat is permitted, the errors are especially significant in the short-period range where the fundamental mode response is essentially limited to the critical base shear making, as argued earlier, the contributions of the higher modes relatively more significant. From a practical point of view, however, the simplified modal analysis procedure provides results that are sufficiently accurate for building design applications.

## Conclusions

It has been demonstrated that a reasonable approximation to the maximum response of a multistory structure can be obtained by assuming that soil-structure interaction and foundation-mat uplift influence only the response contribution of the fundamental mode of vibration; and the contributions of the higher modes can be computed by standard procedures disregarding the effects of interaction and foundation-mat uplift.

Starting with the earthquake response spectrum, a simplified analysis procedure has been presented to compute the maximum value of the response contribution of the fundamental natural mode of vibration. The procedure is applicable to structures attached to a rigid foundation mat which is supported on flexible foundation soil modeled as two spring-damper elements, Winkler foundation with distributed spring-damper elements, or a viscoelastic half space.

It has been demonstrated that the total response can be estimated to a useful degree of accuracy by combining the modal maxima according to the square-root-of-the-sum-of-the-squares (SRSS) formula. The maximum response in the fundamental vibration mode is determined by the aforementioned simplified analysis procedure, and in each of the higher modes by standard procedures, disregarding the effects of soil-structure interaction and foundation-mat uplift.

The foundation parameters are very difficult to evaluate in practice, because they depend on the details of the foundation design including the degree of foundation embedment. Therefore, the structural response analysis should be repeated for a range of foundation parameters to establish design values. Because of its simplicity, the analysis procedure presented in this paper is ideally suited for such repetitive analysis.



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