Noisy Nonlinear Motions of Moored System. Part I: Analysis and Simulations

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Abstract: Nonlinear responses of a submerged moored structure are investigated taking into account the presence of environment random noise. Sources of nonlinearity of the system include a quadratic Morison hydrodynamic damping and a geometrically nonlinear restoring force. The random perturbations are modeled by a white-noise process to examine their effects on nonlinear responses analytically and numerically. The analysis procedure includes a generalized Melnikov process to study response stabilities in a global sense and the Fokker-Planck equation to demonstrate response characteristics from a probabilistic perspective. Rich nonlinear phenomena including bifurcations, coexistence of attractors, and chaos are identified and demonstrated. Probability density functions solved from the Fokker-Planck equation are used to depict (co)existing response attractors on the Poincaré section and demonstrate their probabilistic properties. Noise effects on responses are shown via a generalized Melnikov criterion and the probability density function. It is found that the presence of noise may expand the chaotic domain in the parameter space and also cause transitions between coexisting responses.

INTRODUCTION

Complex responses including chaos of nonlinear deterministic compliant offshore structures have been demonstrated by many recent investigations (Thompson 1983; Bishop and Virgin 1988; Bernitsas and Chung 1990). The richness of such system response under deterministic settings has been elucidated analytically and numerically via period-doubling cascades and the (co)existence of harmonic, subharmonic, and ultraharmonic responses (Gottlieb and Yim 1992). The underlying superstructure in bifurcation sets facilitates numerical search of quasiperiodic, and chaotic responses. Near resonances, coexisting (and competing) nonlinear moored structural responses have been found with different initial conditions. The cascade of local bifurcations (and sudden explosions in some cases) often leads to chaotic responses. These chaotic responses, which seem highly irregular and show no repetition in their time histories, posses a complex attractor structure and are sensitive to initial conditions.

An experimental investigation (Yim et al. 1993) examining nonlinear response behavior of a single-degree-of-freedom (SDOF) moored structural system identifies the existence of subharmonic and ultraharmonic as well as period-doubling bifurcations. The underlying structure in bifurcation sets observed implies the possible existence of higher order nonlinear responses (e.g., quasiperiodic and chaotic). Despite of good agreement between analytical predictions and numerical results, there are experimental observations that cannot be explained using conventional deterministic analysis procedures. Fig. 1 shows the time history of a sample experimental structural response subjected to "deterministic" monochromatic wave excitation. (Details of the experiment will be described in a later paper.) The response resides in a harmonic mode for a relatively long duration [about 120 s, Fig. 1(a)] and then transitions to a subharmonic steady state [Figs. 1(b–c)]. The existence of unexpected transition indicates the presence of random noise caused by imperfect wave conditions (e.g., diffraction, reflection, and re-reflection), which may lead the response trajectory to visit other coexisting attractors (Lin and Yim 1995). Thus this phenomenon and other transitions in responses are to be examined and interpreted in this study by taking into account the presence of inevitable perturbations in the wave excitation. In practice the ocean field environment, including wind, waves, and current, often contains a significant random component, which further necessitates stochastic analysis on randomly perturbed nonlinear responses.

In this paper noisy moored structural responses are analyzed from a stochastic perspective to provide guidelines for interpreting the experimental observations and demonstrate pertinent probabilistic properties of response behavior. Descriptions of the experimental setup, configuration, operation procedure, and data acquisition can be found in the unpublished companion paper. Because of its mathematical simplicity a white-noise model is employed here to approximate possible random perturbations in the wave excitation. In general, correlation time of the random noise is much shorter than the relaxation time of physical systems, which makes the white-noise model a reasonable approximation (Stratonovich 1967). The white-noise model is used here as a first attempt to examine the noise effects on nonlinear moored structural responses, and a filtered white-noise model will be used for simulations to compare with experimental results in the later paper.

The global stability analysis on the deterministic moored system by Gottlieb and Yim (1993) is extended to incorporate a random component in the deterministic excitation. A generalized stochastic Melnikov process taking into account the presence of noise is developed. A mean-square criterion is derived accordingly to identify the noise effects on a possible chaotic domain. Random-noise effects on the nonlinear moored structural responses (regular and chaotic) and their interactive relationship are examined through transient and steady-state probability density functions (PDFs), which are obtained by solving the associated Fokker-Planck equation using path-integral solution (Markov process approach). The unexpected transition in response observed in the experiment is explained from a probabilistic perspective. Analytical and numerical predictions presented here will be calibrated by comparing with experimental results in the later paper.

SYSTEM DESCRIPTION

Assuming surge motion as the dominant system response and is uncoupled with other degrees of freedom, the multipoint

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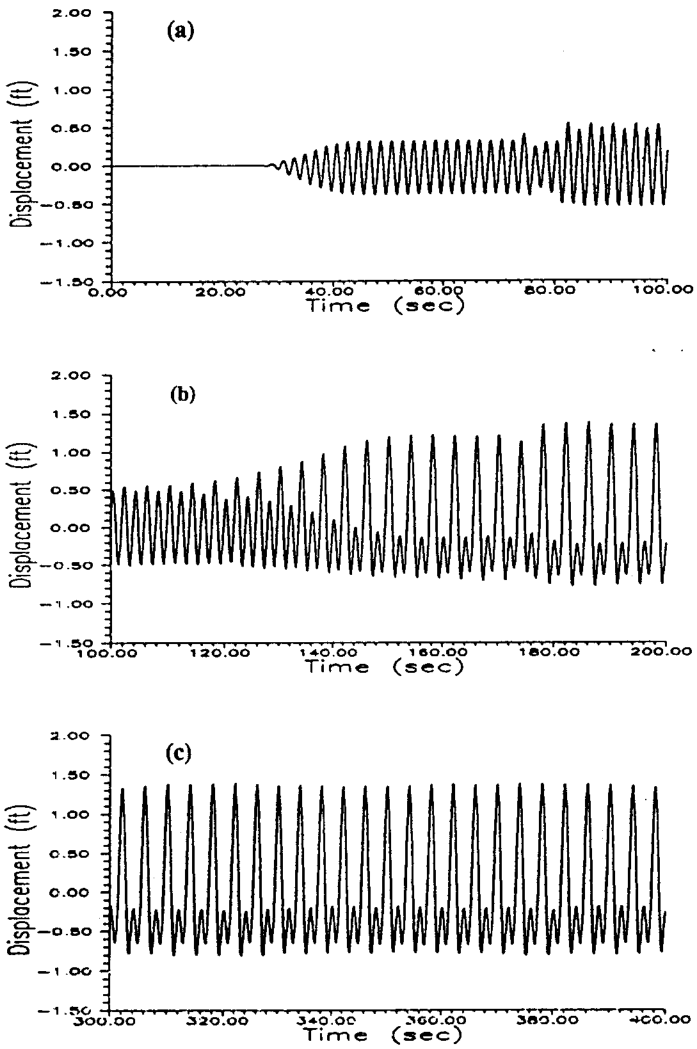

FIG. 1. Transition from Harmonic to Subharmonic in "Deterministic" Moored Structural Response (Test D2): (a) Transient Harmonic; (b) Transition from Harmonic to Subharmonic; (c) Steady-State Subharmonic Response

moored structural system considered can be formulated as a SDOF (in surge) submerged rigid body, hydrodynamically damped and excited nonlinear oscillator (Fig. 2). The elastic mooring cables are assumed to be taut, and the degree of nonlinearity of the restoring force depends on the geometric configuration [mooring angles, Fig. 2(a)]. The exciting force takes into account both nonlinear drag and inertia effects on a submerged axis-symmetric small body (Gottlieb and Yim 1992) using the relative motion Morison formulation (Isaacson 1979). Possible random perturbations in wave excitation are lumped and approximated by a white-noise process.

Governing Equations

The associated nondimensionalized governing equations for a SDOF moored structural system subjected to regular wave excitation with random perturbations is given by

$$\dot{x}_1 = x_2; \quad \dot{x}_2 = -R(x_1) - \gamma x_2 + F_D(x_2) + F_I(x_1, x_2) + \xi(t) \qquad (1a)$$

where x_1 and x_2 = nondimensionalized surge displacement and velocity (Gottlieb and Yim 1992), respectively; and the restoring force R; mooring line lengths $l_{1,2}$; Morison drag force F_D; and inertia force F_I are given by

$$R = \alpha \left[x_1 - \tau \left(\frac{l_1 + l_2}{l_1 l_2} x_1 - \beta \frac{l_1 - l_2}{l_1 l_2} \right) \right];$$

$$l_{1,2} = [1 + (\beta \pm x_1)^2]^{1/2} \qquad (1b)$$

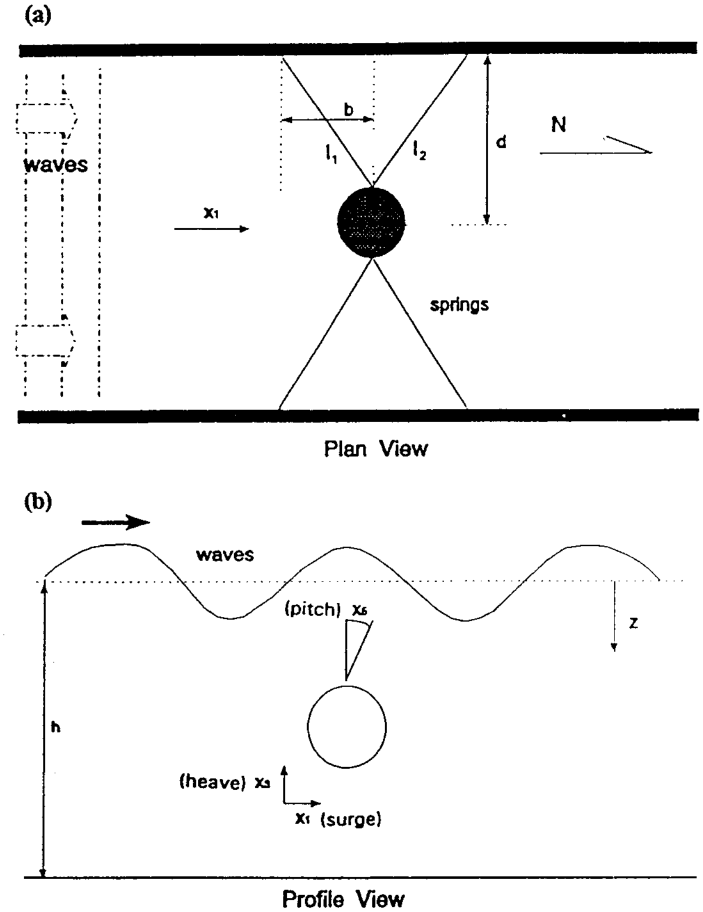

FIG. 2. Definition Sketch of Submerged, Hydrodynamically Damped and Excited Nonlinear Ocean Structural System

$$F_D = \mu\delta(u - x_2)|u - x_2| \qquad (1c)$$

$$F_I = u \left(\frac{\partial u}{\partial t} + (u - x_2) \frac{\partial u}{\partial x_1} \right) \qquad (1d)$$

where α, β, and τ denote the parameters for restoring force R; u = water particle velocity; δ and μ represent the parameters for Morison drag force; and γ = structural damping coefficient. The stochastic excitation component is denoted by a zero-mean, delta-correlated white noise, $\xi(t)$,

$$\langle \xi(t) \rangle = 0; \quad \langle \xi(t)\xi(t') \rangle = \kappa\delta(t - t') \qquad (1e)$$

where κ = noise intensity.

METHODS OF ANALYSIS

Methods used in this study to analyze the noisy nonlinear response include stochastic Melnikov process and Markov approach. The stochastic Melnikov process is applied to identify possible chaotic domains in the parameter space based on the associated averaged system under the assumption of slow variations in response amplitude and phase. With a white-noise approximation, the Markov approach is used and the resulting transient and steady-state joint PDFs can depict global information about the response behavior. Analytic predictions of the responses are confirmed via numerical simulations in the time domain.

Melnikov Approach

The Hamiltonian system corresponding to (1*a*) has only one fixed point, a center, and no homoclinic or heteroclinic orbits. Nevertheless, homoclinic connections may exist near the primary resonance in the associated averaged system (Gottlieb and Yim 1993). For convenience and demonstration purpose, the restoring force is approximated by a two-term, odd-order polynomial, and the averaged system is accordingly derived. With the averaged system, a generalized Melnikov criterion is developed and represented in a mean-square sense by taking into account the presence of random noise.

Existence of Homoclinic Orbits

By treating time as a state variable, θ, employing a polynomial approximation for restoring force ($\alpha_1 x_1 + \alpha_3 x_1^3$) (good agreement to the taut restoring force is shown in Gottlieb 1991) and inertia force $F_I(\omega, \theta) = -\mu\omega^2 a_1' \sin\theta$, (1*a*) can be rewritten in an autonomous form

$$\dot{x}_1 = x_2; \quad \dot{x}_2 = -\gamma x_2 - \alpha_1 x_1 - \alpha_3 x_1^3 - \mu\omega^2 a_1' \sin\theta$$

$$+ F_D(x_2, \omega, \theta) + \xi(\omega, \theta); \quad \dot{\theta} = \omega \tag{2}$$

The white-noise component, $\xi(\omega, \theta)$, in (2) can be approximated by a Rice noise representation (Kapitaniak 1988)

$$\xi(\omega, \theta) \cong \xi_r(\omega, \theta) = \sum_{j=1}^{N} a_j'' \cos(\omega_j t - \psi_j) \tag{3}$$

Defining the detuning parameter $\varepsilon\Omega' = \omega^2 - (m/n)^2\alpha_1$ and Van der Pol coordinate transformation, and further integrating and averaging the governing equations over period $2m\pi/n$, the averaged system can be written in *u-v* coordinates

$$\dot{u} = -\frac{\varepsilon n}{2m\omega}\left\{-\Omega' v + \frac{3}{4}\left(\frac{m}{n}\right)^2 \alpha_3'(u^2 + v^2)v + \frac{m\omega}{n}\gamma' u + I_s - \mu\omega^2 a_1 \delta_{r,1} + \frac{1}{2}\left(\frac{m}{n}\right)^2 a' \sin\psi\right\} \tag{4a}$$

$$\dot{v} = -\frac{\varepsilon n}{2m\omega}\left\{\Omega' u - \frac{3}{4}\left(\frac{m}{n}\right)^2 \alpha_3'(u^2 + v^2)u + \frac{m\omega}{n}\gamma' u + I_c + \frac{1}{2}\left(\frac{m}{n}\right)^2 a' \cos\psi\right\} \tag{4b}$$

and $\alpha_3 = \varepsilon\alpha_3'$; $\gamma = \varepsilon\gamma'$; $a_1' = \varepsilon a_1$; and $a_j'' = \varepsilon a'$. Where $\delta_{r,1}$ = Kronecker delta function ($\delta_{r,1} = 1$ for $r = 1$ and $\delta_{r,1} = 0$ otherwise), and $I_s(u, v)$ and $I_c(u, v)$ represent the averaged drag force over period $2n\pi/m$ (Gottlieb and Yim 1993). The symbols a' and ψ denote the amplitude and random phase of the noise component, which is in tune with the dominant frequency. By employing a nonlinear polar transformation (Meirovitch 1970), $J = 1/2(u^2 + v^2)$ and $\Phi = \tan^{-1}(v/u)$, the averaged system [(4*a*) and (4*b*)] can be written as a perturbed Hamiltonian system. Fixed points can be obtained by solving the Hamiltonian H_0 (i.e., the unperturbed case). The complex expressions for H_0 and the solution procedure are described in detail in Gottlieb and Yim (1993). A jump bifurcation defines the criterion of existence of a unique center [$a_1 > \beta_c^H$] or two coexisting centers and a saddle [$a_1 < \beta_c^H$] (Gottlieb and Yim 1993), with

$$\beta_c^H = \frac{16}{27}\frac{(\omega^2 - \alpha_1)}{\mu\omega^2}\left(\frac{\omega^2 - \alpha_1}{\alpha_3}\right)^{3/2} \tag{5}$$

A pair of homoclinic orbits within the averaged system can be determined based on the Bendixson criterion (Jordan and Smith 1987). For small structural damping the system near primary resonance contains homoclinic loops defined by the stable and unstable manifolds of the saddle. The pair of homoclinic orbits $[q_\pm^0(\gamma)] = [J_\pm(\theta), \Phi_\pm(\theta)]$ is given by

$$\theta - \theta_0 = \int_{J_\pm(\theta_0)}^{J_\pm(\theta)} \frac{dJ}{\sqrt{2Jf^{*2} - (H_0 - \Omega^* J + \alpha_3^* J^2)^2}} \tag{6a}$$

and

$$\Phi = \sin^{-1}\frac{H_0 - \Omega^* J + \alpha_3^* J^2}{\sqrt{2J}f^*} \tag{6b}$$

where $J_\pm(\theta)$ and Φ are functions of θ and θ_0, and constants α_3^*, Ω^*, and f^* are closely related to α_1, α_3, μ, ω, a_1, *m* and *n* in (2) and (4). Based on the existence of homoclinic connections, the generalized stochastic Melnikov function is formulated in the following section.

Stochastic Melnikov Process

With the presence of random perturbations, the distance between the stable and unstable manifolds is given by Lin and Yim (1995).

$$M(\theta_0) = \int_{-\infty}^{\infty} F(q_\pm^0(\theta) \wedge G(q_\pm^0(\theta), \theta + \theta_0)\, d\theta$$

$$= \int_{-\infty}^{\infty} (F_1 G_2 - F_2 G_2)\, d\theta \tag{7}$$

where F_1 and F_2 denote the components of a two-dimensional (2-D) Hamiltonian vector field; and G_1 and G_2 represent the perturbations. Note that (7) represents the Melnikov function for an autonomous system, which does not depend on time explicitly. For autonomous 2-D vector fields either the stable and unstable manifolds of a hyperbolic fixed point coincide or they do not interset at all, and the presence of a fixed point in the averaged system indicates the existence of a periodic orbit in the original dynamical system (Wiggins 1990). A zero (fixed point) in the autonomous Melnikov function implies the possible existence of a periodic (time-dependent) Melnikov function with infinite transverse intersections between stable and unstable manifolds in the original system. Hence, when $M(\theta_0) = 0$ in (7) chaotic response may exist and a criterion for the chaotic domain can be thus defined.

Noise effects on occurrence of chaotic responses are to be identified in the Melnikov sense here by isolating the damping term related to the chaos threshold and regrouping the perturbation effects into deterministic and random components. Accordingly, (7) can be rewritten as

$$\gamma^* \int_{-\infty}^{\infty} m_1^d\, d\theta = \int_{-\infty}^{\infty} m_2^d\, d\theta - A^* \int_{-\infty}^{\infty} m_1^r\, d\theta + A^* \int_{-\infty}^{\infty} m_2^r\, d\theta \tag{8a}$$

where

$$\gamma^* = \frac{\varepsilon\pi\omega\gamma}{2\mu\delta}; \quad A^* = \frac{m\pi\varepsilon a}{4n\mu\gamma} \tag{8b}$$

$$m_1^d = 2J_\pm\left(-\Omega^* + 2\alpha_3^* J_\pm - \frac{f^* \sin\Phi_\pm}{\sqrt{2J_\pm}}\right) \tag{8c}$$

$$m_2^d = \frac{4}{3}\sqrt{\Gamma_\pm}\left[2J_\pm(-\Omega^* + 2\alpha_3^* J_\pm)\left(1 + \frac{a_1 \sin\Phi_\pm}{\sqrt{2J_\pm}}\right) - f^*\sqrt{2J_\pm}\sin\Phi_\pm - a_1 f^*\right] \tag{8d}$$

$$m_1^r = 2J_\pm f^* \cos\Phi_\pm(\theta)\cos[\Phi_\pm(\theta + \theta_0) + \psi] \tag{8e}$$

$$m_2^r = \left(-\Omega^* + 2\alpha_3^* J_\pm - \frac{f^* \sin\Phi_\pm}{\sqrt{2J_\pm}}\right) \cdot \sqrt{2J_\pm}\sin[\Phi_\pm(\theta + \theta_0) + \psi] \tag{8f}$$

$$\Gamma_\pm = a_1^2 + 2a_1\sqrt{2J_\pm}\sin\Phi_\pm + 2J_\pm; \quad J_\pm = J_\pm(\theta, \theta_0); \quad \Phi_\pm = \Phi_\pm(\theta, \theta_0) \tag{8g}$$

with superscripts d and r corresponding to the deterministic and random perturbations, respectively. Due to the complexity of the integrant [see (8)], (8a), is not explicitly solved here. Nevertheless, the noise effects on the Melnikov criterion are demonstrated qualitatively.

The criterion based on the Melnikov function (7) provides a necessary condition for the existence of chaotic response (Yim and Lin 1992), and the noise effects should be represented in a mean-square sense

$$\gamma^*\left\langle\left(\int m_1^d\,d\theta\right)^2\right\rangle = \left\langle\left(\int m_2^d\,d\theta\right)^2\right\rangle + \sigma_{M_1^r}^2 + \sigma_{M_2^r}^2$$
$$+ 2\left\langle\left(\int m_2^d\,d\theta\right)\left(A^*\int m_1^r\,d\theta\right)\right\rangle + 2\left\langle\left(\int m_2^d\,d\theta\right)\right.$$
$$\left.\cdot\left(A^*\int m_2^r\,d\theta\right)\right\rangle + 2\left\langle\left(\int m_1^r\,d\theta\right)\left(\int m_2^r\,d\theta\right)\right\rangle \quad (9)$$

Because m_1^r and m_2^r are cosine and sine functions of random phase shift ψ, respectively, they are statistically uncorrelated. Therefore, (9) can be simplified as

$$\gamma^*\left(\int m_1^d\,d\theta\right)^2 = \left(\int m_2^d\,d\theta\right)^2 + \sigma_1^2 + \sigma_2^2 \quad (10)$$

where σ_1^2 and σ_2^2 = variances associated to m_1^r and m_2^r, respectively. The positive terms, σ_1^2 and σ_2^2, indicate that the presence of noise lowers the threshold for chaos and enlarges the possible chaotic domain (mean-square sense) in the parameter space.

Markov Approach

By assuming the noisy nonlinear response is a function of only the most recent (prior and post) probability states, a Markov process approach can be applied to obtain the evolution of response PDF. The associated Fokker-Planck equation (FPE) is derived and solved here by a solution procedure based on the path-integral solution (PIS).

Fokker-Planck Equation and Path-Integral Solution

The FPE corresponding to (1a) is given by

$$\frac{\partial P(X, t)}{\partial t} = -\frac{\partial}{\partial x_1}[x_2P(X, t)] - \frac{\partial}{\partial x_2}[(-R(x_1)$$
$$- \gamma x_2 + F_D(x_2, t) + F_I(t))P(X, t)] + \frac{\kappa}{2}\frac{\partial^2 P(X, t)}{\partial x_2^2} \quad (11)$$

with $X = [x_1x_2]^T$ and the short-time propagator $\Gamma(X', X, t; \tau)$ is given by

$$\Gamma(X', X, t; dt) = (2\pi dt)^{-2}\kappa^{-1/2}\exp\left[-R(x_1) - \gamma x_2 + F_D\right.$$
$$\left. + F_I - \frac{x_2' - x_2}{dt}\right]\delta\left[x_2 - \frac{x_1' - x_1}{dt}\right] \quad (12)$$

where X' and X represent the post-state and the prior-state, respectively. The Dirac delta function in (12) indicates randomness is present only in the excitation force, hence the noise vector in the corresponding Langevin equation is degenerated (Risken 1984). The PDF at the desired time can be obtained by applying the short-time propagator iteratively

$$P(X, t) = \lim_{\substack{dt\to 0\\ N\to\infty\\ Ndt\to t-t_0}} \prod_{i=0}^{N-1}\int\cdots\int$$
$$\exp\left[-dt\sum_{j=0}^{N-1}\Gamma(X_{j+1}, X_j, t_j; dt)P(X_0, t_0)\right]dX_i \quad (13)$$

STOCHASTIC ANALYSIS OF NOISY NONLINEAR RESPONSES

Due to the presence of inevitable random noise in engineering systems, the noise effects on nonlinear (large) structural responses are of practical interest. With noise intensity considered as the controlling parameter, highly nonlinear phenomena (e.g., transitions between coexisting attractors and chaos) relating to the instability of responses are examined from a probabilistic perspective. In the numerical examples shown in this section different approximation procedures and system parameters are selected to demonstrate the salient feature of the nonlinear system response.

Coexisting Attractors

Bifurcations in deterministic moored structural response have been examined via local stability analysis, and possible coexisting responses are identified (Gottlieb and Yim 1992). Coexisting periodic responses may be obtained near resonances with various initial conditions. Fig. 3(a) shows two coexisting periodic responses (small- and large-amplitude) in the phase plane with initial conditions at (0, 0) and (1, −2) respectively, where symbol * denotes the corresponding Poincaré points. Coexisting response attractors can be demonstrated in the probability domain via steady-state PDFs. The evolution of the PDF is depicted by repeatedly sampling its transient [Figs. 3(b–d)] and steady state [Figs. 3(e,f) on a Poincaré section (integer multiple of excitation period). The system is excited under quiescent initial conditions, i.e., (0, 0) [Fig. 3(b)], the response trajectories reside in a periodic attractor (small amplitude) after four cycles of the forcing period [(Fig. 3(c)]. The response trajectories start to shift from the attractor (small amplitude) to the coexisting attractor (large amplitude) after eight cycles of the forcing period [Fig. 3(d)]. The probability of the trajectories to stay in the coexisting attractor (large amplitude) increases after 12 cycles of the forcing period [Fig. 3(d)]. The steady-state PDF is observed after 16 cycles of the forcing periods [Figs. 3(e,f)] and it clearly depicts coexisting periodic responses. Thus with the presence of noise the precision of the initial conditions becomes less significant to observe the coexisting responses, and the associated PDF provides global information about the system behavior. For large time t, there is theoretically only a single domain of attraction, but with two distinct high-strength "attractors."

The noise effects on the coexisting nonlinear responses are examined in Fig. 4. When the intensity of the noise present is low (κ = 0.003), the PDF is concentrated at the domain of small-amplitude response [Fig. 4(a)], indicating its stronger attracting strength relative to the coexisting large response attractor, Thus, in this case, the moored structural system oscillator mainly (most of the time) with small motions will have a small probability of excursion to larger amplitude motions (i.e., rarely visits the large amplitude domain). When the noise intensity is increased (κ = 0.007), the domains of attraction of the coexisting responses begin to bridge as shown in Fig. 4(b). Characteristics of both attractors (small and large amplitudes) may exhibit in the moored structural response behavior. When the noise intensity is moderate (κ = 0.02), the attractors further emerge into a single attraction domain [Fig. 4(c)], in which no obvious periodic response can be distinguished. The variation of the boundary of the emerged attraction domain is stabilized when the noise intensity further increases [Figs. 4(d–f)]. The "orderness" still can be observed in the PDF with κ = 0.1 [Fig. 4(d)], which indicates that the periodicity may exhibit in the system response. The smooth curve and the stationary boundary of the PDF [Figs. 4(e,f)] implies strong randomness in the system behavior.

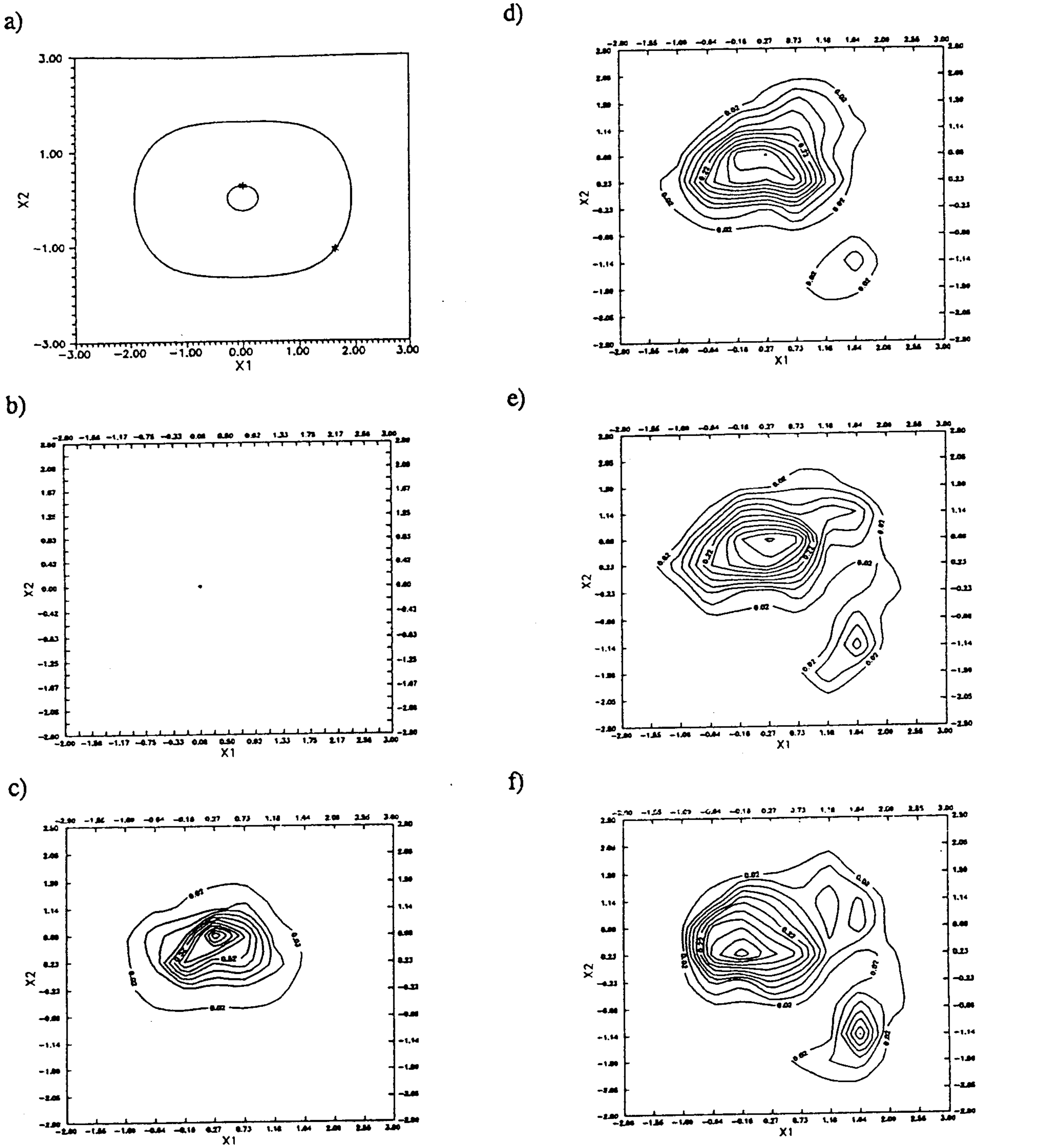

FIG. 3. Coexisting Periodic Responses: (a) Phase Portrait and Poincaré Points (*), Evolution of Corresponding PDF; (b) Quiescent Initial Conditions; (c) 4th Cycle; (d) 12th Cycle; (e) 16th Cycle; (f) 20th Cycle; $(\alpha, \beta, \gamma, \delta, \mu, A, \omega, \kappa) = (0.25, 0.3, 0.01, 0.05, 1.0, 0.2, 1.0, 0.007)$

Numerical results presented here demonstrate the effects of noise intensity on coexisting nonlinear moored structural responses, which may explain the transition in the experimental response (Fig. 1). The presence of inevitable noise in the wave flume plays a bridging role between coexisting attractors as demonstrated [Fig. 4(a)]. The system was initially excited in a harmonic attraction domain. Due to the presence of very weak noise, the structural response transitions to the stronger subharmonic attractor and stays after a relatively long transient.

Noise-Induced Chaotic Response

It is well known that, by varying system parameters, a deterministic nonlinear response may shift from one attractor to another, even to chaos (Moon 1987). It is indicated, based on criterion from the stochastic Melnikov process, the presence of random noise may expedite the occurrence of chaotic response in the parameter space. Effects on response transitions by these two parameters (excitation amplitude and noise intensity) are demonstrated and discussed here.

As an example, for the parameters specified in Fig. 5 in the deterministic moored structural system, by increasing the excitation amplitude, the response shifts from a period-2 subharmonics, to a higher order subharmonics, to a chaotic attractor, and then to another chaotic attractor [Figs. 5(a–d), respectively]. In a noisy environment, with the same parameters as its deterministic counterpart specified in Fig. 5(a), a similar transition among response attractors can be observed

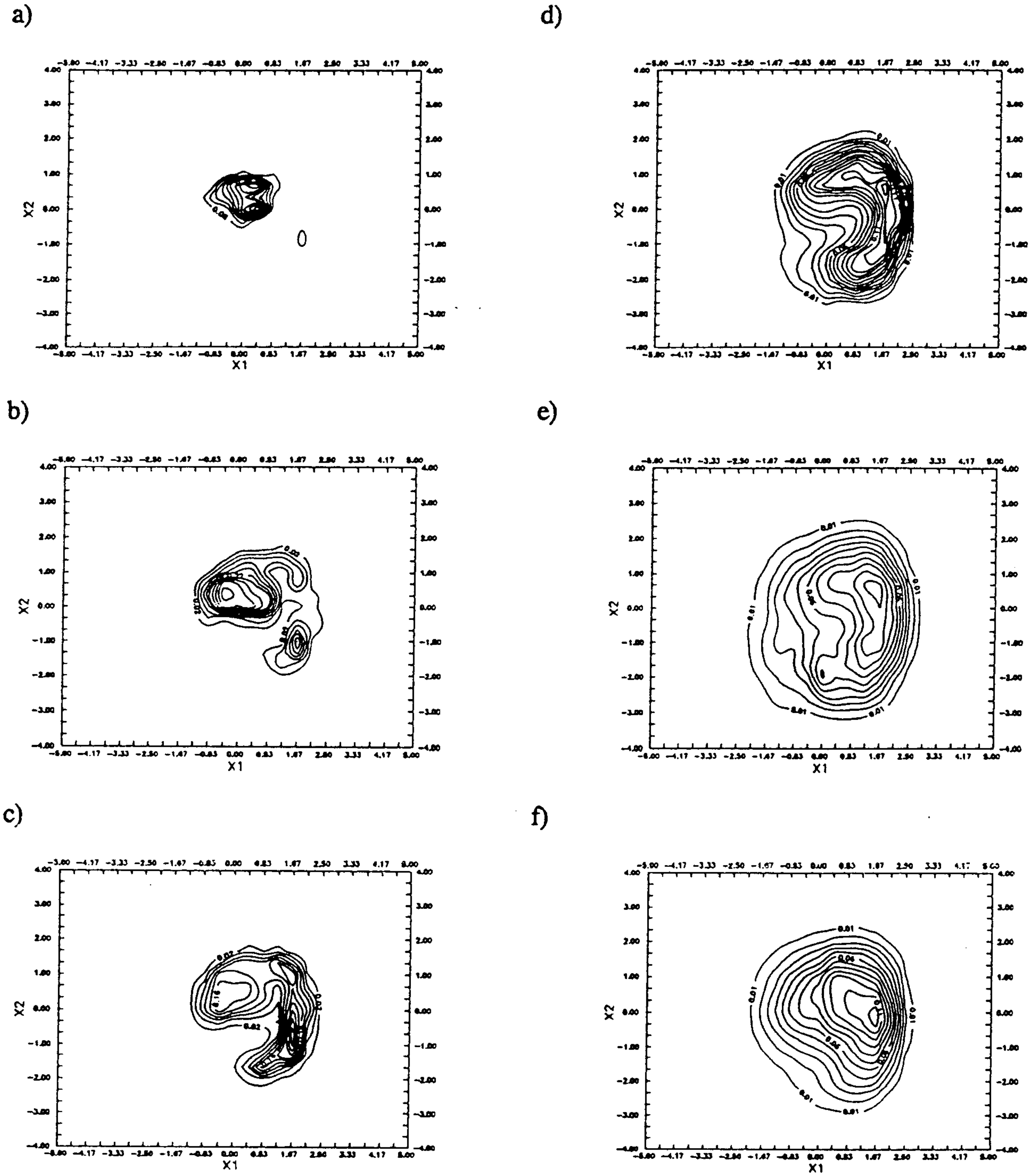

FIG. 4. Coexisting Periodic Responses with Increasing Noise Intensity (Joint PDFs): κ = (a) 0.003; (b) 0.007; (c) 0.02; (d) 0.1; (e) 0.3; (f) 0.4 (α, β, γ, δ, μ, A, ω) = (0.25, 0.3, 0.01, 0.05, 1.0, 0.2, 1.0)

when the noise intensity is increased (Fig. 6). Fig. 6(a) [which is identical to Fig. 5(a)] shows a deterministic period-2 subharmonic response ($\sigma^2 = 0.0$), and under the same system parameters, the moored structural response becomes chaotic with the presence of weak noise [$\sigma^2 = 0.01^2$, Fig. 6(b)]. When the noise intensity is further increased ($\sigma^2 = 0.02^2$), the response shifts to another chaotic attractor [Fig. 6(c)]. When the noise intensity is relatively strong, increasing noise intensity in the excitations increases the randomness in the response [$\sigma^2 = 0.07^2$, Fig. 6(d)].

Note that the chaotic attractors observed via Poincaré map in both cases—deterministic [Figs. 5(b,c)] and noisy [Figs. 6(b,c)]—are approximately of the same shape, indicating transitions induced by both excitation and noise parameter variations follow approximately the same route. Thus the noise intensity can also be considered as a controlling parameter between different response attractors in the parameter space.

Noise Effects on Nonlinear Responses

The occurrence of deterministic chaotic response of the moored structural system has been predicted by Gottlieb and Yim (1992). With the presence of noise, the probabilistic properties of this nonlinear response and the corresponding noise-induced transition to a random state are examined here via transient and steady-state PDFs.

Fig. 7(a) shows a sample deterministic chaotic moored structural response on Poincaré maps (obtained by sampling data points from the response time history at integer multiples of the forcing period). The evolution of the corresponding PDF

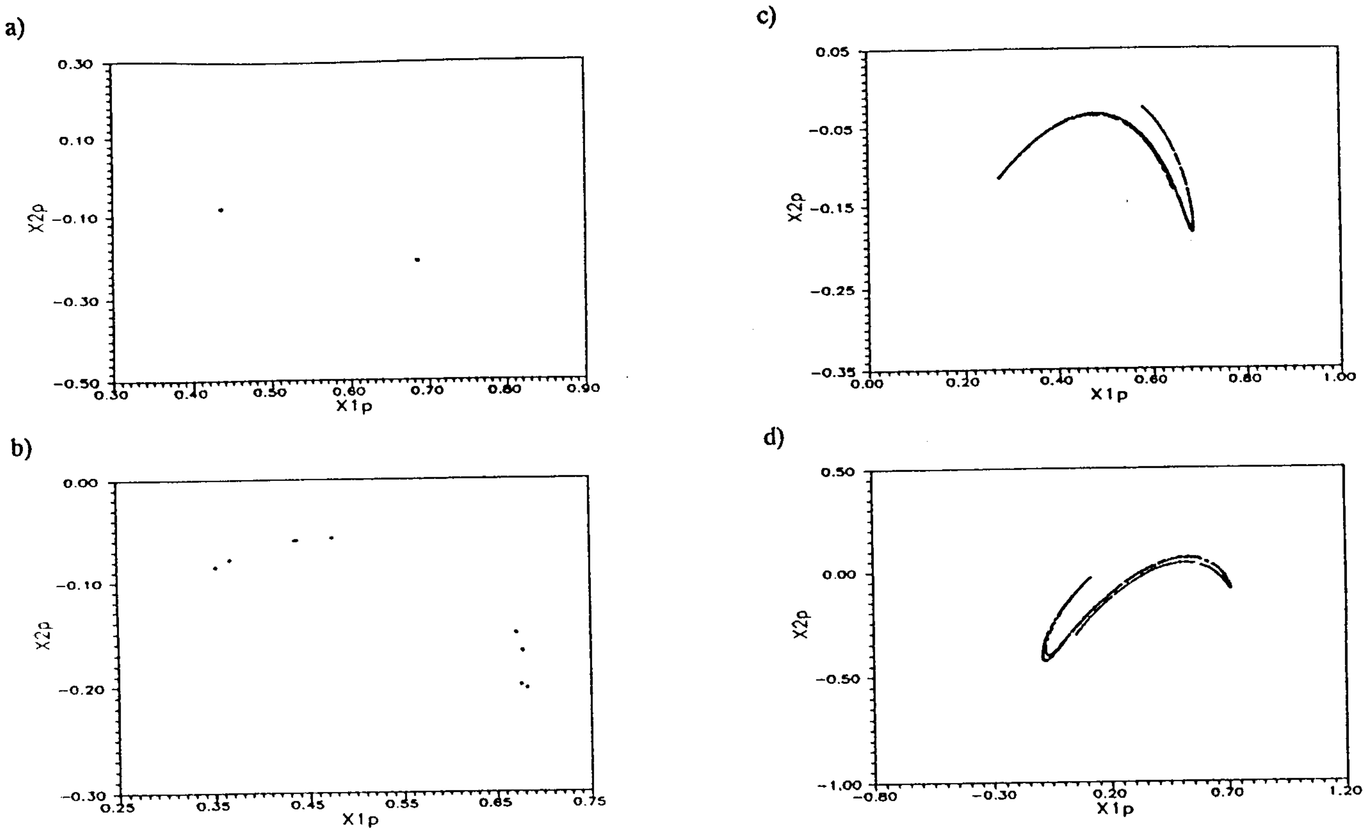

FIG. 5. Transition of System Response with Increasing Excitation Amplitude: A = (a) 15; (b) 16; (c) 17; (d) 20 ($\alpha, \beta, \gamma, \delta, \mu, \omega$) = (4.0, 0.0, 0.001, 0.01, 1.0, 0.32)

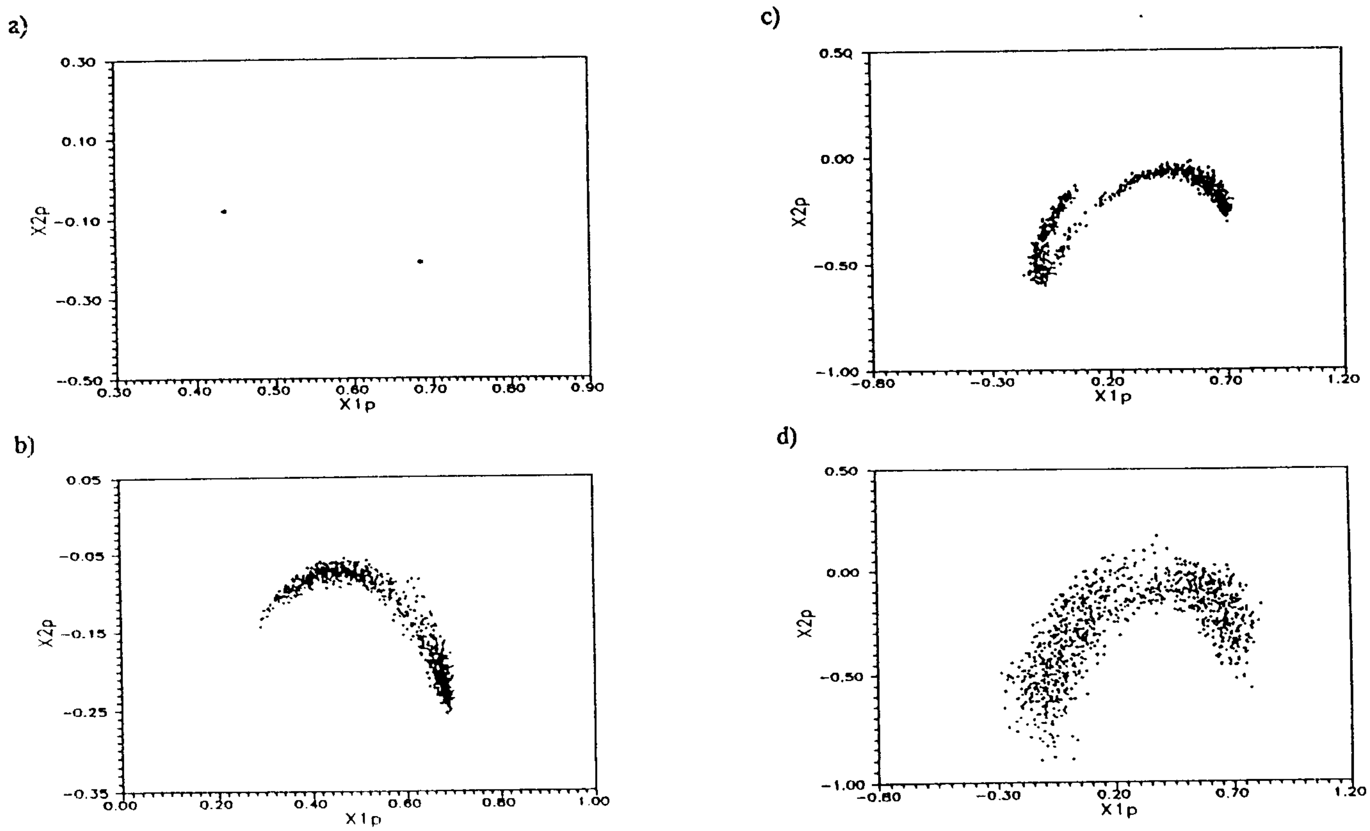

FIG. 6. Noise-Induced Transition of System Response: σ^2 = (a) 0.0^2; (b) 0.01^2; (c) 0.02^2; (d) 0.07^2 ($\alpha, \beta, \gamma, \delta, \mu, A, \omega$) = (4.0, 0.0, 0.001, 0.01, 1.0, 15.0, 0.32)

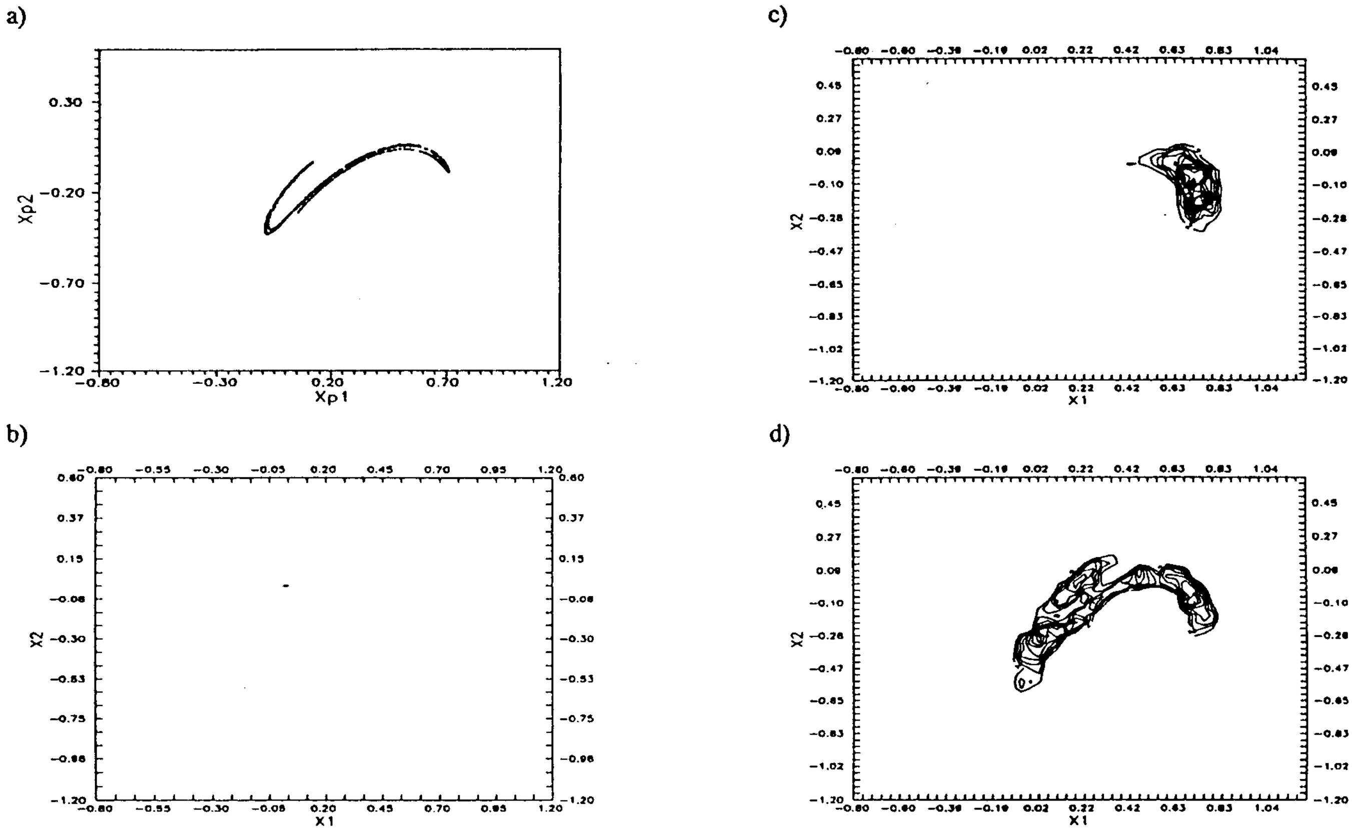

FIG. 7. Chaotic Response Attractor: (a) Poincaré Map, Evolution of Corresponding PDF; (b) Quiescent Initial Conditions; (c) 2nd Cycle; (d) 10th Cycle of the Forcing Period $(\alpha, \beta, \gamma, \delta, \mu, A, \omega) = (4.0, 0.0, 0.001, 0.001, 1.0, 20.0, 0.32)$

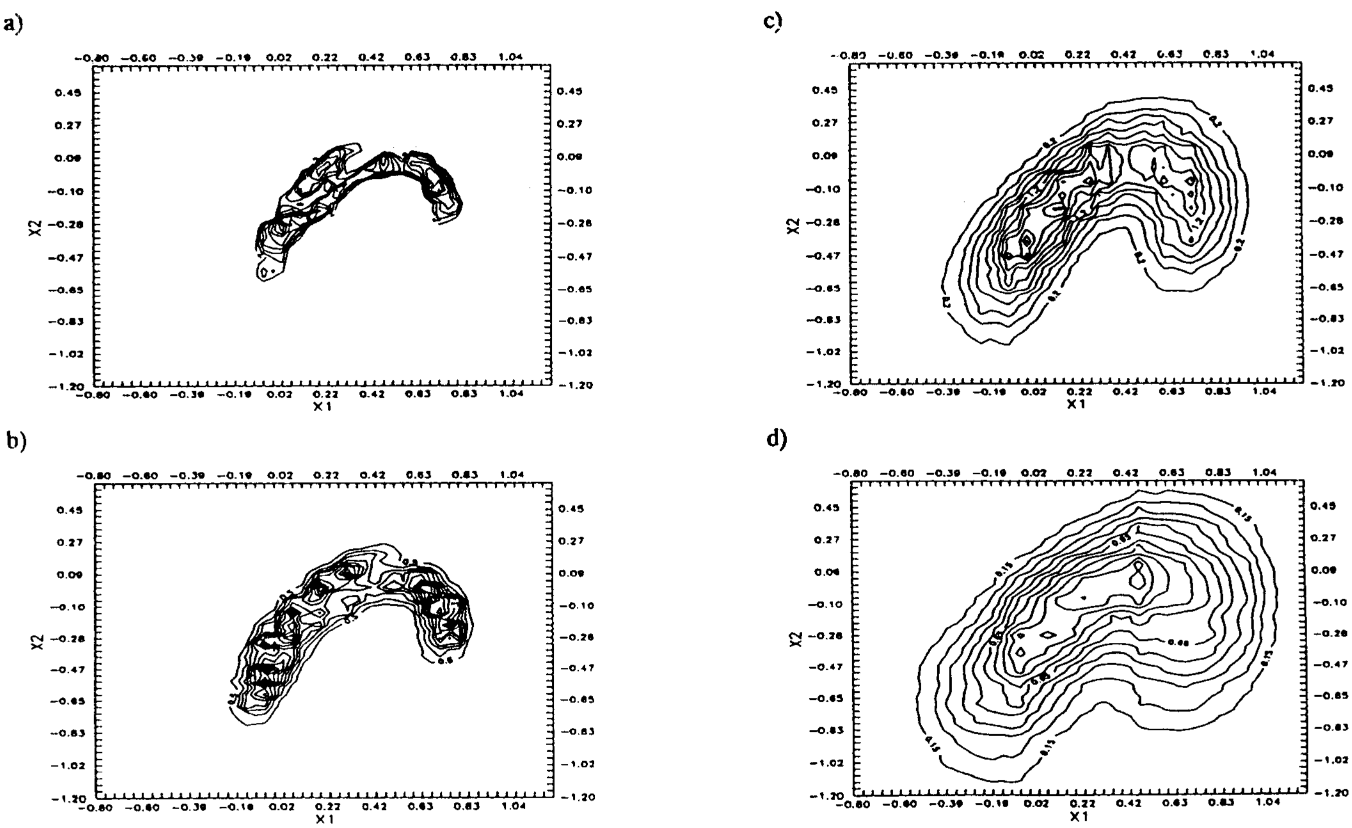

FIG. 8. Transition of PDF with Increasing Noise Intensity: κ = (a) 0.0003; (b) 0.001; (c) 0.007; (d) 0.02 $(\alpha, \beta, \gamma, \delta, \mu, A, \omega) = (4.0, 0.0, 0.001, 0.01, 1.0, 20.0, 0.32)$

is demonstrated in Figs. 7(b–d). The moored structural system response starts with deterministic quiescent conditions, i.e., a delta function at (0, 0) in the probability domain [Fig. 7(b)]. Initially, the joint PDF spreads with time and moves toward the chaotic attractors and begins to cover the attracting domains [Fig. 7(c)]. After 10 cycles of the forcing period, the PDF reaches its steady state [Fig. 7(d)] and reflects the fractal details of the deterministic chaotic attractors on the Poincaré section.

The noise effects on a sample chaotic attractor are examined by varying the noise intensities (Fig. 8). The fractal details of the chaotic attractor is preserved [Fig. 8(a)] when the additive noise is of low intensity ($\kappa = 0.0003$). The boundary of the chaotic attractor becomes obscured as the noise intensity increases to 0.001 [Fig. 8(b)]. When the noise intensity κ is further increased to 0.007, the domain of the attractor becomes more obscured and the fractal details of the boundary of the attractor diminish. In fact, it becomes difficult to identify the chaotic attractor from the shape of the observed boundary [Fig. 8(c)]. Randomness in the response increases further as the noise intensity increases to 0.02 [Fig. 8(d)]. Note that the structure of the attracting domain is smoothed even with very low noise intensity ($\kappa \geq 0.001$), which indicates the strong sensitivity to the random noise and the weak stability of the chaotic attractor.

CONCLUDING REMARKS

An analytical investigation of noise-induced transitions among nonlinear response attractors of a hydrodynamically excited multipoint moored system is motivated by an "unexpected" response transition observed in the corresponding experiment. The analytical and numerical results are summarized as follows:

1. Taking into account the presence of random noise, a generalized Melnikov criterion is derived to identify possible chaotic domains. Results, confirmed by numerical simulations, indicate that the presence of low-intensity noise perturbations expedites possible occurrence of chaotic response.
2. The Fokker-Planck equation governing the evolution of response probability density function is developed and solved using a path-integral solution procedure. Coexisting response attractors are depicted by the associated steady-state joint PDF, which also provides global information of system behavior.
3. Noise intensity is considered a controlling parameter of transition between coexisting response attractors here. When the noise intensity is low the response trajectories stay in the stronger attractor with low probability of exiting to the other attractor. The system response exhibits mainly the characteristics of the stronger attractor. When the noise intensity is moderate the domains of coexisting attractors are bridged and the system response exhibits combined characteristics of coexisting attractors. When the noise intensity is strong the coexisting attractors are further bridged and smoothed, and merge into a single domain of attraction. The corresponding system response appears random.
4. Numerical results show that fluctuations in the noise intensity may result in transitions in neighboring different response modes.
5. The "unexpected" response transition observed in experiment is interpreted in light of the analytical results, which further assesses that the presence and effects of noise may need to be taken into account for the analysis and design of engineering system.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

F_D = Morison drag force;
F_I = inertia force;
l_1, l_2 = lengths of mooring lines;
M = Melnikov function;
P = probability density function;
R = restoring force;
t = time;
u = fluid particle velocity;
X = state vector ($=[x_1, x_2]^T$);
x_1 = nondimensionalized surge displacement;
x_2 = nondimensionalized surge velocity;
α = nondimensionalized mooring stiffness;
β = projected length of mooring lines in surge direction;
Γ = short-time propagator;
γ = structural damping coefficient;
δ, μ = parameters for nondimensionalized hydrodynamic forces;
κ = intensity parameter of stochastic excitation component;
ξ = stochastic excitation component;
τ = pretensioned parameter in mooring lines; and
ω = wave frequency.