A ADDITIONAL EXAMPLE APPLICATIONS AND RESULTS

In this appendix, we provide additional results of applying our multiscale analysis framework to a number of applications.

A.1 Sullivan and Noise

Describing rotating fluids is a demanding research topic in fluid mechanics. The Sullivan vortex is a classical model that is a solution to the Navier Stoke's equation. The horizontal velocity component starts with a zero magnitude at the center and then spirals out radially. These local velocity fields near the center can be visible sometime for tornados or hurricanes through the damage on their paths. Knowing the gradient change of the vector fields therefore is of great importance. Here, we demonstrate our eigenvector manifold visualization that captures the pure rotation CCW (red), and anisotropic stretching CCW (pink) until the radius is smaller than 2.667. Then due to noise present in data, it gets split into anisotropic stretching CCW (pink) and anisotropic stretching CW (cyan) producing 245 regions (see Figure 12(a)). Scaling it up to scale level 1000 under the metric described in section 5 brings the field to pure rotation and anisotropic stretching both in CCW with two degenerate points as depicted in exact solution (see Figure 12(b)). Furthermore, we simulate the phenomenon where some type of noise (Gaussian) is added to the rotation, and verify the effectiveness of our multi-scale framework on denoising. We can see that Gaussian noise with mean 1 and standard deviation 0.3 produces 2 extra degenerate points and 456 regions all together that are again scaled up through scale level 1000 under the same metric. At that stage, we regain the pure rotation and anisotropic stretching both in CCW along with two degenerate points of the Sullivan vortex (see Figure 12(d)).

A.2 Cooling Jacket

In general, one of the major causes of engine failure can result from over-heating [18]. The complex shape of the cooling jacket is influenced by multiple factors including the shape of the engine block and optimal temperature at which the engine runs. A very large cooling jacket would be effective in transporting heat away from the engine cylinders, however, too large of a geometry results in extra weight to be transported. Also, engineers would like the engine to reach its optimal operating temperature quickly.

The cooling jacket geometry consist of three components: the cylinder head (top half), the bottom called the cylinder block, and a thin component connecting the cylinder head with the block called the gasket. The cylinder head (top) is responsible for transferring heat away from the intake and exhaust ports at the top of the engine block. The cylinder block is responsible for heat transfer from the engine cylinders and for even distribution of flow to the head. This cooling jacket is used



Fig. 12: These figures illustrate the multi-scale analysis of eigenvector graph of Sullivan Vortex at the cost-to-benefit ratios (a) 0 and (b) 1042 as well as, the multi-scale analysis of eigenvector graph of Sullivan Vortex with added Gaussian noise at the cost-to-benefit ratios (c) 0 and (d) 2462.

with a four cylinder engine block. Between the cylinder head and block lies the cooling jacket gasket. The gasket consists of a series of small holes that act as conduits between the block and head. These ducts can be quite small relative to the overall geometry but nonetheless are very important because they are used to govern the motion of fluid flow through the cooling jacket.

There are two main components to the flow through a cooling jacket: a longitudinal motion lengthwise along the geometry and a transversal motion from cylinder block to head and from the intake (left) to the exhaust side (right).

Four main design goals are essential for the mechanical engineers:

- 1. Obtaining an even distribution of flow to each engine cylinder.
- 2. Avoiding regions of stagnant flow.
- 3. Avoiding very high velocity flow.
- 4. Minimizing the fluid pressure loss between the inlet and the outlet.

The first design goal, an even distribution of fluid to each cylinder, is intuitive. An even distribution of flow should result in an even rate of heat transfer away from each cylinder, intake port, and exhaust port. The second goal, avoiding regions of stagnant flow is very important. Stagnant flow does not transport heat away and can lead to boiling conditions. Boiling fluid can indicate potential problem areas in the cooling jacket geometry that lead ultimately to overheating. We note that the optimal cooling jacket temperature is about 90C or 363K.

The third goal, to avoid regions of velocity too high in magnitude is less obvious. High velocity flow can lead to cavitation of the formation of low-pressure bubbles, such as those resulting from the rotation of a marine propeller. Firstly, cavitation wastes energy in the form of noise. Secondly, cavitation can also lead to damage to the walls of the cooling jacket itself over the long term. Cavitation is associated with explosions and unnecessary vibration. Erosion of the boundary surfaces can result in a shorter product lifetime.

The fourth design goal is to minimize pressure loss across the cooling jacket geometry. The water pump (not shown) located at the cooling jacket's inlet (left) is responsible for maintaining a specified pressure at the inlet. The greater the pressure drop between the cooling jacket's inlet and outlet, the more energy the water pump requires in order to maintain the desired pressure. An ideally straight pipe with an inlet and outlet of equal size would exhibit no pressure loss across its geometry, thus a water pump would require much less energy in this case. Generally, the smaller the cooling jacket gasket, the larger the pressure loss. Curves in the geometry can also cause pressure losses.

The main variable in cooling jacket design lies in the gasket. Engineers adjust the number, location, and size of the conduits in their pursuit of the ideal fluid motion. However, the many conduits in the cooling jacket geometry lead to a lot of noise when we try to visualize the flow as in Figure 13 (left). These noisy flow patterns can draw attention away from the major orientations of flow which are the longitudinal and transversal dominant motions described above. As observed in Figure 16, the multi-scale topology visualization method can reduce the amount of noisy flow resulting from the gaskets and let the engineer focus on the properties of the major longitudinal and transversal flow.

Furthermore, the multi-scale tensor field visualization reveals more properties of the cooling jacket than traditional flow visualization techniques. For example, Figure 13 reveals two layers of flow characterized by anisotropic stretching along the entire length of the cylinder block. We can also notice regions dominated by clockwise rotation in the cylinder block. Removing the noisy flow behavior from this simulation was simply not possible without the technique presented here.



Fig. 13: These figures illustrate the multi-scale analysis of the eigenvector graph at the cost-to-benefit ratio (a) 0 and (b) 3615 for the cooling jacket flow.